## Definitions: Heuristics \& biases

Heuristic strategies:
Shortcuts and approximations used in probability judgement and prediction, especially if correct answers are not known or would take too long to construct. Using heuristics is useful and necessary, but may lead to biases in judgement.

Bias: (in this context)
Differences arising from the use of heuristics or other nonnormative strategies.

## Question 10: Base rate neglect

## Question from Tversky \& Kahneman 1973

## Type a:

A panel has interviewed and administered personality tests to 70 doctors and 30 lawyers, all successful in their respective fields. On the basis of this information, thumbnail descriptions of the 70 doctors and 30 lawyers have been written. Below is a description, chosen at random from the 100 available descriptions.

Dick is a 30-year-old man. He is married with no children. A man of high ability and high motivation, he promises to be quite successful in his field. He is well liked by his colleagues.

What is the probability that Dick is one of the 70 doctors in the sample?

## Type b:

The same except numbers swapped (30 doctors and 70 lawyers)

## Subjects at study at University of Oregon 1973 neglect the base rates: Answer to Type a and Type b had similar distributions.

## What about Warwick students (in Maths and in Stats depts)?

ST222’14@Warwick:
\#\#\#\#\#\#\#\#\# Question 10 \#\#\# \# Q10: doctors
par (mfrow=c (2,1))
hist(D[a,13],
probability=T, main="Type a
(70 doctors, 30 lawyers)", $x \lim =c(0,1), \quad b r e a k s=20$, xlab="Probability Dick
is a doctor")
hist(D[b, 13],
probability=T,
main="Type b
(30 doctors, 70 lawyers)",
$x \lim =c(0,1)$, breaks=20, xlab="Probability Dick is a doctor")

Type a (70 doctors, 30 lawyers)


Type b (30 doctors, 70 lawyers)


## ST222’15@Warwick:

Type a (30 doctors, 70 lawyers):
> round(100*table(D[1:38,11])/n, digits=1)

| 0. | 0.5 | 0.6 | 0.7 | 0.8 | 0.85 | 0.875 | 0.9 | 1.0 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5. | 10.6 | 2.6 | 65.8 | 5.2 | 2.6 | 2.6 | 2.6 | 2.6 | \% |

## Type b (30 doctors, 70 lawyers):

> round(100*table(D[39:76,11])/n, digits=1)

| 0.0 | 0.1 | 0.3 |
| :--- | :--- | ---: |
| 5.2 | 5.2 | 71.0 |

ST222@Warwick: Huge majority used base rate, and of those who did not, some gave reasons.

## Neglect of base rates: cab problem (T \& K, I982)

A cab was involved in a hit and run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:

1. $85 \%$ of the cabs in the city are Green and $15 \%$ are Blue.
2. a witness identified the cab as Blue. The court tested the reliability of the witness under the same circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colors $80 \%$ of the time and failed $20 \%$ of the time.

What is the probability that the cab involved in the accident was Blue rather than Green?
A. Tversky, D. Kahneman, Evidential impact of base rates, in Judgement under uncertainty: Heuristics and biases, D. Kahneman, P. Slovic, A. Tversky (editors), Cambridge University Press, 1982

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What is the probability that the cab involved in the accident was Blue rather than Green?

Most people answer probability for blue is $50 \%-80 \%$.
But the answer using normative theory is about $41 \%$. (See next page).

## Most people answer probability for blue is 50\%-80\%.

(Cabs are 85\% green, I5\% blue)

## Answer using conditional probabilities:

Let $G$ be the event of the delinquent being Green. Let $B$ be the event of the delinquent being Blue. Finally, let $W$ be the witness' report. Clearly,

$$
\begin{aligned}
\frac{p(B \mid W)}{p(G \mid W)} & =\frac{p(W \mid B)}{p(W \mid G)} \cdot \frac{p(B)}{p(G)} \\
& =\frac{0.8}{0.2} \cdot \frac{0.15}{0.85} \\
& =\frac{12}{17}
\end{aligned}
$$

$$
\begin{array}{r}
\frac{x}{y}=r, \quad x+y=1 \\
x+x / r=1 \\
x=\frac{r}{r+1}
\end{array}
$$

Since $p(G \mid W)+p(B \mid W)=1$, it follows that

$$
p(B \mid W)=\frac{12}{12+17} \approx 0.41
$$

$$
\frac{12 / 17}{12 / 17+1}=\frac{12}{12+17}
$$

meaning that, in spite of the witness testimony, the hit-and-run cab is more likely to be Green than Blue.

