ST222 2016 GAMES, DECISIONS AND BEHAVIOUR SHEET 2

(1) **Balls in bags.** A bag contains n_r red, n_b blue and n_g green balls in it. Specify the event space related to a single draw and its atoms. If the draw is made at random what are the probabilities of each of the atoms? Now add n_y yellow balls. What is the new event space and its atoms. What are the probabilities of the atoms in the new space? Explain how you would use the axioms of probability to specify the probabilities of other events in this second event space.

(2) **Client's believes.** A client tells you that for two disjoint events B and C, $P(B) + P(C) > P(B \cup C)$ but $0 \leq P(B) \leq P(C) \leq P(B \cup C) \leq 1$. Use the ball in bag method to construct a Dutch Book against your client's agent who is instructed to trade with these probabilities. Note that this together with the Dutch Book given in the notes demands that for any three elicited probabilities your client should always set probabilities such that $P(B) + P(C) = P(B \cup C)$ whenever B and C are disjoint.

(3) **Clinical trials.** A new treatment for a disease is being tested, to see whether it is better than the standard treatment. The existing treatment is effective on 50% of patients. It is believed initially that there is a 2/3 chance that the new treatment is effective on 75% of patients, and a 1/3 chance that the new treatment is effective on 50% of patients. In a pilot study, the new treatment is given to 20 patients selected at random, and turns out to be effective for 14 of them.

(a) Given this information, what is the probability that the new treatment is better than the standard treatment?

(b) A second study is done later, giving the new treatment to 20 new patients selected at random. Given the results of the first study, what is the PMF for how many of the new patients the new treatment will be effective on? *(Letting p be the answer to the previous question your answer can be left in terms of p.)*

(4) **Insurance policy.** Remember the insurance example described in lectures. In reality, most insurance policies have an excess. Consider the following decision problem. The policy costs c, your worldly goods are of value v and the insurance policy has an excess e (meaning that if you make a claim you receive an amount e less than the amount stolen; you may assume that $e < v/10$). The three possible outcomes are:

$$x_1 = \{\text{No thefts}\} \quad x_2 = \{\text{Small theft (loss } 0.1v)\} \quad x_3 = \{\text{Serious burglary (loss } v)\}$$

You may assume that the probabilities elicited in lectures still hold:

$$p(x_1) = 0.946 \qquad p(x_2) = 0.043 \qquad p(x_3) = 0.011$$

(a) Write down the loss function for this problem.

(b) Calculate the expected loss associated with each decision.

(c) Obtain inequalities in terms of c, e and v which describe the region of parameter space in which buying insurance is the optimal strategy in an EMV sense.

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(5) **Investing into a machine.** A manager must decide whether to lease a small or large machine for one year. The small machine will cost them £10,000; the large machine £30,000. The small machine can produce up to 500 units a month; the large machine 3000 units. If they find a distributor (an event which they believe has a probability p) then there will be a market for 2200 units per month over the year; otherwise just 400 units. Whichever machine they lease, they will manufacture precisely the number of units which they sell. The profit per unit sold is £5.

  (a) For what values of p is the optimal EMV solution to lease the small machine?
  (b) Assume the manager also has the option to indulge in industrial espionage. For £10,000 they can determine whether or not a distributor will be found in advance of making the decision. What is the new EMV strategy as a function of p?
  (c) Draw a decision tree for the previous problem.
  (d) If the espionage has a probability q of being successful if a distributor is found and probability r of being successful if they are not (i.e. with these probabilities, the espionage provides correct information and with probabilities $1 - q$, $1 - r$ the spy gets the wrong answer) then how does this change things?
    (i) Draw a new decision tree for the revised problem.
    (ii) What is the EMV strategy (note that the optimal strategy must depend upon the unknown probabilities)?

(6) **Maximise life expectation in medical treatment decision.** On his twentieth birthday a patient is brought in to a hospital with an illness which is either Type I (with probability 0.4) or Type II (with probability 0.6). Independent of the type of the illness, without treatment he will die on that day with probability 0.8 and otherwise survive and have normal life expectancy. The surgeon may take one of three possible courses of action:

- d_1 : not to treat the patient;
- d_2 : to give the patient drug L at once;
- d_3 : to operate on the patient at once.

She cannot both operate and administer the drug. Both operating and administering the drug are dangerous to the patient. Independent of the type of illness, operating will kill the patient with probability 0.5, while the drug will kill him with probability 0.2. Should the patient survive the effects of the drug, if he has Type I illness then it will cure him with probability 0.5 and otherwise they will die. If he has Type II illness then it will have no effect. Should the patient survive the effects of the operation, if he has Type II illness then it will cure him with probability 0.8 and otherwise they will die. If he has Type I illness then it will have no effect. In all cases survival will give the patient 70 years life expectancy (measured from birth).

  (a) Draw a decision tree to represent the surgeon's decision problem.
  (b) Calculate her best strategy assuming she wishes to maximize her patients life expectancy.

(7) (*Not examinable*) **Pasadena game.** Philosophers of probability have observed some strange goings-on in Pasadena. Similarly to the St Petersburg game, a fair coin is tossed until it lands heads for the first time. But the payoffs are less generous. In St Petersburg they were 2^n, where n is the number of trials to the first heads. In Pasadena, they reduced and on top that rewards take turns with losses: the payoff is $(-1)^{n-1}2^n/n$. (Assume money is linear in utility; if not replace currency unit with utiles.) How much would you play to play this game? What is the answer according to the EMV approach in decision theory? Is the question well-posed?
Note: Philosophers are still arguing about the Pasadena game among themselves. We will not post a solution to these questions, but suggest you compare your thoughts to theirs, e.g. in Alan Hájek and Harris Nover, "Perplexing Expectations", Mind 115 (July 2006) and Alan Hájek, "Unexpected Expectations", Mind 123 (April 2014).