L1

Ex 1. Job offers

Choose between two jobs, 10h per week,

A: £8/h for 10 weeks

B: £7/h for 2 weeks and for the next 8 weeks £10/h if employer is very satisfied, otherwise stay at £7/h.

* What is better?

A: No uncertainty. $£8/h \cdot 10h \cdot 10 = £800.$

B: Best case: $£7/h \cdot 10h \cdot 2 + £10/h \cdot 10h \cdot 8$
$= £140 + £800 = \underline{£940}$

Worst case: $£7/h \cdot 10 \cdot 10 = \underline{£700}$

More refined?

p = proba employer is very satisfied

B: expect $\underbrace{£7/h \cdot 10h \cdot 10}_{\text{get this either way}} + p \cdot \underbrace{£3/h}_{\text{extra}} \cdot 10h \cdot 8$

$= £700 + p£240 =: (*)$

Risk B worth it $\Leftrightarrow (*) \geq £800$ (from A)

$\Leftrightarrow p \geq \frac{£100}{£240}$

$\Leftrightarrow p \geq 5/12$ (≈ 0.42)

L1

Ex: Sport betting

Game with teams A and B one of which will win.

- Kim believes A will win
  How strongly?
  Elicitation with a bet: $P_{Kim}(A \text{ wins})$
- Cam believes B will win
  $P_{Cam}(A \text{ wins}) \neq P_{Kim}(A \text{ wins})$
  Proba is subjective
- Tom likes them both.
  He tells Kim $P_{Tom}(A \text{ wins}) = 0.7$
  He tells Cam $P_{Tom}(B \text{ wins}) = 0.6$
- Kim and Cam talk ...
  Kim offers Tom bet $b(£1, A) = \begin{cases} £1 & A \text{ wins} \\ £0 & A \text{ doesn't win} \end{cases}$
  for the price of £0.7
  Cam offers Tom $b(£1, B)$ for £0.6

* What happens to Tom?
A wins: Tom has $£1 - £0.7 - £0.6 = \underline{-£0.3}$
B wins: same result

* What happens to Kim & Cam?
A wins: $-£1 + £0.7 + £0.6 = \underline{£0.3}$
B wins: same result

→

Kim & Cam : Sure wins
Tom : Sure loss
"Dutch book"

Ex : Medical treatment (non-emergency)

Choices : do nothing, treatment; more invasive treatment etc

Outcomes : full recovery, complications, death etc

Nothing is certain, outcomes have probabilities

Ex for this in clinical practice :

- vaccination
- surgery to stop chronic pain
- cosmetic surgery
- adjuvant chemotherapy

L1

Ex St Petersburg game

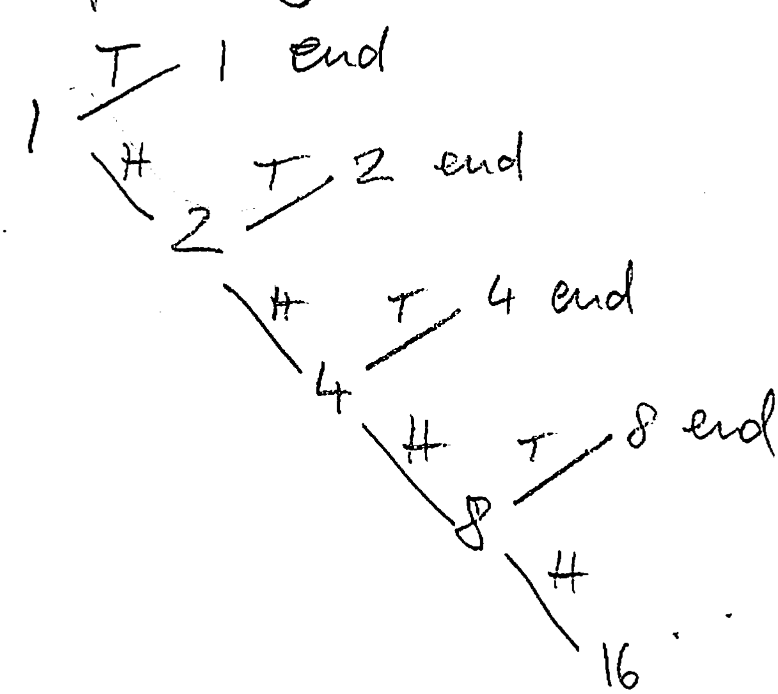
in pot: 1

toss coin if heads double pot, continue game

if tails cash out, end game

How much would you willing to pay to play?

Expected gain?



assume: fair coin

$$\frac{1}{2}\cdot 1+\left(\frac{1}{2}\right)^2 2+\left(\frac{1}{2}\right)^3\cdot 4+\left(\frac{1}{2}\right)^4\cdot 8+\ldots$$

$$=\sum_{i=1}^{\infty}\left(\frac{1}{2}\right)^i 2^{i-1}=\frac{1}{2}\sum_{i=0}^{\infty}\left(\frac{1}{2}\right)^i 2^i=\frac{1}{2}\cdot\sum_{i=0}^{\infty}1=\infty$$

Would you pay any amount of money to play?

Why not?

$\rightsquigarrow$ utility

L1

Ex: Rock - Paper - Scissors (Part II of ST222)

Show game with hands

- Mathem. representation?

Matices!

$$\begin{array}{cc|ccc} & & & \text{Player 2} & \\ & & R & P & S \\ \hline & R & 0 & -1 & 1 \\ \text{Player 1} & P & 1 & 0 & -1 \\ & S & -1 & 1 & 0 \end{array} = M_1 \quad \text{Player 1's pay-off}$$

$\neq$ " 2's " ? $= -M_1$

- Advantages? → zero-sum game
- Are there superior moves?
- In which sense?

→ dominant moves, seperability ...

Ex: Monte Carlo 18.8.1913 (Part III of ST222)

- Roulette game produced 26 times black in a row
- Proba for this is very small $(18/37)^{26}$
- Gambler's lost scores betting against black

"Gambler's fallacy": Belief that after a streak of one kind of outcome the other outcome becomes more likely."

More about this and other fallacies in Week 8; 9.