On normative theory

"Give me an axiom, and I'll design the experiment that refutes it."

Amos Tversky

Conjunction: Linda

T & K series of experiments involving making probability (or rank) judgments about people's profession based on short profiles.

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Please rank-order the following statements by their probability, using 1 for the most probable and 8 for the least probable.

Tversky A and Kahneman D, *Extentional versus intuitive reasoning: The conjunction fallacy in probability judgement.* Psychological Review 90 (1983), 293-315.

Linda: Within-subject design (direct test)

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Please rank-order the following statements by their probability, using 1 for the most probable and 8 for the least probable.

Linda is a teacher in elementary school.

Linda works in a bookstore and takes Yoga classes.

Linda is active in the feminist movement.

Linda is a psychiatric social worker.

Linda is a member of the League of Women Voters.

Linda is a bank teller.

Linda is an insurance salesperson.

Linda is a bank teller and is active in the feminist movement.

Linda: Within-subject design (direct test)

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Please rank-order the following statements by their probability, using 1 for the most probable and 8 for the least probable.

	Linda is a teacher in elementary school.
	Linda works in a bookstore and takes Yoga classes.
F	*Linda is active in the feminist movement.*
	Linda is a psychiatric social worker.
	Linda is a member of the League of Women Voters.
B	*Linda is a bank teller.*
	Linda is an insurance salesperson.
B & F	*Linda is a bank teller and is active in the feminist movement.*

Linda: Rules of probability versus empirical evidence

B & F *Linda is a bank teller and is active in the feminist movement.*

F *Linda is active in the feminist movement.*

B *Linda is a bank teller.*

Normative rules of probability:

$$B \cap F \subseteq B \quad \text{implies} \quad P(B \cap F) \leq P(B)$$
$$B \cap F \subseteq F \quad \text{implies} \quad P(B \cap F) \leq P(F)$$

Empirical findings:

Very dominant response (86% in initial study) pattern is to rank

$$P(B) < P(B \cap F) < P(F)$$

Why? What do people think? Created many years of discussions…

Linda: Replication and variation

Results confirmed many times under various conditions:

- Addressing potential issues of the design:
  e.g. between-subject (indirect) and within-subject (direct)
- Alternative stories
- Different levels of statistical sophistication
- Even ST222@Warwick!

Linda: Between-subject design (indirect test)

Half of participants receive type a questionnaire, other half type b.

type a

Linda is a teacher in elementary school.
Linda works in a bookstore and takes Yoga classes.
Linda is a psychiatric social worker.
Linda is an insurance salesperson.
B & F *Linda is a bank teller and is active in the feminist movement.*
Linda is a member of the League of Women Voters.

type b

Linda is a teacher in elementary school.
Linda works in a bookstore and takes Yoga classes.
F *Linda is active in the feminist movement.*
Linda is a psychiatric social worker.
B *Linda is a bank teller.*
Linda is an insurance salesperson.
Linda is a member of the League of Women Voters.

Linda: Meet also Bill!

Bill is 34 years old. He is intelligent, but unimaginative, compulsive, and generally lifeless. In school, he was strong in mathematics but weak in social studies and humanities.

Please rank-order the following statements by their probability, using 1 for the most probable and 8 for the least probable.

	Bill is a physician who plays poker for a hobby.
	Bill is an architect.
A	*Bill is an accountant.*
J	*Bill plays jazz for a hobby.*
	Bill surfs for a hobby.
	Bill is a reporter.
A & J	*Bill is an accountant who plays jazz for a hobby.*
	Bill climbs mountains for a hobby.

Linda: Level of statistical sophistication

Experiment was conducted on subjects with different levels of statistical training. Surprisingly, this had negligible effect.

