Methodology: Normative theory versus descriptive theory

Normative theories of decision making:

- How people *should* behave when taking decisions
- Based on an idealised form of human being
- Methods: Mathematical axioms and optimisation

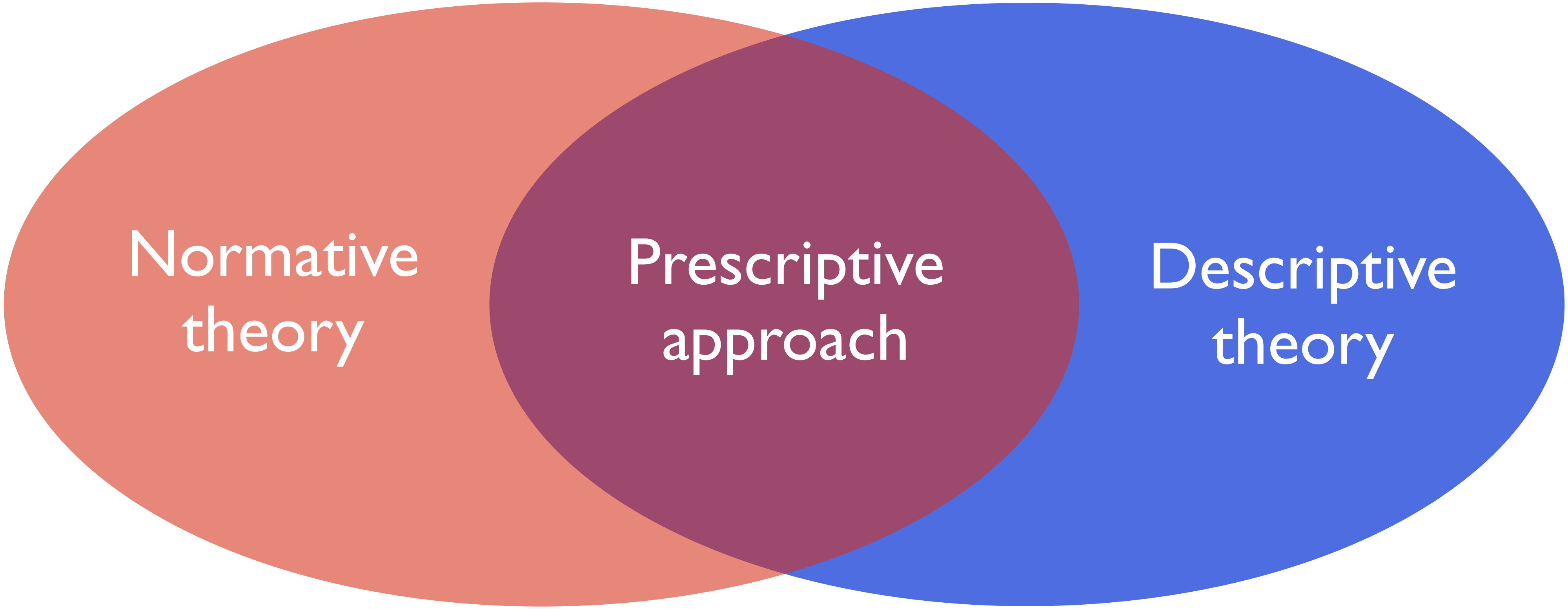
Descriptive theories of decision making

- How *people actually make* decisions
- Based on observation (empirical studies)
- Methods: empirical studies, revised models

Why is normative theory not enough?

Empirical studies have demonstrated that people do not always follow the axioms of probability (biases, fallacies, heuristics).

Methodology: Complementary theories of decision making



Why is normative theory not enough?

Empirical studies have demonstrated that people do not always follow the axioms of probability (biases, fallacies, heuristics).

Examples: Empirically shown deviations from normative theory

- Gambler’s fallacy, inverse gambler’s fallacy, belief in hot hand
- Random sequences generation biases (starting value, runs)
- Clustering illusion
- Anchoring bias (with related and unrelated information)
- Framing effect

Are there other typical deviations from normative theory? **Yes!**

Next few lectures:

- Empirical research by Allais, Ellsberg, research programme by Kahneman and Tversky and many other researchers in past and present.
- Replications of studies with Warwick students!

On data

“I have no data yet.
It's a capital mistake to theorise before one has data.
Insensibly one begins to twist facts to suit theories,
instead of theories to suit facts.”

Who said this?

On data

“I have no data yet.
It's a capital mistake to theorise before one has data.
Insensibly one begins to twist facts to suit theories,
instead of theories to suit facts.”