## Concepts: Availability heuristic

- How do people predict a frequency (or probability)?

Calculation may be too difficult. They may lack information.

- In such situations, cannot use objective measures and follow normative rules.
- Instead, they judge frequency (or probability) by assembling stored information, a mental shortcut called availability heuristics.
- That can lead to availability bias/fallacy.
- More severely when other factors (e.g. recency, emotional association) selectively interfere with the effectiveness of the individual's memory.

Example: Estimated word frequencies (see previous lecture)
Now another example...

## Experiment: K\&T's Permutation study

Question 2 (in 2014 version)
Consider the two structures, $(A)$ and $(B)$, which are displayed below.
(A)

$X X X X X X X X \quad X X$
$X X$
$X X$
$X X$
$X X$
$X X$
$x X$

A path in a structure is a line that connects an element in the top row to an element in the bottom row, and passes through one and only one element in each row. In which of the two structures are there more paths? How many paths do you think there are in each structure?

In which of the two structures are more paths?
How many paths do you think there are in each structure?

Kahneman D \& Tversky A, On the psychology of prediction. Psychological Review 80 (1973), 237-5 I. Study 4.

Common intuitive judgement:
(A) has more paths than (B)

K\&T's data: 46 out or 54 participants said (A) has more paths. $\begin{aligned} & x x \\ & x x\end{aligned}$
Explanation: Differential availability of paths in the two structures for a number for reasons including:

- most obvious paths are columns, and (A) has 8 while (B) has only 2
- paths in (A) are more distinctive and less confusable than those in (B)
- path in (A) are shorter and hence easier and faster to visualise

Criticism: Given the phrasing "which has more" used in the question, the answer "same" may feel inappropriate or oppositional discouraging subjects to use it - despite it being correct!

Test this on Warwick Students from Maths and Stats...

## Question: Permutations

## We added explicit answer choices adding the option "same" in half of the questionnaires (type a, type b) to see if this effects answers.

## Question 2

Consider the two structures, (A) and (B), which are displayed below.

| (A) | (B) |
| :---: | :---: |
| X X X X X X X | X X |
|  | X X |
| X X X X X X X | X X |
|  | X X |
|  | X X |
|  | X X |
|  | XX |
|  | XX |
|  | X X |

A path in a structure is a line that connects an element in the top row to an element in the bottom row, and passes through one and only one element in each row. In which of the two structures are there more paths? How many paths do you think there are in each structure?

More in (A) More in (B) Same
Estimated number of paths in (A):
Estimated number of paths in (B):

## ST222@Warwick results:

type a (same not given as answer choice): type b (same given):
(A) (B) same 44\% 19\% 38\%
$33 \% ~ 21 \% ~ 46 \%$

Interpretation:

- For type b, where the correct answer was an option, a (weak) majority picked it.
- For type a, where the correct answer not an option, a majority selected (A), but $38 \%$ of the respondents just added the missing option!
- For both types, the remaining answers showed a strong majority for type (A) in line with the availability heuristic. The relative proportions of remaining answer are:
type a:
(A) $71 \%$
(B) $31 \%$
type b: (A) 61\%
(B) $39 \%$


## Decision process: Reason-based choice

Shafir, Simonson and Tversky asked questions like:

- What is the role of uncertainty in decision making?
- What happens when choices are in conflict with each other?

Two approaches are available to answer these questions:

- Reason-based analysis used in theoretical explanations
- Value-based approaches involving experimental data

Search for a comprehensive view.
Can the two approaches be combined through experiments that also elicit the role of reasons in making choices?

Shafir, E., Simonson, I., and Tversky, A., Reason-based choice, Cognition, 49 (2), I993, pp. I I-36

## Discussion: Reason-based choice

## Motivation for this research as described in the papers:

"The need to choose often creates conflict: we are not sure how to trade off one attribute relative to another or, for that matter, which attributes matter to us most. It is a commonplace that we often attempt to resolve such conflict by seeking reasons for choosing one option over another."
"Conflict plays no role in the classical theory of choice. In this theory, each option $x$ has a value $u(x)$ such that, for any offered set, the decision maker selects the option with the highest value. In particular, a person is expected to search for additional alternatives only if the expected value of searching exceeds that of the best option currently available.

