Example *) *A patient has eye disease x_1 if unable to distinguish 2 kinds of image. Before experimetation prob. a patient has this disease is p A patient is presented with n such images. Given the patient has the disease x_1 or not - atom x_2 - the n events of the success of her guess will be independent. The prob. a normal person classiifes any of these n images correctly is 0.8.& for a diseased person 0.5.Immediate treatment can be given (d_1) at a cost of £1,000 whether or not she has the disease. Alternatively treatment can be deferred (d_2) and if $X = x_2$ there will be no cost . On the other hand if treatment delayed when $X = x_1$ treatment cost will be £10,000 .Calculate the doctor's EMV strategy as a function of p, and the no. r out of n correctly classified images.*

Answer

- Let R be the number of correctly classified images. Then given $X = x_1$, $R \backsim Bi(n, 0.5)$ has binomial distribution whose pmf

  $$p(r|x_1) = \binom{n}{r}(0.5)^r(0.5)^{n-r}$$

  If $X = x_2$ $R \backsim Bi(n, 0.8)$,

  $$p(r|x_2) = \binom{n}{r}(0.8)^r(0.2)^{n-r}$$

*) from Jim Smith' lecture notes

- $L(d_1)$ does not depend on X so $\overline{L}(d_1) = £1,000$.

- OTOH

  $$\overline{L}(d_2) = 10,000 \times p(x_1|r) + 0 \times p(x_2|r) = 10,000 \times p(x_1|r)$$

Question How do we calculate $p(x_1|r)$?

Answer Use Bayes Rule!

$$p(x_1|r) = \frac{\binom{n}{r}(0.5)^n p}{\binom{n}{r}\left\{(0.8)^r(0.2)^{n-r}(1-p) + (0.5)^n p\right\}} = \frac{5^n p}{5^n p + 8^r 2^{n-r}(1-p)}$$

$\overline{L}(d_2) \leq \overline{L}(d_1) \Leftrightarrow p(x_1|r) \leq 0.1 \Leftrightarrow$ so delay if

$$10\frac{5^n p}{5^n p + 8^r 2^{n-r}(1-p)} \leq 1$$

$$\begin{aligned} 10(5^n p) &\leq 5^n p + 8^r 2^{n-r}(1-p) \\ 9(5^n p) &\leq 8^r 2^{n-r}(1-p) \end{aligned}$$

$$\begin{aligned} 9\left(\left(\frac{5}{2}\right)^n \frac{p}{1-p}\right) &\leq 4^r \\ (\log 4)^{-1}\left\{\log 9 + n\log 2.5 + \log\frac{p}{1-p}\right\} &\leq r \end{aligned}$$