Arthur Conan Doyle (Sherlock Holmes)

Survey in Week 1 of this module in 2015

ST222 **Lecturer: Dr Julia Brettschneider**

This is a collection of questions about decision making in a variety of situations. This is not a test. The intention is to give you some concrete experience with making decisions, so the methodology we study will become more meaningful.

Please answer the questions quietly on your own and return this sheet in about 20 min. The questions will later be posted on the module website, so you can discuss answers with your class mates and friends.

ST222@Warwick: 12 questions, some in two versions

ST222@Warwick: The data file

surveyWeek1.csv

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	Version	1	2.1	2.2	2.3	3	4	5	6	7	8	9	10	11	12	Dept	Origin	Gender	Comments	
2	1	y	a	80	64	t	y	s	bd	ad	5	a	0.7	b	463875	m	h	m		
3	1	y	s	512	512	t	y	s	bd	ad	8	a	0.7	b	453621	m	h	m	q2 "same" though not	
4	2	y	s	512	512	t	y	e	ad	ad	5	a	0.3	a	3425671	m	h	m	changed q8 to ing	
5	2	y	s	512	512	t	y	s	bd	ad	16	b	0.3	b	2714563	m	h	m	changed q8 to ing	
6	1	y	s	512	512	t	y	s	bd	ad	5	a	1	b	534621	m	h	m	q2 "same" though not	
7	4	y	s	500	500	t	y	s	bd	ad	16	a	0.3	b	2413876	o	h	m	degree CS	
8	4	y	b	13	9	t	n	e	bd	ad	16	a	0.1	b	0	s	n	f		
9	4	y	s	512	512	t	n	s	bd	ad	11	n	0.3	a	5312674	m	h	m		
10	3	y	s	512	512	t	y	s	ad	ad	8	a	0.8	a	324651	m	h	m	q2 ticked both a, b	
11	2	y	b	5	10	t	y	l	bd	ad	5	a	0.3	b	4513881	s	n	f	q12 ranks impossible	
12	1	n	a	24	18	t	y	s	bd	ad	5	b	0.7	a	652431	s	n	f		
13	4	y	a	24	18	c	y	s	bd	ad	16	n	0.3	b	4712653	s	n	f		
14	3	n	a	512	100	t	y	s	bd	ad	16	a	0.7	a	342651	s	n	m		
15	3	y	s	512	512	t	y	s	bd	bd	1	a	0.7	b	342516	m	h	m	q2 no answer for first	
16	4	y	s	512	512	t	y	e	ad	ad	8	b	0.3	b	2461537	m	h	f		
17	1	y	a	512	512	t	y	s	ad	ad	3	a	0.7	a	251634	m	h	m	q2 a but same numbe	
18	3	y	b	NA	8	t	y	s	ad	ad	5	b	0.7	a	80010	s	n	f	q12 missing get 0	
19	2	y	b	500	1000	t	y	s	bd	ad	3	a	0.3	b	10000	s	n	m	q12 missing get 0	
20	1	y	s	512	512	t	y	s	bd	ad	16	b	0.7	b	253641	m	h	m	q2 "same" though not	
21	4	n	s	512	512	t	y	s	ad	ac	16	b	0.8	b	1524673	m	h	m		
22	3	y	b	NA	NA	t	y	s	bd	ad	11	a	0.5	b	123654	m	h	m		
23	2	y	b	200	250	t	y	s	ad	ad	11	b	0.3	b	3615872	m	h	m		
24	2	y	a	272	256	t	n	s	bd	ad	11	a	0.3	b	2315764	m	h	f		

ST222@Warwick: Beginning of R-session

```
########## Initial settings etc

dirAnalysis<-"/Users/juliab/Dropbox/ST222/Questionnaire/"
dirData<-"/Users/juliab/Dropbox/ST222/Questionnaire/"
dirPlots<-"/Users/juliab/Dropbox/ST222/Questionnaire/Plots/"
setwd(dirAnalysis)

D<- read.table(file="surveyWeek1.csv",sep=",", header=T)
#D<-scan(file="surveyWeek1.csv",sep=",", what=c(0,"","",0,0,"","","","","",0,"",0,"",0,"","","",""))
doesn't work

# Versions
counts <- table(D[,1])
barplot(counts, main="Versions", xlab="")

V<-D[,1]
a <- V==1 | V==3
b <- V==2 | V==4
> sum(a)
[1] 51
> sum(b)
[1] 49
```

Question 5: Judging sample variation

Question from Kahneman & Tversky's 1970s program on probability judgement

Question 5 - type a (type b)

A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50% of all babies are boys. However, the exact percentage varies from day to day. Sometimes it may be higher than 50%, sometimes lower. For a period of 1 year, each hospital recorded the days on which more than 60% of the babies born were boys. Which hospital do you think recorded more such days?

