ST222 2017 GAMES, DECISIONS AND BEHAVIOUR
EXERCISE SHEET 5

(1) **Runs in random sequences.** In the lecture, we calculated the expected number of runs in sequences of fair coin tosses. This exercises asks the same question, but without assuming the coin is fair.

Model a sequence of n fair coin tosses by a sequence of independent random variables X_i ($i = 1, \ldots, n$) with values in $\{0, 1\}$ and with $P(X_i = 1) = p$ and $P(X_i = 0) = 1 - p = q$ for all $i = 1, \ldots, n$.

Let Z_r be the number of runs of heads or of tails of length r in n tosses of the coin. Calculate the expectation of Z_r.

(2) **Error rates in terrorist identification.** (Somewhat based on Wikipedia entry about *base rate fallacy.*) In a city with N inhabitants let there be k terrorists and $N - k$ non-terrorists. For simplification it is assumed that all people present in the city are inhabitants. Thus, the base rate probability of a randomly selected inhabitant of the city being a terrorist is $\pi = k/N$.

In an attempt to catch the terrorists, the city installs an alarm system with a surveillance camera and automatic facial recognition software.

Let p be the probability that the camera sends an alarm when it scans a terrorist (*sensitivity*). Let q be the probability that the camera does not send an alarm when it scans a non-terrorist (*specificity*).

The software can fail in two ways:

- *False negative:* the system does not send an alarm despite the individual being scanned is a terrorist
- *False positive:* the system sends an alarm, despite the individual being scanned is not a terrorist

The false negative rate α and the false positive rate β are the probabilities associated with these events.

Answer the following questions in the context of this alarm system in this city.

(a) The company that sells the software claims says that $p = 0.95$ and $q = 0.99$ and hence the false negative rate is 5% and the false positive rate is 1%. Is this true? Can you say which are the correct percentages? Discuss how the company may have derived these percentages.

(b) Derive formulas for α and for β.

(c) What are α and β for $k = 0$? First explain what the answer should be intuitively and then calculate it.

(d) What are α and β for $k = N$? First explain what the answer should be intuitively and then calculate it.

(e) Assume the sensitivity p is 95% and the specificity q is 0.99 and that the city in question has 100 terrorists among 1 million inhabitants. Calculate α and β. Comment on the numbers you obtained.

(f) (*Optional*) If you have a computer with R (or another suitable other programme), write some code to do the previous exercise for a range of values of k.

(g) Give an lower bound for q as a function of p and π such that β is at most 1%. Calculate the threshold explicitly for $p = 0.95, N = 1,000,000$ and $k = 100$.

(h) (*Optional*) If you have a computer with R (or another suitable other programme), write some code to draw the false negative rate β as functions of the specificity q with fixed sensitivity $p = 0.95$ and fixed prevalence $\pi = 0.0001$. Show a few such curves for different prevalences between $\pi = 0.01$ and $\pi = 0.000001$. Interpret the results.

(i) (*Optional*) If you have a computer with R (or another suitable other programme), write some code to draw the false positive rate β as functions of the prevalence for fixed sensitivity $p = 0.95$ fan fixed specificity $q = 0.999$. Show a few such curves for different specificities between $q = 0.9$ and $q = 0.99999$. Interpret the results.

(3) **Empirical study on affect heuristic.** You want to empirically study the *affect heuristic* in the context of how people perceive and evaluate risk. Here is some scientific background:

> "Humans perceive and act on risk in two fundamental ways. Risk as feelings refers to individuals' instinctive and intuitive reactions to danger. Risk as analysis brings logic, reason, and scientific deliberation to bear on risk management. Reliance on risk as feelings is described as the affect heuristic."

Source: Paul Slovic and Ellen Peters, Risk Perception and Affect, Current Directions in Psychological Science, Vol. 15, No. 6, pp. 322-325, 2006.

You have the opportunity to collect data in 8 classes of young people between 13 and 15 at your old secondary school. You have 30 minutes of class time, a set-up for showing videos and tablet computers for each students are available.