Table 3.1 Tests of the conjunction rule in likelihood rankings

		Direct test				Indirect test		
Subjects	Problem	V	R (A & B)	R (B)	*N*	R (A & B)	R (B)	Total *N*
Naive	Bill	92	2.5	4.5	94	2.3	4.5	88
	Linda	89	3.3	4.4	88	3.3	4.4	86
Informed	Bill	86	2.6	4.5	56	2.4	4.2	56
	Linda	90	3.0	4.3	53	2.9	3.9	55
Sophisticated	Bill	83	2.6	4.7	32	2.5	4.6	32
	Linda	85	3.2	4.3	32	3.1	4.3	32

Note. V = percentage of violations of the conjunction rule; R (A & B) and R (B) = mean rank assigned to A & B and to B, respectively; *N* = number of subjects in the direct test; Total *N* = total number of subjects in the indirect test, who were about equally divided between the two groups.

A for the characteristic that does not fit the previous story
(Bill: accountant, Linda: bank teller)

B for the characteristic that does fit the previous story
(Bill: jazz, Linda: feminist)

How do ST222@Warwick answer this question?

Rcode from the analysis: (*not examinable!*)

```
> d <- D[a,16]
> mean(d,na.rm=T)
> tab <- tabulate(d)
> tab<-c(tab,0)
> barplot(tab, main="Feminist and bank teller", xlab="",
names.arg=c(1:6), col="violet")
> d <- D[b,17]
> mean(d,na.rm=T)
> tab <- tabulate(d)
> tab<-c(tab,0)
> barplot(tab, main="Feminist", xlab="",
names.arg=c(1:7),col="blue")
> d <- D[b,18]
> mean(d,na.rm=T)
> tab <- tabulate(d)
> barplot(tab, main="Bank teller", xlab="",
names.arg=c(1:7), col="red")
```

ST222'14@Warwick:

Between-subject design

$$P(B) < P(B \cap F) < P(F)$$

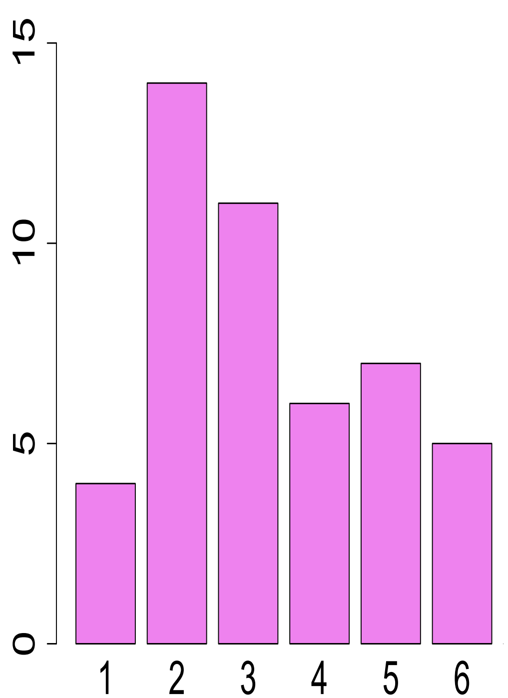
contradicting normative rules

mean rank B & F = **3.3,**
rescaled: 3.0*7/6 = 3.85

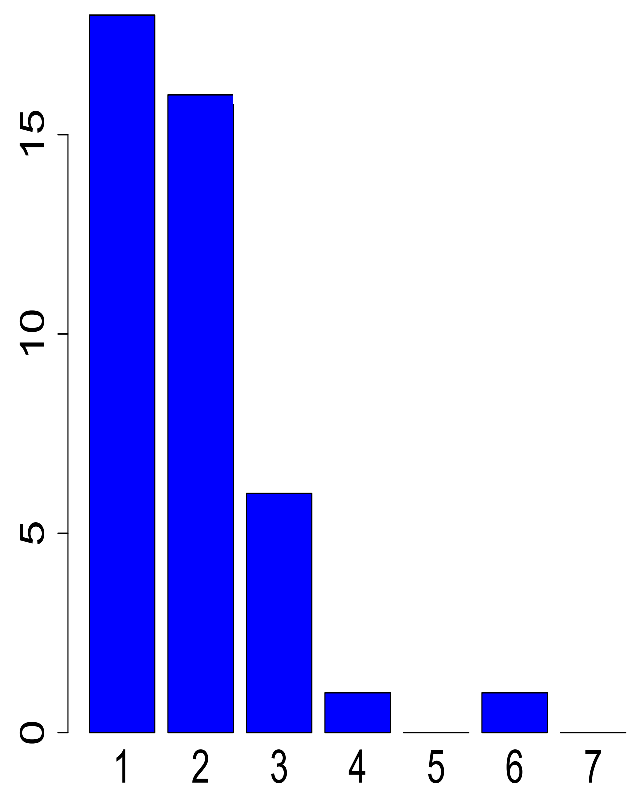
mean rank F =**1.9**

mean rank B =6.1

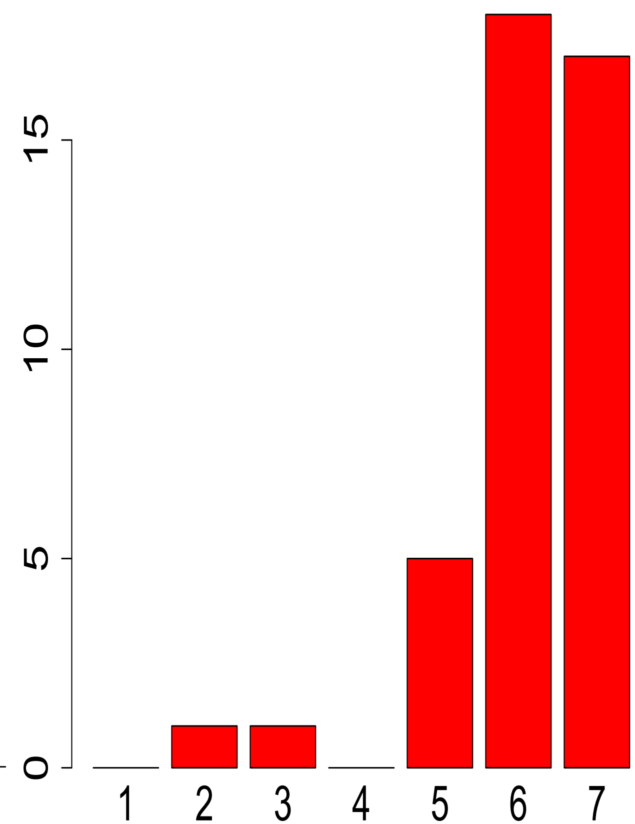
Feminist and Bank Teller



Feminist



Bank Teller



ST222'15@Warwick:

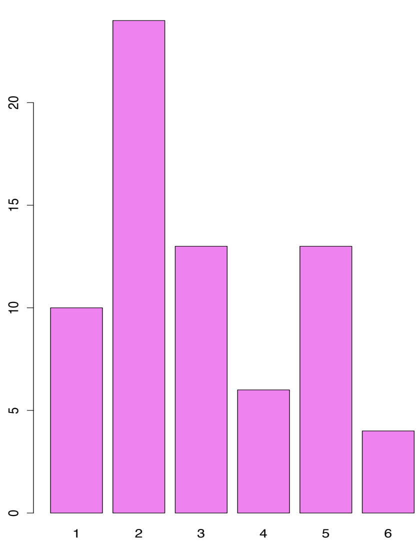
Between-subject design

$$P(B) < P(B \cap F) < P(F)$$

contradicting normative rules

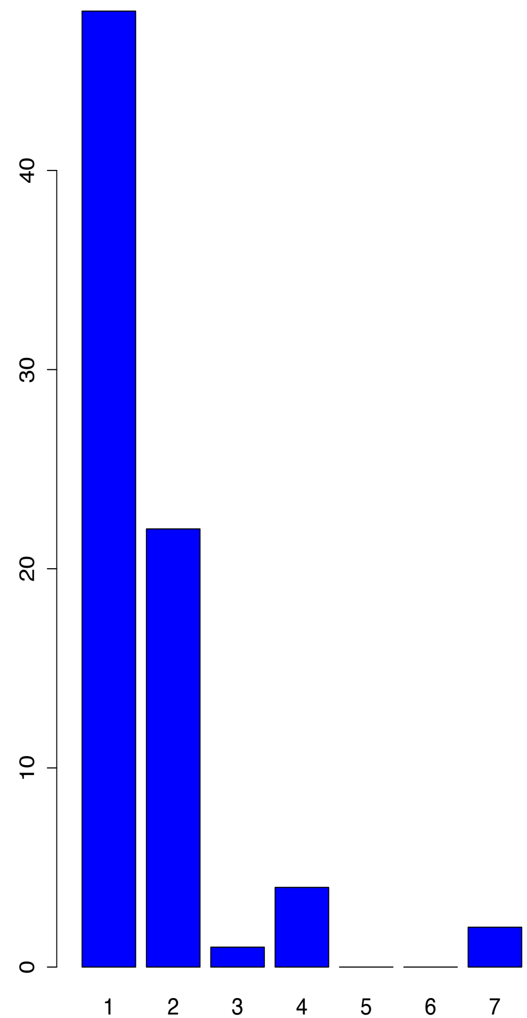
mean rank B & F = **3.0,**
rescaled: 3.0*7/6 = 3.5

