ST222 Week 4 - LECTURE 2

- Example Oil drilling: model, tree, EMV solution
- Value of Information
- Value of information in oil drilling example
- Various decision tasks in product development, agriculture and everyday live

EXAMPLE: OIL DRILLING

Relates to lecture notes, Section 5.3

Decision options and rewards

- You may drill (at a cost of £31M) in one of two sites: field A and field B.
  - If there is oil in site A it will be worth £77M.
  - If there is oil in site B it will be worth £195M.
- Or you may conduct preliminary trials in either field at a cost of £6M.
- Or you can do nothing. This is free.

This gives a set of 5 decisions to make immediately. If you investigate site A or B you must then, further, decide whether to drill there, in the other site or not at all (we'll make things simpler by neglecting the possibility of investigating both).

Subjective probabilities

- The probability that there is oil in field A is 0.4.
- The probability that there is oil in field B is 0.2.
- If oil is present in a field, investigation will advise drilling with probability 0.8.
- If oil is not present, investigation will advise drilling with probability 0.2.

Subjective probabilities

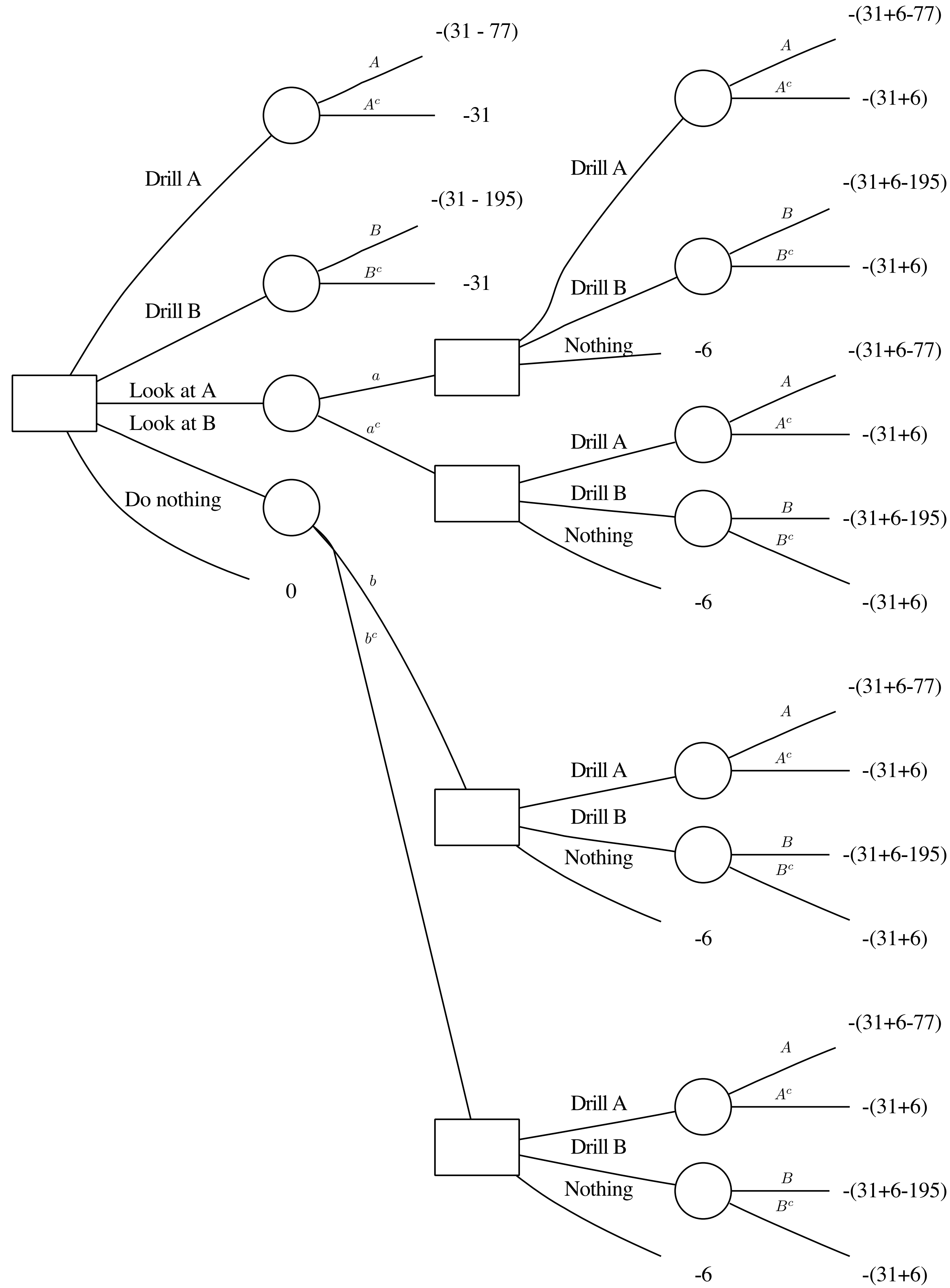
- The probability that there is oil in field A is 0.4.
- The probability that there is oil in field B is 0.2.
- If oil is present in a field, investigation will advise drilling with probability 0.8.
  event: a
- If oil is not present, investigation will advise drilling with probability 0.2.
  event: b

