Ex Eye disease & screening

X disease status $\quad x_0$ = no disease

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\; x_1$ = has/will get disease

D decision $\quad d_1$: wait & watch $\quad L(d_1, x_0) = £0$

$\quad\quad\quad\quad\quad\quad d_2$: treat all now $\quad L(d_1, x_1) = £10K$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad L(d_2, \cdot) = £1K$

$p = P(X = x_1)$

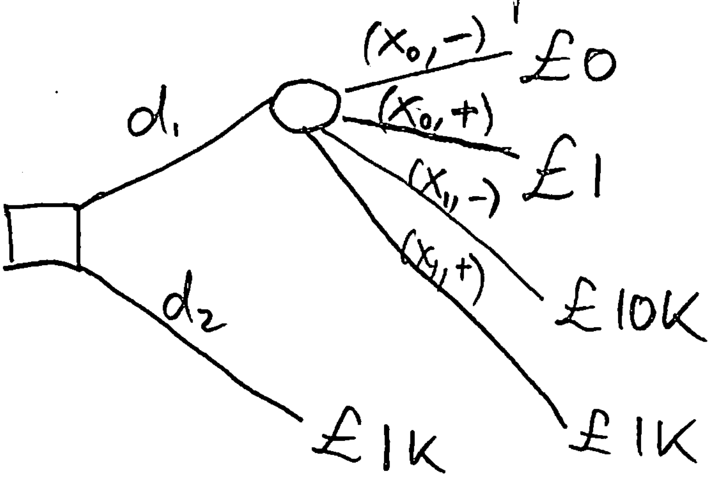
EMV $\quad d_1$ better $\Leftrightarrow p < 1/10$ $\quad$ (see last week)

More personalised approach, if there is a screening test

T test result $\quad$ + = positive test $\quad$ (predicting $X = x_1$)

$\quad\quad\quad\quad\quad\quad\quad\quad$ − = negative test $\quad$ (predicting $X = x_0$)

Tree now more complex:



$\Omega = \{x_0, x_1\} \times \{+, -\}$, $\quad \mathcal{F} = \mathcal{P}(\Omega)$, $\quad P$?

$$P(x_0, -) = P(T = - \mid X = x_0) P(X = x_0)$$
$$P(x_0, +) = P(T = + \mid X = x_0) P(X = x_0)$$
$$P(x_1, -) = P(T = - \mid X = x_1) P(X = x_1)$$
$$P(x_1, +) = P(T = + \mid X = x_1) P(X = x_1)$$

test char. $\quad$ ✓

$P(T = - \mid X = x_0) = q_0$

$P(T = + \mid X = x_1) = q_1$

$P(X = x_1) = p$

W4 - L1

$$\mathcal{L}(d_2) = £1K$$

$$\mathcal{L}(d_1) = \underbrace{q_0 \cdot (1-p) \cdot £0}_{\text{perfect}} + \underbrace{(1-q_0)\cdot(1-p)\cdot £1K}_{\text{overspent (unnecessary)}}$$
$$+ \underbrace{(1-q_1)\cdot p \cdot £10K}_{\text{bad (init. missed)}} + \underbrace{q_1 \cdot p \cdot £1K}_{\text{perfect}}$$

d_1 or d_2 preferred depends on p, q_0, q_1

Limitations of EMV approach / Alternatives

Example Farmer asks

Which crop should I plant given profit predictions? (harvest)

	Weather good	fair	bad	
Crop A	11	1	-3	"risky"
Crop B	7	5	0	
Crop C	2	2	2	"robust"

Answer depends on risk attitude.

Optimist: Expect best conditions and choose best option then

	good
A	11
B	7
C	2

$\Rightarrow$ plant A

Pessimist: Expect worst conditions and choose best option then

	bad
A	-3
B	0
C	2

$\Rightarrow$ plant C

W4-L1

Discussion:

- doesn't need proba for outcomes
- could even accommodate qualitative rewards as long as ordered ("a lot", "nothing")
- allows subjective approaches (optimist, pessimist)
- driven by extremes, even if very unlikely
- instable (adding different option, even with very small proba, can entire change decision)

Now develop a revised approach of EMV rule that incorporates some of the above without giving up the advantages of a quantitative approach.