A reliance on reasons, on the other hand, entails that we should be more likely to opt for an available option when we have a convincing reason for its selection, and that we should be more likely to search further when a compelling reason for choice is not readily available."

## Empirical study: Reason-based choice

Decisional conflict and search for options:

- Seeking for additional options requires time and effort, and may result in loosing previously available options.
- Does presence of decisional conflict increase seeking for additional options?


## Experiment:

- Subjects are university students
- Subject were presented with pairs of options
- Two domains: bets with different probabilities/payoffs, appartments with different monthly rent/distance from campus
- At each trial, subject can either choose one option or request another option from a known catalogue, at some cost


## Some questions from the study: Featuring appartments

## Conflict condition: (Half of the subjects)

Imagine that you face a choice between two apartments with the following characteristics:
(x) $\$ 290$ a month, 25 minutes from campus
(y) $\$ 350$ a month, 7 minutes from campus

Both have one bedroom and a kitchenette. You can choose now between the two apartments or you can continue to search for apartments (to be selected at random from the list you reviewed). In that case, there is some risk of losing one or both of the apartments you have found.

## Dominance condition: (Other half of the subjects)

Similar question except replaced (y) by ( $x^{\prime}$ ) as below:
(x) $\$ 290$ a month, 25 minutes from campus
(x') $\$ 330$ a month, 25 minutes from campus

## Potential choice behaviours and underlying motivations:

- Principle of value maximisation would suggest: Subjects search for additional alternatives if and only if the expected value of searching exceeds that of the best alternative currently available.
- Because the best alternative offered in the dominance condition is also available in the conflict condition, value maximisation implies that the percentage of subjects who seek an additional alternative cannot be greater in the conflict than in the dominance condition.
- In conflict situations: Subjects may feel paralysed because they can not justify their choice. This may lead to requesting additional alternatives to delay decision making.


## Study results:

Subjects requested an additional alternative $64 \%$ of the time in the conflict condition, and only $40 \%$ of the time in the dominance condition ( $\mathrm{p}<0.05$ ).

Study show that people tend to ask for more options when the choice was harder to rationalise.

Discussion:

- Data contradicts the principle of value maximisation.
- Requesting additional alternatives may be motivated by the difficulty of choosing among the options under consideration.
- People may request additional alternatives for the sake of improving the justification for their choice rather than to improve the outcome of the decision itself

Next question:
What if people had a justification, but one that is conditional on some random event?

## Empirical study: Reason-based choice

Definitive versus disjunctive reasons:
"People sometimes encounter situations of uncertainty in which they eventually opt for the same course of action, but for very different reasons, depending on how the uncertainty is resolved."

Experiment: (with UG students)
Three groups:
Uncertainty pass/fail
Passed
Failed


## Question text versions

## Disjunctive version: (First group of subjects [66])

Imagine that you have just taken a tough qualifying examination. It is the end of the fall quarter, you feel tired and run-down, and you are not sure that you passed the exam. In case you failed you have to take the exam again in a couple of months - after the Christmas holidays. You now have an opportunity to buy a very attractive 5-day Christmas vacation package in Hawaii at an exceptionally low price. The special offer expires tomorrow, while the exam grade will not be available until the following day.
Would you?:
(a) buy the vacation package.
(b) not buy the vacation package.
(c) pay a $\$ 5$ non-refundable fee in order to retain the rights to buy the vacation package at the same exceptional price the day after tomorrow after you find out whether or not you passed the exam.

## Question versions from the study

Fass/fail versions:
(Second group of subjects [67], half pass half fail condition)
Imagine that you have just taken a tough qualifying examination. It is the end of the fall quarter, you feel tired and run-down, and you find out that you [passed the exam. / failed the exam. You will have to take it again in a couple of months - after the Christmas holidays.] You now have an opportunity to buy a very attractive 5-day Christmas vacation package in Hawaii at an exceptionally low price. The special offer expires tomorrow. Would you?:
(a) buy the vacation package.
(b) not buy the vacation package.
(c) pay a $\$ 5$ non-refundable fee in order to retain the rights to buy the vacation package at the same exceptional price the day after tomorrow.