Design an experiment to compare how students perception and evaluation of risk depends on their mood. Your experiment should start with an exposure to risk that is supposed to impact the subjects feelings one way or another, followed by assessing their perception and evaluation of risk through questions or tasks.

Explain the set-up of the experiment and give reasons for your choices. Acknowledge potential difficulties and limitations of the design (e.g. logistical challenges, biases, variation between subjects, questions regarding conclusions for a general population etc).

(4) **Properties of the probability weighting function.** In response to empirical findings dismissing *homo economics* as a model organism for human behaviour, Kahneman and Tversky have proposed a revised version of (subjective) expected utility theory. Prospect theory models decision making under uncertainty using a probability distortion function w and certain types of utility function asymmetric with respect to a reference point. Kahneman and Tversky defined the probability distortion function

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}} \qquad (p \in [0,1])$$

for parameters $\gamma > 0$.

(a) Show that w has the *subcertainty property,* that is,

$$w(p) + w(1-p) < 1 \text{ for all } p \in (0,1).$$

(b) For commonly used values of γ, w is monotone. However, this is not the case for small γ. Using analysis and graphs (or numerical methods), determine *approximately* for which γ the function w is monotone.

(5) **Medical treatment choice with additional option.** In a survey (Redelmeier DA, Shafir E. Medical Decision Making in Situations That Offer Multiple Alternatives. JAMA 1995) of a random sample of 373 family physicians in Ontario, two almost identical patient descriptions were used in a decision task. The descriptions are printed below with differences marked in bold. The physicians were randomly assigned to one of the two groups.

Group A: *The patient is a 67-year-old farmer with chronic right hip pain. The diagnosis is osteoarthritis. You have tried several nonsteroidal anti-inflammatory agents (e.g., aspirin, naproxen, and ketoprofen) and have stopped them because of either adverse effects or lack of efficacy. You decide to refer him to an orthopedic consultant for consideration for hip replacement surgery. The patient agrees to this plan.*
Before sending him away, however, you check the drug formulary and find that there is one nonsteroidal medication that this patient has not tried (ibuprofen). What do you do?

Group B: *The patient is a 67-year-old farmer with chronic right hip pain. The diagnosis is osteoarthritis. You have tried several nonsteroidal anti-inflammatory agents (e.g., aspirin, naproxen, and ketoprofen) and have stopped them because of either adverse effects or lack of efficacy. You decide to refer him to an orthopedic consultant for consideration for hip replacement surgery. The patient agrees to this plan.*
Before sending him away, however, you check the drug formulary and find that there are two nonsteroidal medications that this patient has not tried (ibuprofen and piroxicam). What do you do?

In Group A, 53% chose the default option of proceeding to surgery, and in Group B it were 72%. (Running standard statistical tests shows a statistically significant difference. The response rate in this study was 77%, so the results are meaningful.) Give a possible explanation for the difference. Use the terminology introduced in lecture.

Hint: Check in the lecture slides for experiments creating a similar decision situation and discuss the physicians in this study may behave along lines.

(6) **Choices involving different types of risk.** Consider the following lab experiment. Fifty-six undergraduates were given the following question:

You have two lotteries to win \$250. One offers a 5% chance to win the prize and the other offers a 30% chance to win the prize.
A: You can improve the chances of winning the first lottery from 5 to 10%.
B: You can improve the chances of winning the second lottery from 30 to 35%.
Which of these two improvements, or increases, seems like a more significant change?

The majority of respondents (75%) viewed option A as the more significant improvement.

The same respondents were also given a different question.

You have two lotteries to win \$250. One offers a 65% chance to win the prize and the other offers a 90% chance to win the prize.
C: You can improve the chances of winning the first lottery from 65 to 70%.
D: You can improve the chances of winning the second lottery from 90 to 95%.
Which of these two improvements, or increases, seems like a more significant change?

In the second question, only 37% of the participants viewed option C as a more significant improvement.

(a) Give reasons why respondents chose option A over B.
(b) Give reasons why respondents chose option D over C.
(c) Explain how this apparent incoherence can be explained.