Feminist and Bank Teller



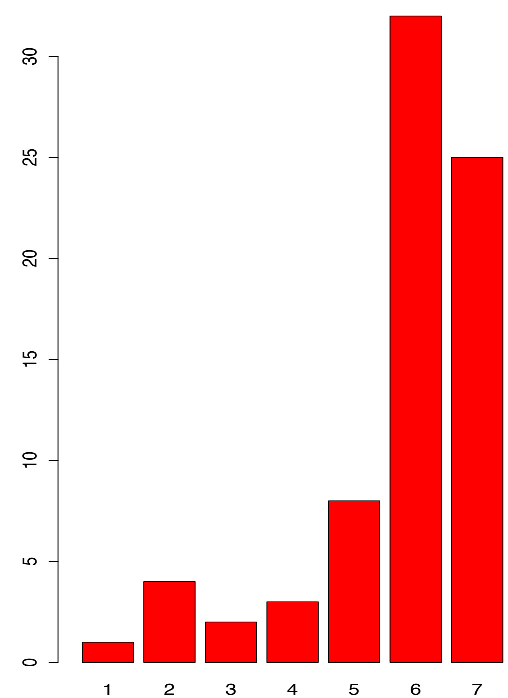
mean rank F =**1.7**

Feminist



mean rank B = **5.9**

Bank Teller



Concept: Conjunction fallacy

Definition

The *conjunction fallacy* is a behavioural bias reflecting the belief that the probability of the joint event A & B is bigger than the probability of one of the individual events.

More generally, it reflects that specific conditions are more probable than a single more general one.

Explanation

Depends on context. For the Linda experiment, T & K argue that it is due to the representativeness heuristic:

People think:	*B is less typical for Linda than B&F, and* *B&F is less typical for Linda than F*
People conclude:	*B is less probable than B&F, and* *B&F is less probable than F*

Conjunction fallacy: Forecasting of events

- T & K's questionnaire at the *Second International Congress on Forecasting* in July of 1982
- **Subjects:** 115 professional analysts, employed by industry, universities, or research institutes
- Two *different* experimental groups
- Asked to rate the probability of two different statements
- Scale: <0.01%, 0.1%, 0.5%, 1%, 2%, 5%, 25%
- Each group seeing only one statement (indirect design)

"A complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983."

"A Russian invasion of Poland, and a complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983."

Results: Estimates of probability were low for both statements, but significantly *lower* for the first group than the second ($p < 0.01$ by Mann-Whitney).

"A complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983."

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Results: Estimates of probability were low for both statements, but significantly *lower* for the first group than the second ($p < 0.01$ by Mann-Whitney).

Also tried variations (and obtained similar results), e.g.:

"A 30% drop in the consumption of oil in the US in 1983"

"A dramatic increase of oil prices and a 30% drop in the consumption of oil in the US in 1983"

"A complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983."

"A Russian invasion of Poland, and a complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983."

Results: Estimates of probability were low for both statements, but significantly *lower* for the first group than the second ($p < 0.01$ by Mann-Whitney).

Discussion: Conjunctions involving a (hypothetical) cause are particularly prone to fallacies. Why? People may intuitively asses the probability of the effect given the *cause* rather than the joint probability, e.g.:

P(*suspension of US-SU relationship* | *Russian invasion of Poland*)

P(*suspension of US-SU relationship* & *Russian invasion of Poland*)

Conjunction: Dice playing

Replace explicit mentioning of probabilities by a reward in game.