The larger hospital The smaller hospital About the same (within 5% of each other)

Correct answer: The smaller hospital.
Reason: Smaller samples are more variable. Hence they record more days with over 60% boys.

Kahneman D & Tversky A, *Subjective probability: A judgement of representativeness.* Cognitive Psychology, 3 (1972), 430-454

Question 5: Judging sample variation

Question 5 - type a (type b)

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The larger hospital The smaller hospital About the same (within 5% of each other)

Original study was on Stanford UG students without training in proba/stats: They answered mostly wrong

Are trained Warwick UG students better?

```
########################### Question 5
# Q5: D[,8] (which hospital?)
```

```
> table(D[a,8])
 e  l  s
 2  2 47
```

Type a question

larger	smaller	equal	
3.9%	**92.2%**	3.9%	# though "equal" option was not available!

```
> table(D[b,8])
 e  l  s
 7  3 39
```

Type b question

larger	smaller	equal
6.1%	**79.6%**	14.3%

Question 8: Word frequencies

Question from Kahneman & Tversky's 1970s program on probability judgement

Question 8 - type a (n)
In four pages of a novel (about 2,000 words), how many words would you expect to find that have the form _ _ _ _ _ **n** _ ? Indicate your best estimate by circling one of the values below:

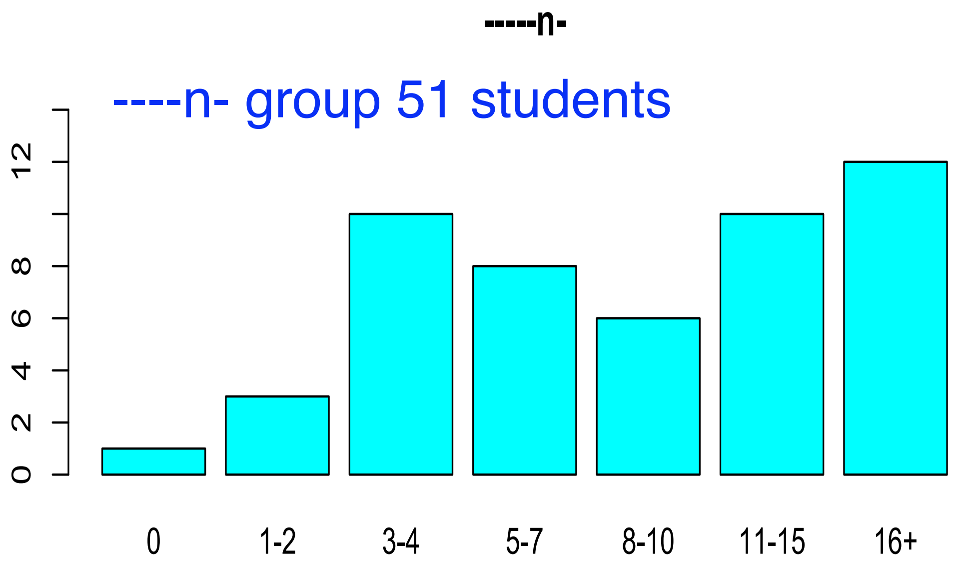
0 1-2 3-4 5-7 8-10 11-15 16+

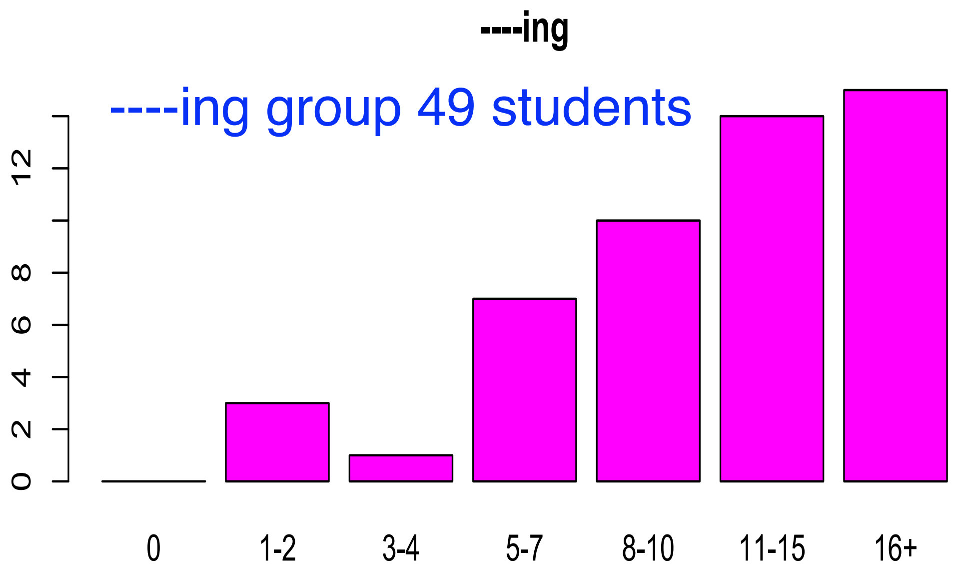
Question 8 - type b (ing)
In four pages of a novel (about 2,000 words), how many words would you expect to find that have the form _ _ _ _ **i n g** (seven-letter words that end with "ing")? Indicate your best estimate by circling one of the values below:

0 1-2 3-4 5-7 8-10 11-15 16+

Kahneman D & Tversky A, *On the psychology of prediction.* Psychological Review 80, *237-51*.

ST222@Warwick:





The *less* restrictive condition creates fewer words!

Violates normative rules of probability:

For A subset of B,

P(A) < P(B)

Is this normal? Why?

Confirms result form the literature:

Judging frequency (question as above)

----n-: median 2.3 ---ing: median 6.4

Creating as many as possible words in 60 sec:

----n-: median 4.7 ---ing: median 13.4

Similar results obtained comparing word groups -----l- and -----ly

Latter classes produced more words despite being contained in former!

What are explanations for this incoherence?

Availability heuristics:

Increased efficiency of memory search offsets reduced extension of target class.

Example: Searching for "-ing" may lead to the words "timing", "resting", "drawing", "going", "talking" faster than searching for "-n-"

Example: Allais paradox

First experiment:

S1: 1M for sure
R1: 5M with 0.10, 1M with 0.89, 0M with 0.01

Second experiment:

S2: 1M with 0.11, 0M with 0.89
R2: 5M with 0.10, 0M with 0.90

Allais conjecture:

S1 > R1: certain outcome
S2 < R2: huge difference in gain
(small difference in proba)

Allais, M. (1953), *Le Comportement de l'Homme Rationnel devant le Risque: Criticue des Postulats et Axiomes de l'Ecole Americaine,* Econometrica 21: 503 – 546.

Example: Allais paradox

First experiment:

S1: [1M, 1.00]
R1: [5M, 0.10], [1M, 0.89], [0M, 0.01]

Mathematically equivalent to

S1': [1M, 0.89], [1M, 0.11]
R1': [1M, 0.89], [0M, 0.01], [5M, 0.10]

Second experiment:

S2: [1M, 0.11], [0M, 0.89],
R2: [5M, 0.10], [0M, 0.90],

S2': [0M, 0.89], [1M, 0.11]
R2': [0M, 0.89], [0M, 0.01], [5M, 0.10]

Allais conjecture about preferences:

S1 > R1: certain outcome
S2 < R2: huge difference in gain
(small difference in proba)

If E[u(S1')] > E[u(R1')]
then E[u(S2')] > E[u(R2')]
(addends cancel out)

INCONSISTENT with expected utility theory, independence axiom

Example: Allais paradox

First experiment:

S1: 1M for sure
R1: 5M with 0.10, 1M with 0.89, 0M with 0.01

Second experiment:

S2: 1M with 0.11, 0M with 0.89
R2: 5M with 0.10, 0M with 0.90

Allais conjecture:

S1 > R1: because certain outcome is preferred
S2 < R2: huge difference in gain
(small difference in proba)

Empirical evidence confirms Allais conjecture

Numerous studies using hypothetical, monetary and health outcomes

Example: Allais paradox

First experiment:

S1: 1M for sure
R1: 5M with 0.10, 1M with 0.89, 0M with 0.01

Second experiment:

S2: 1M with 0.11, 0M with 0.89
R2: 5M with 0.10, 0M with 0.90

Allais explanation for incoherence: Preferences are *not independent.*
10% of getting 5M carries 1% risk of getting nothing (feeling disappointed), in contrast to sure gain of 1M (feeling of certainty).

Is expected utility theory (EUT) wrong?

How do Warwick UG students answer this question?

Question 6: Allais paradox

Question 6

You are asked to choose between the following 2 gambles below. Circle your preference.

[S1] A. A 100% chance of receiving $1 million.
[R1] B. A 10% chance of receiving $5 million, an 89% chance of receiving $1 million, and a 1% chance of receiving nothing.