Ex 5 St Peterburg's game

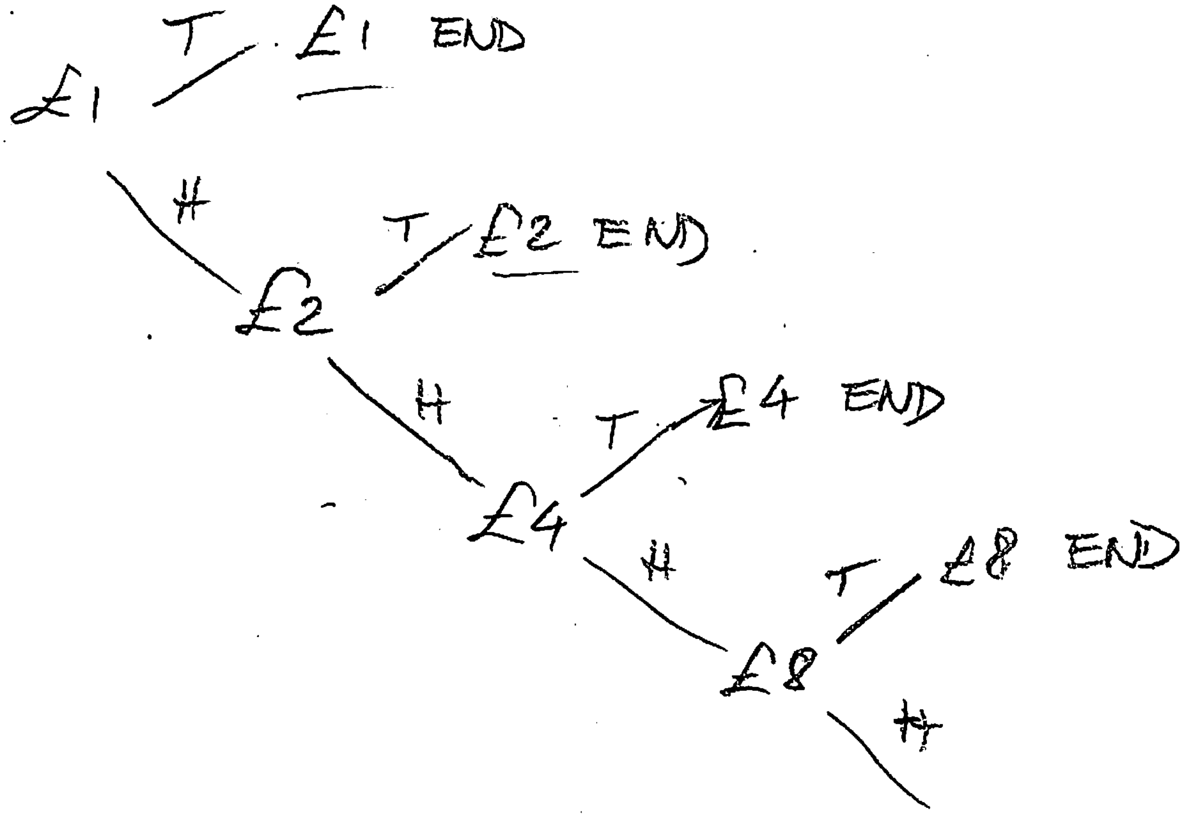
£1 in pot

If coin shows heads double pot

and if tails cash out (and end game)

How much would you be willing to pay to play?

Expected gain?



$$\frac{1}{2}\cdot £1 + \frac{1}{2}\cdot\frac{1}{2}\cdot £2 + \left(\frac{1}{2}\right)^3\cdot £4 + \ldots \; !$$

$$= \sum_{i=1}^{\infty}\left(\frac{1}{2}\right)^{i+1}\underbrace{£2^i}_{\text{replace by } u(\ldots)} = \frac{1}{2}\sum_{i=1}^{\infty}\left(\frac{1}{2}\right)^i £2^i = \frac{1}{2}\sum_{i=1}^{\infty} £1 = £\infty$$

Would you pay £∞ to play this game?!

Or any finite amount? Why not?

Utility changes (non-linearly)