In short:

- $\mathbb{P}(A) = 0.4$
- $\mathbb{P}(B) = 0.2$
- $\mathbb{P}(a|A) = \mathbb{P}(b|B) = 0.8$
- $\mathbb{P}(a|A^c) = \mathbb{P}(b|B^c) = 0.2$

FULL TREE FOR OIL DRILLING

Relates to lecture notes, Section 5.3

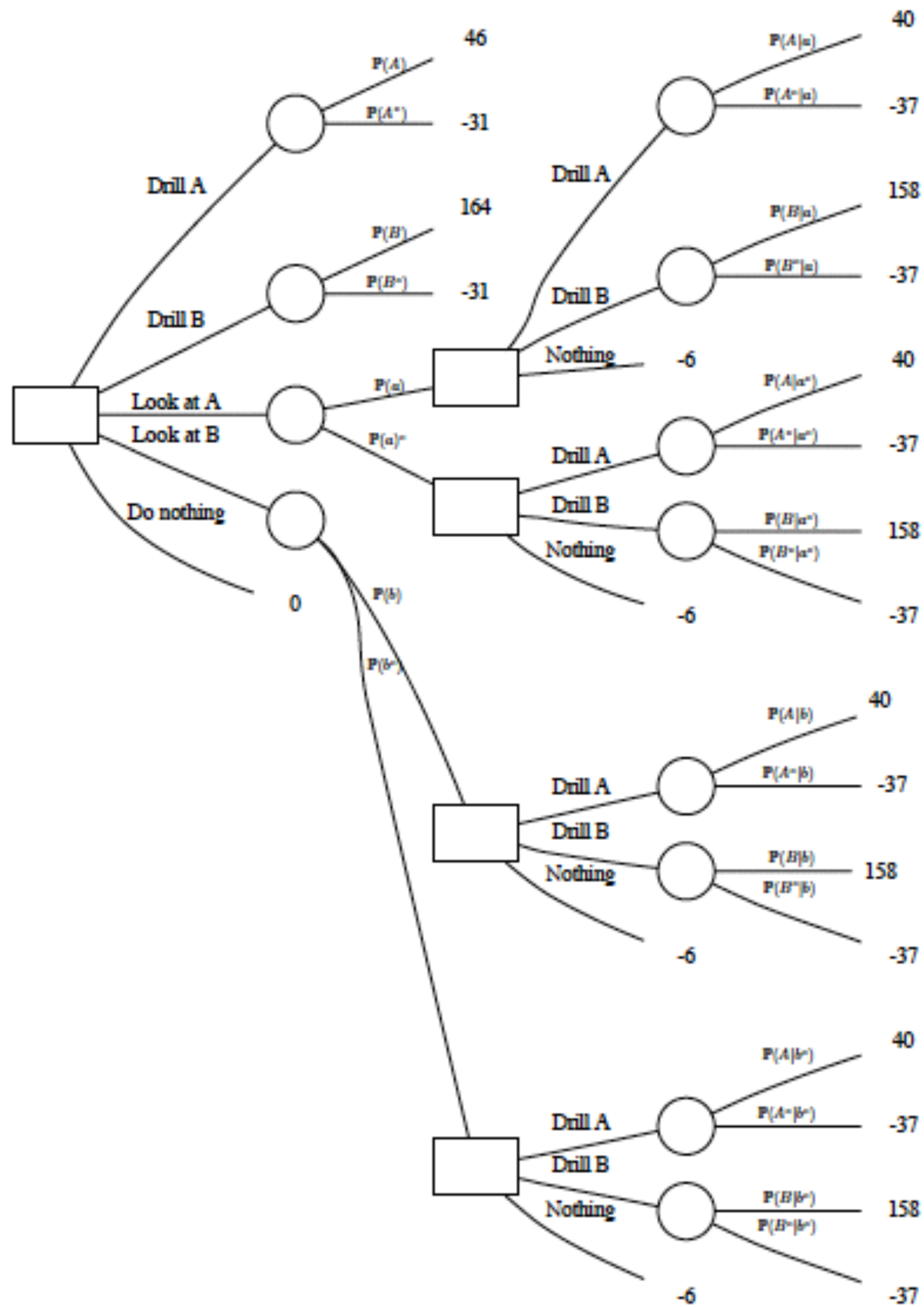


FULL TREE FOR OIL DRILLING

