ST222 2017 GAMES, DECISIONS AND BEHAVIOUR SHEET 4

(1) **Ellsberg paradox.** There are some criticisms of utility-based decision making. This question illustrates one of them.

An urn contains 90 balls. 30 balls are red and the remainder are black or yellow. You do not know how many are black and how many are yellow.

In experiment 1 you are given a choice between two bets:

- d_1: Win £100 if a red ball is drawn.
- d_2: Win £100 if a black ball is drawn.

In experiment 2 you are given a choice between two other bets:

- d_1': Win £100 if a red or yellow ball is drawn.
- d_2': Win £100 if a black or yellow ball is drawn.

A friend asserts that they would prefer d_1 to d_2 but they would prefer d_2' to d_1' because in both cases this maximises a lower bound on their probability of winning.

Show that this is inconsistent with the expected utility approach to decision making. One way to do this is to show that no utility function could lead to this preference ordering.

(2) **Allais paradox.** In the lectures we already saw another example for how utility-based decision is not always consistent with people's behaviour. Consider the combination of lotteries proposed by Allais (*see Lecture notes Week 8 Friday and Week 9 Tuesday*).

(a) Show that no utility function can explain the preferences conjectured by Allais (and confirmed empirically for a majority of people). (To keep things shorter in the lecture, this set up in lecture under the additional assumption that $u(0) = 0$, but you can show it for the general case.)

(b) **(*)** Discuss that the preferences conjectured by Allais are incompatible with the independence axiom.

(3) **Separability.**

(a) Consider the following reward matrix in a zero-sum game:

$$\begin{pmatrix} \text{a} & \text{b} \\ \text{c} & \text{d} \end{pmatrix}$$

Determine under which conditions this game is separable. For the case where it is separable, determine the solution(s).

(b) Is the following zero-sum game separable?

$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

If yes, determine the solution(s).

(c) Is the following zero-sum game separable?

$$\begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}$$

If yes, determine the solution(s).

(d) Is the rock-paper-scissors game is a zero-sum game with payoff matrix

$$\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

(*see also Lecture notes Week 6 Friday at 4pm*). Is it separable? If yes, determine the solution(s).

(4) **Game with dominant strategies.** Consider a game with the following payoff matrix:

	α	β	γ
a	(5,6)	(3,4)	(2,5)
b	(3,7)	(6,10)	(8,11)
c	(4,8)	(8,5)	(10,4)

Using iterated elimination of dominated strategies show that there is a single strategy which the players should, under the assumption that *rationality is common knowledge*, adopt deterministically.

(5) **Prisoner's trilemma.** Consider a prisoner's trilemma in which, as well as the option of staying silent or betraying their partner, they can confess — admitting that both of them were involved in the crime. This leads to a payoff matrix:

	S	B	C
S	(-1,-1)	(-5,0)	(-5,-4)
B	(0,-5)	(-4,-4)	(-4,-4)
C	(-4,-5)	(-4,-4)	(-4,-4)

Using an argument involving domination and/or separability, explain what a rational player should do, and why.

(6) **Zero-sum game.** Consider a zero sum game with the following payoff matrix (for player 1; remember player 2 has payoffs corresponding to the negative of those of player 1 in a zero sum game):

	δ_1	δ_2	δ_3	δ_4	δ_5
d_1	0	1	5	6	8
d_2	10	6	5	2	3

(a) What is player 1's maximin mixed strategy? *You may find it helpful to use a graphical method.*

(b) What is player 2's maximin mixed strategy?

(c) What is the value of this game?

(7) **(*) Plus one.** Each player chooses a number from $\{1, 2, ..., n\}$ and writes it down on a piece of paper; then the players compare the two numbers. If the numbers differ by one, the player with the higher number wins \$1 from the other player. If the players? choices differ by two or more, the player with the higher number pays \$2 to the other player. In the event of a tie, no money changes hands.

(a) What is the payoff matrix of the game?
(b) Use the concept of domination to reduce the game to a 3 by 3 game.
(c) Solve the reduced game.

(8) **(*) Row and column swapping.** Let M be the pay-off matrix for a zero sum game in which $D = \Delta$ and $|D| = |\Delta| = n$. Denote by $M((i_1, i_2), (j_1, j_2))$ the pay-off matrix obtained by permuting rows i_1 , i_2 and columns j_1 , j_2 in M . Suppose that for each relabelling of rows swapping i_1, i_2 there exists a relabelling of columns j_1, j_2 such that

$$M((i_1, i_2), (j_1, j_2)) = M.$$

(a) Show that if P1 has a unique maximin mixed strategy then it must be of the form $(1/n, ..., 1/n)$.
(b) What is the value of this game if $M = (M_{ij})$ and $M_{ij} = -M_{ji}$ for all i, j?