After you have made your choice, you are asked to choose between the following two gambles. Circle your preference.

[S2] C. An 11% chance of receiving $1 million, and an 89% chance of receiving nothing.
[R2] D. A 10% chance of receiving $5 million, and a 90% chance of receiving nothing.

ST222@Warwick (details next slide):
About half of this class behaved consistent with EUT preferring R1 and R2 over S1 and S2. That means, you value certainty about outcomes less then typical subjects in existing studies.

Question 6

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After you have made your choice, you are asked to choose between the following two gambles. Circle your preference.

[S2] C. An 11% chance of receiving $1 million, and an 89% chance of receiving nothing.
[R2] D. A 10% chance of receiving $5 million, and a 90% chance of receiving nothing.

```
################ Question 6
# Q6: D[,9] (Allais)

table(D[,9])
ac  ad  bc  bd
 5  43   3  49     (out of 100 total)
```

ST222@Warwick:

S1>R1 & S2<R2 43%
Allais paradox

S1<R1 & S2<R2 49%
Consistent(!) with EUT

Question 7: Ellsberg paradox

Suppose you have an urn containing 30 red balls and 60 other balls that are either black or yellow. (You don't know how many black or how many yellow balls there are, but that the total number of black balls plus the total number of yellow equals 60.) The balls are well mixed so that each individual ball is as likely to be drawn as any other. You are given a choice between the two gambles below. Circle the one you prefer.

A. You receive £100 if you draw a red ball.

B. You receive £100 if you draw a black ball.

After the urn has been put back into its original state, you are given the choice between the two gambles below. Circle the one you prefer.

C. You receive £100 if you draw a ball that is not black.

D. You receive £100 if you draw a ball that is not red.

Good question: What is *original state*?! Question text from literature... They do not mean the exact physical arrangement of the balls, but refer to the *state* in which *each individual ball is as likely to be drawn as any other.*

30 red balls, 60 other balls that are either black or yellow.

A. You receive £100 if you draw a red ball.
B. You receive £100 if you draw a black ball.

After the urn has been put back into its original state.

C. You receive £100 if you draw a ball that is not black.
D. You receive £100 if you draw a ball that is not red.

How to approach this?

Prefer A>B since proportion of red balls is known.

Alternatively, make (implicit) assumptions about proportions black/yellow, e.g. 30/30.

Ellsberg: Assume you settle on A>B. Then you should choose D>C for the same reason (preference for known probability).

Empirical studies show that a strong majority of people do indeed have these preferences (A>B, D>C).

30 red balls, 60 other balls that are either black or yellow.

A. You receive £100 if you draw a red ball.
B. You receive £100 if you draw a black ball.

After the urn has been put back into its original state.

C. You receive £100 if you draw a ball that is not black.
D. You receive £100 if you draw a ball that is not red.

What does Expected utility theory (EUT) say?

Let M=u(£100), 0=u(£0).

E[u(A)] = 30/90 * M

E[u(B)] = Black/90 * M

E[u(C)] = (30+60-Black)/90 * M

E[u(D)] = 60/90 * M

E[u(A)] - E[u(B)] = (30-Black)/90 * M

E[u(C)] - E[u(D)] = (30+60-Black-60)/90 * M = (30-Black)/90 * M

EUT says A>B is equivalent to C>D.

See also exercise sheet 4

30 red balls, 60 other balls that are either black or yellow.

A. You receive £100 if you draw a red ball.
B. You receive £100 if you draw a black ball.

After the urn has been put back into its original state.

C. You receive £100 if you draw a ball that is not black.
D. You receive £100 if you draw a ball that is not red.

What do ST222 students at Warwick say:

############ Question 7

ad bc contradict EUT
ac bd compatible with EUT

ac	ad	bc	bd	
12%	81%	2%	5%	100 of ST222'14@Warwick
8%	84%	3%	4%	76 of ST222'15@Warwick

ST222@Warwick: Huge majority behaved as predicted by Ellsberg, i.e. they are **not** following expected utility theory (EUT).

Compare: Allais paradox and Ellsberg paradox

Allais paradox:
Different levels of uncertainty regarding the ***outcomes***.
All probabilities are known. They have different levels, including even probability of 1 (certainty).

Certainty effect:
Prefer the option that offers certain win to avoid disappointment of no win at all (even if probability very small).

Ellsberg paradox:
Uncertainty regarding the ***probabilities*** that govern the outcomes.
Specifically, the amounts of black and yellow balls are not given.

Ambiguity aversion:
Preference for known risks over unknown risks.