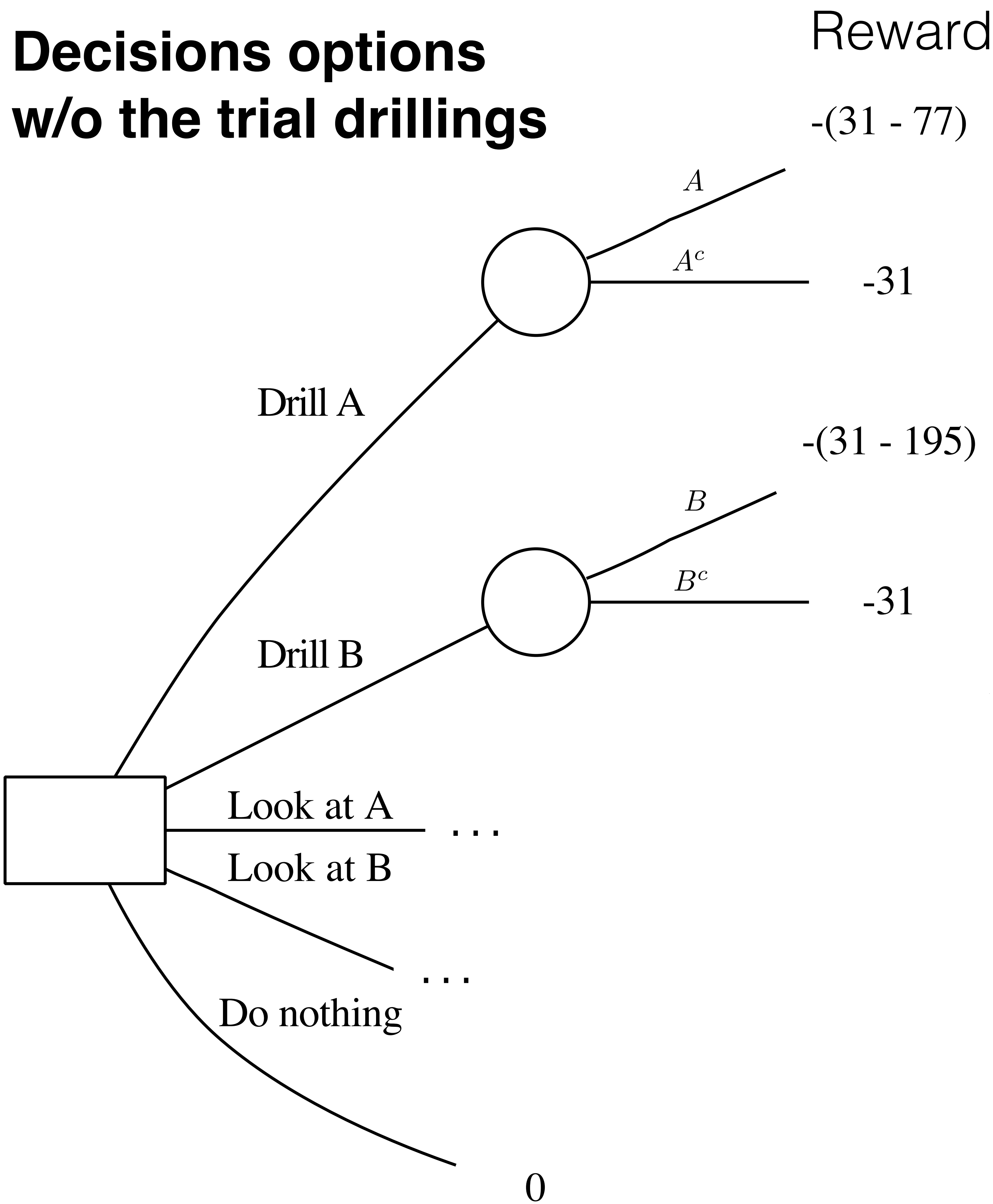
Relates to lecture notes, Section 5.3

Tree with probabilities

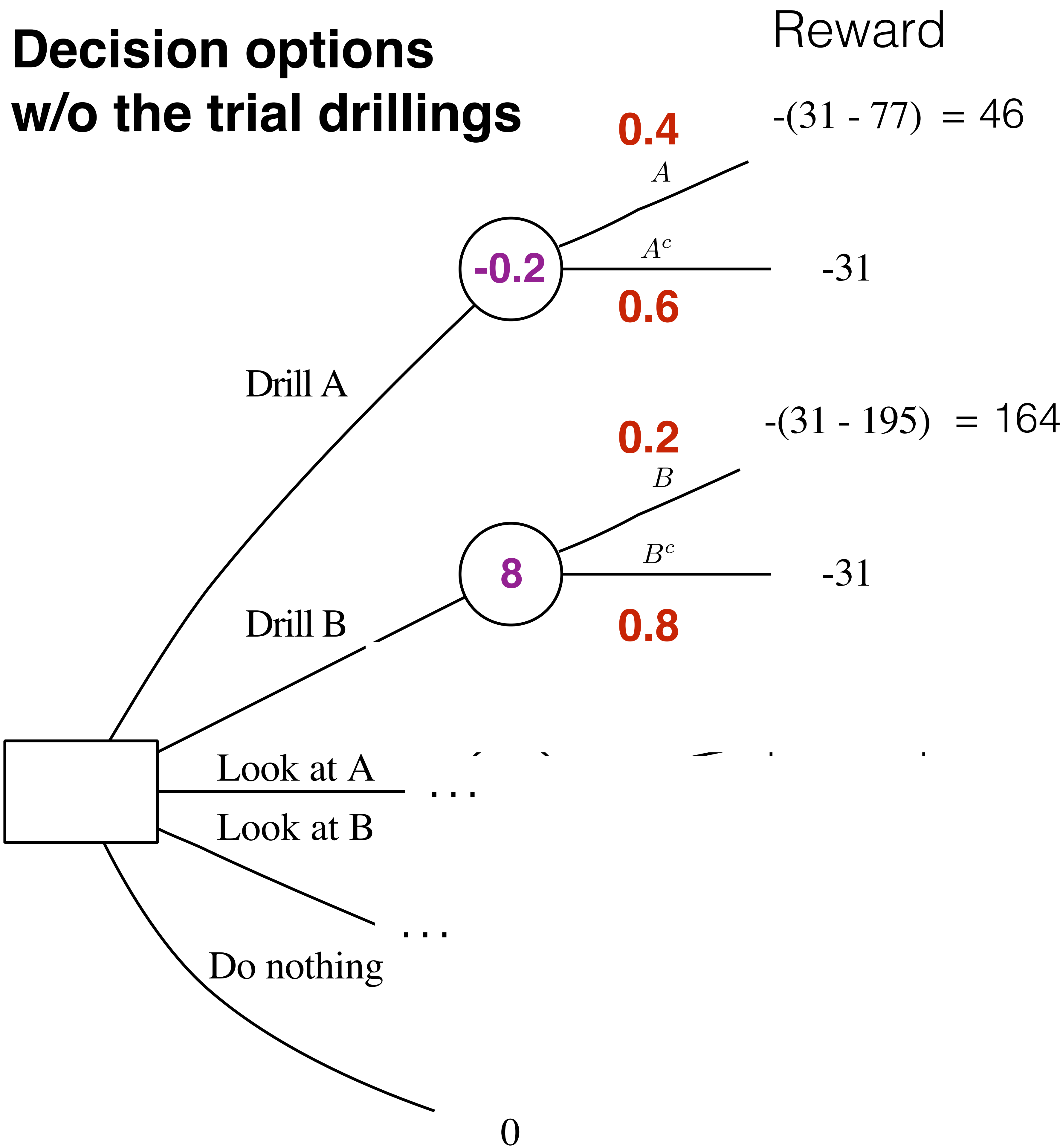
