ST222 2014 GAMES, DECISIONS AND BEHAVIOUR
EXERCISE SHEET 4

(1) Consider a game with the following payoff matrix:

	α	β	γ
a	(5,6)	(3,4)	(2,5)
b	(3,7)	(6,10)	(8,11)
c	(4,8)	(8,5)	(10,4)

Using iterated elimination of dominated strategies show that there is a single strategy which the players should, under the assumption that *rationality is common knowledge*, adopt deterministically.

(2) Consider a prisoner's trilemma in which, as well as the option of staying silent or betraying their partner, they can confess — admitting that both of them were involved in the crime. This leads to a payoff matrix:

	S	B	C
S	(-1,-1)	(-5,0)	(-5,-4)
B	(0,-5)	(-4,-4)	(-4,-4)
C	(-4,-5)	(-4,-4)	(-4,-4)

Using an argument involving domination and/or separability, explain what a rational player should do, and why.

(3) Consider a zero sum game with the following payoff matrix (for player 1; remember player 2 has payoffs corresponding to the negative of those of player 1 in a zero sum game):

	δ_1	δ_2	δ_3	δ_4	δ_5
d_1	0	5/6	1/2	5/6	1
d_2	1	1/2	3/4	3/4	3/4

(a) What is player 1's maximin mixed strategy? *You may find it helpful to use a graphical method.*
(b) What is player 2's maximin mixed strategy?
(c) What is the value of this game?

(4) Let M be the pay-off matrix for a zero sum game in which $D = \Delta$ and $|D| = |\Delta| = n$. Denote by $M((i_1, i_2), (j_1, j_2))$ the pay-off matrix obtained by permuting rows i_1 , i_2 and columns j_1 , j_2 in M . Suppose that for each relabelling of rows swapping i_1, i_2 there exists a relabelling of columns j_1, j_2 such that

$$M((i_1, i_2), (j_1, j_2)) = M.$$

(a) Show that if P1 has a unique maximin mixed strategy then it must be of the form $(1/n, ..., 1/n)$.
(b) What is the value of this game if $M = (M_{ij})$ and $M_{ij} = -M_{ji}$ for all i, j?