Consider a regular six-sided die with four green faces and two red faces. The die will be rolled 20 times and the sequences of greens (G) and reds (R) will be recorded. You are asked to select one sequence, from a set of three, and you will win $25 if the sequence you chose appears on successive rolls of the die. Please check the sequence of greens and reds on which you prefer to bet.

1. RGRRR
2. GRGRRR
3. GRRRRR

125 undergraduates at UBC and Stanford, monetary rewards.
65% of the subjects chose sequence **2**.

Tversky A and Kahneman D, *Extentional versus intuitive reasoning: The conjunction fallacy in probability judgement.* Psychological Review 90 (1983), 293-315.

Conjunction: Dice

1. RGRRR
2. GRGRRR
3. GRRRRR

125 undergraduates at UBC and Stanford, monetary rewards. 65% of the subjects chose sequence 2. More than sequence 1!

Normative for probability:

GRGRRR is the conjunction of RGRRR and another event, hence *less likely* to be observed than just RGRRR:

$$P(\mathrm{GRGRRR}) \leq P(\mathrm{RGRRR})$$

Subjects perceive GRGRRR are *more representative* of the die and wrongly concluded it was *more probably*.

Tversky A and Kahneman D, *Extentional versus intuitive reasoning: The conjunction fallacy in probability judgement.* Psychological Review 90 (1983), 293-315.

Concept: Representativeness

Representativeness is a directional relation between two objects:

Model M ⟵ Event X

Typical questions:

- Is X or Y more representative of M?
- Is X more representative of M or N?

Examples:

- Is sample S representative of population P?
- Is person X representative of the stereotype of librarians?

Kahneman D, Slovic P & Tversky A (eds.), *Judgement under Uncertainty: Heuristics and Biases*. CUP, Cambridge, 1982. Chapter 6.

Concept: Representativeness

Representativeness is a directional relation between two objects:

Model M ⟵ Event X

Type 1

X value of variable defined in class M.

Most representative value mean (or median, mode) or the distribution of the variable. Relation determined by knowledge of that distribution.

Examples: M=students in this class, X=height.
M=UK population, X=person's salary.

Kahneman D, Slovic P & Tversky A (eds.), *Judgment under Uncertainty: Heuristics and Biases*. CUP, Cambridge, 1982. Chapter 6.

Concept: Representativeness

Representativeness is a directional relation between two objects:

Model M ⟵ Event X

Type 2

X instance of class M.

Representativeness reflects degree of how central characteristics of X are for M. Does not necessarily reflect frequency.

Examples: a robin is a more representative but less frequent bird than a chicken), New York is more representative (a prototype) for an American city, though Cincinnati is more typical.

Prototypical elements of a category are better learned/recalled/ recognized, even if less frequent.

Representativeness can cause bias. Example: "_ing" versus "__n_").

Concept: Representativeness

Representativeness is a directional relation between two objects:

Model M ⟵ Event X

Type 3

X is a subset of class M.

Example: M=Warwick UG students.
Which of the below is most representative of M?
X=all Maths students, Y=all MORSE students, Z=all Econ students

Criteria for representativeness now includes not only central tendency, but also range and variability.

Example: Is X= "A doctor is also a father, brother, son…" This is not representative of the class M of roles of doctors that include both males and females.

Concept: Representativeness

Representativeness is a directional relation between two objects:

Model M ⟵ Event X

Type 4

X is potential consequence of causal system M.

Examples: M=economy, X=inflation rate.
M=pneumonia, X=high fever.

Here, representativeness is controlled by a *system of causal beliefs* (valid or not).

Concepts: Representativeness & probability

Evaluation of probability is a complex process including:

1. interpretation of the question
2. search for relevant information
3. algorithm combining the information

Representativeness may cause bias in Step 2.

Despite this, why do people still use this relationship to elicit subjective probabilities?

- accessible, easy to evaluate
- representativeness often correlates with probability
- people overestimate this correlation, though...