## Results

|  | Disjunctive | Fail | Pass |
| :---: | :---: | :---: | :---: |
| (a) | $32 \%$ | $54 \%$ | $57 \%$ |
| (b) | $7 \%$ | $16 \%$ | $12 \%$ |
| (c) | $61 \%$ | $30 \%$ | $31 \%$ |

Answer preferences hardly differ between pass and fail conditions.
Interpretation: Whether they are actually pass or fail is not that important for the decision outcome, but it has an effect on how people conduct their decision process.
They do differ between disjunctive vs pass/fail condition.
Not knowing their exam result, over $60 \%$ were willing to pay a fee to postpone the decision. Only half of that fraction would do so knowing the result, but regardless of what the actual result actual is.

Criticism (on method): Result is about on an aggregate level. Population fractions compared, not individuals' actions. (Unclear how to design an experiment assessing the latter.)

## Interpretation

- Once the outcome of the exam is known, the student has good - albeit different - reasons for taking the trip:
Having passed the exam, the vacation is presumably seen as a reward following a hard but successful semester; having failed the exam, the vacation becomes a consolation and time to recuperate before a re-examination.
- Not knowing the outcome of the exam, however, the student lacks a definite reason for going to Hawaii.
- Uncertainty leads to delaying decisions, irrespectively of whether or not the missing information would actually change the outcome of the decision.

Disjunction effect
A disjunction of different reasons is often less compelling than either definite reason alone.

## Question 4: Lost ticket

Imagine that you have decided to go to a play where admission is $£ 20$ per ticket. As you enter the theatre you discover that you have lost a $£ 20$ bill. Would you still pay $£ 20$ for a ticket for the play?

Yes No

## Studies in the literature: Majority of subjects says "No"

Explanation: Regret, not making peace with the loss.

## Question 4: Lost ticket

Imagine that you have decided to go to a play where admission is $£ 20$ per ticket. As you enter the theatre you discover that you have lost a £20 bill. Would you still pay $£ 20$ for a ticket for the play?
Yes No

Studies in the literature: Majority of subjects says "No"
Explanation: Regret, not making peace with the loss.
ST222’I4@Warwick: Majority of subjects says "Yes" (86\%)!
\#\#\#\#\#\#\# Question 4 \#\#\#\#\#\#\#
\# Q4: regret, pay again $D[, 7]$
> table(D[,7])
n y
1486

## Question 4: Lost ticket - a variation ('I 5 )

## Type a:

Imagine that you have decided to go to a play where admission is $£ 20$ per ticket. As you enter the theatre you discover that you have lost a $£ 20$ bill. Would you still pay £20 for a ticket for the play?

Yes No

## Type b:

Imagine that you have decided to go to a play and paid the admissions price of £20 per ticket. As you enter the theatre you discover that you have lost the ticket. The seat was not marked and the ticket can not be recovered. Would you pay £20 for another ticket?

Yes No

## Question 4: Lost ticket - a variation ('I5)

## Type a:

Imagine that you have decided to go to a play where admission is $£ 20$ per ticket.
As you enter the theatre you discover that you have lost a $£ 20$ bill. Would you still pay $£ 20$ for a ticket for the play?

Yes No
86.8\% I3.2\% ST222’15@Warwick (sample of 38)

## Type b:

Imagine that you have decided to go to a play and paid the admissions price of $£ 20$ per ticket. As you enter the theatre you discover that you have lost the ticket. The seat was not marked and the ticket can not be recovered. Would you pay £20 for another ticket?

Yes No
65.8\% 31.6\% ST222’15@Warwick (sample of 38)

Some comments on the questionnaires: "Depends on the play!", "If l'm with friends"

## Question 3: Missed plane

## Question 3

Mr Cane and Mr Tees were scheduled to leave the airport on different flights, at the same time. They traveled from town in the same cab, were caught in a traffic jam, and arrived at the airport 30 minutes after the scheduled departure time of their flights. Mr Cane is told that his flight left on time. Mr Tees is told that his flight left 5 min ago. Who is more upset? Mr Cane Mr Tees

Studies in the literature: Majority of subjects says "Mr Tees"
Explanation: Perceived control (could have made it if...), missing plane partly attributed partly to personal failure

ST222@Warwick: Huge majority of subjects says "Mr Tees":
ST222’।4@Warwick: 86\%
ST222’15@Warwick: 92.1\%
Other answers given: "Mr Cane","neither"/"same"