B is less typical for Linda than B&F, and B&F is less typical for Linda than F

Incorrect conclusion through analogy probably=representative:
B is less probable than B&F, and B&F is less probable than F

Definition: Heuristics

Heuristic strategies:

Shortcuts, tools, approximations used in

- probability judgement
- prediction
- decision making under uncertainty

Examples: Base rate neglect, availability heuristics, anchor effect, gambler's fallacy, hot hand…

More likely to occur with inhibited ability to construct correct answers. Potential reasons:

- lack of resources (information, time, attention, priority, ability, knowledge, memory capacity)
- inability to process information (emotions, intoxication)

May lead to biases and fallacies.

Heuristics & biases: Confirmation bias

Tendency to search for, interpret, or prioritise information in a way that confirms one's beliefs or hypotheses through

- gathering and remembering information selectively
- use of ambiguous evidence

Explanations:

- Wishful thinking
- Overconfidence in personal beliefs
- Personality types with strong preference for order avoiding cognitive dissonance/doubt

Implications:

- Cognitive and systematic error of inductive reasoning
- Stable models of the world, persistence of prejudices

Wason, Peter C. (1960), *On the failure to eliminate hypotheses in a conceptual task*, *Quarterly Journal of Experimental Psychology* (Psychology Press) **12** (3): 129–140,

Overview: Heuristics & biases in finance

A list of links of common biases in financial decision making with links to definitions and research papers has been compiled by ABFE (Academy of Behavioral Finance & Economics) at www.behaviouralfinance.net/

Introduction	History	Bibliography	Links	Glossary
BF or BS?	Important Publications	Mailing List	People	Universities

Adaptive Attitudes	Affect Heuristic	Aversion to Ambiguity	Anchoring (and Adjustment)	Attention Anomalies
Attribution Substitution Heuristic	Availability Heuristic	Barn Door Closing	Base Rate Neglect	Bid-Ask Spread
Book-to-market	Bubbles	Calendar Effects	Cascades	Causality Heuristic
Certainty Effect	Choosing by Default Heuristic	Choosing By Liking Heuristic	Closed end	Clustering
Cognitive Dissonance	Cognitive Diversity	Communal Reinforcement	Confirmation Bias	Conjunction Fallacy
Conservatism Bias	Contagions	Contrarian	Culture	Curse of knowledge
Daylight Saving Anomaly	Demand	Disappointment	Disjunction Effect	Disposition Effect
Dividends	Dotcom	Earnings	Endowment Effect	Equity premium
Event Selection	Evolutionary	Expected Utility Hypothesis	False Consensus	Favourite-Longshot Bias
Fear	Fluency Heuristic	Framing	Frequency Illusions	Fungibility
Gamblers Fallacy	Gambling/Speculation	Geomagnetic Storms	Glamour vs. Value	Global/Domestic
Growth	Halloween	Herding/Crowd	Heuristics	Hindsight Bias
Holidays	Hot Hand	Illusion of Control	Illusion of Knowledge	Illusion of Validity
Intraday Effects	Intramonth Effects	Irrelevance of History	January Barometer	January Effect
Law of Small Numbers	Loss Aversion	Loss Realization	Lunar	Magical Thinking
Mean Reversion	Mental Accounting	Mental Compartments	Momentum	Monday effect
Money Illusion	Mutual Funds	Noise	Optimism	Outrage Heuristic
Overconfidence	Overreaction	Over- and Underreaction	Persistence	Persuasion Effect
Pessimism and Doubt	Press Coverage	Price Reversals	Probability and Statistics	Prospect Theory
Prototype Heuristic	Quarterly Effects	Rational	Recognition Heuristic	Regret
Representativeness Heuristic	Reward Pursuit	Risk	Saving	Selective Thinking
Self Control	Self-attribution Bias	Self Deception	Sentiment	Serial Correlation
Similarity Heuristic	Size Effect	Solar	Sport	Status Quo Bias
Style	Sunk Cost	Super Bowl	Surprise Heuristic	Survival
Technical Analysis	Touchy-feely Syndrome	Trend	Uncertainty	Underreaction
Unpacking Effect	Value	Weather	Weekend Effect	Window Dressing
Winner's Curse				