Now, calculate this step by step…



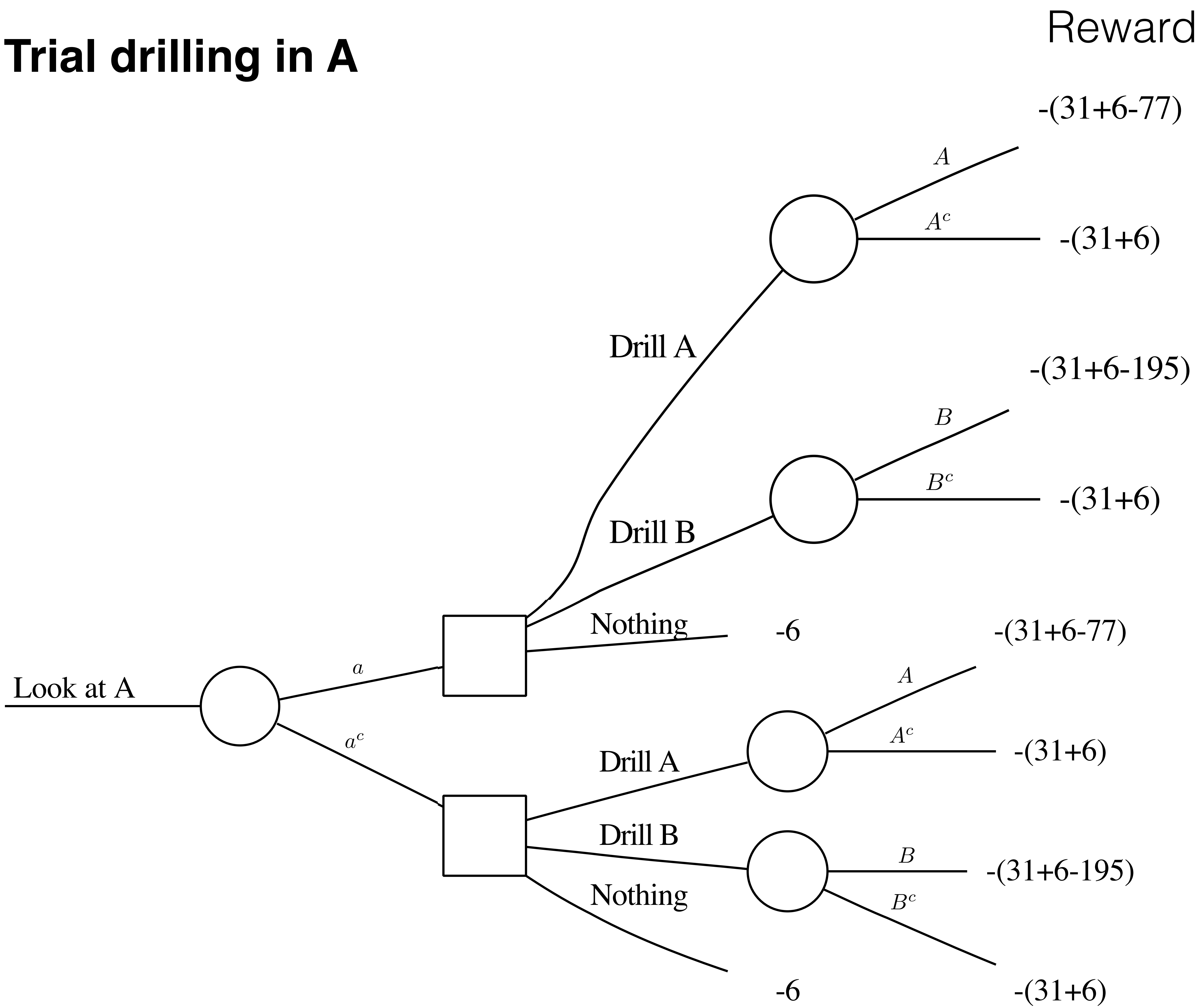
Decisions options w/o the trial drillings



Decision options w/o the trial drillings

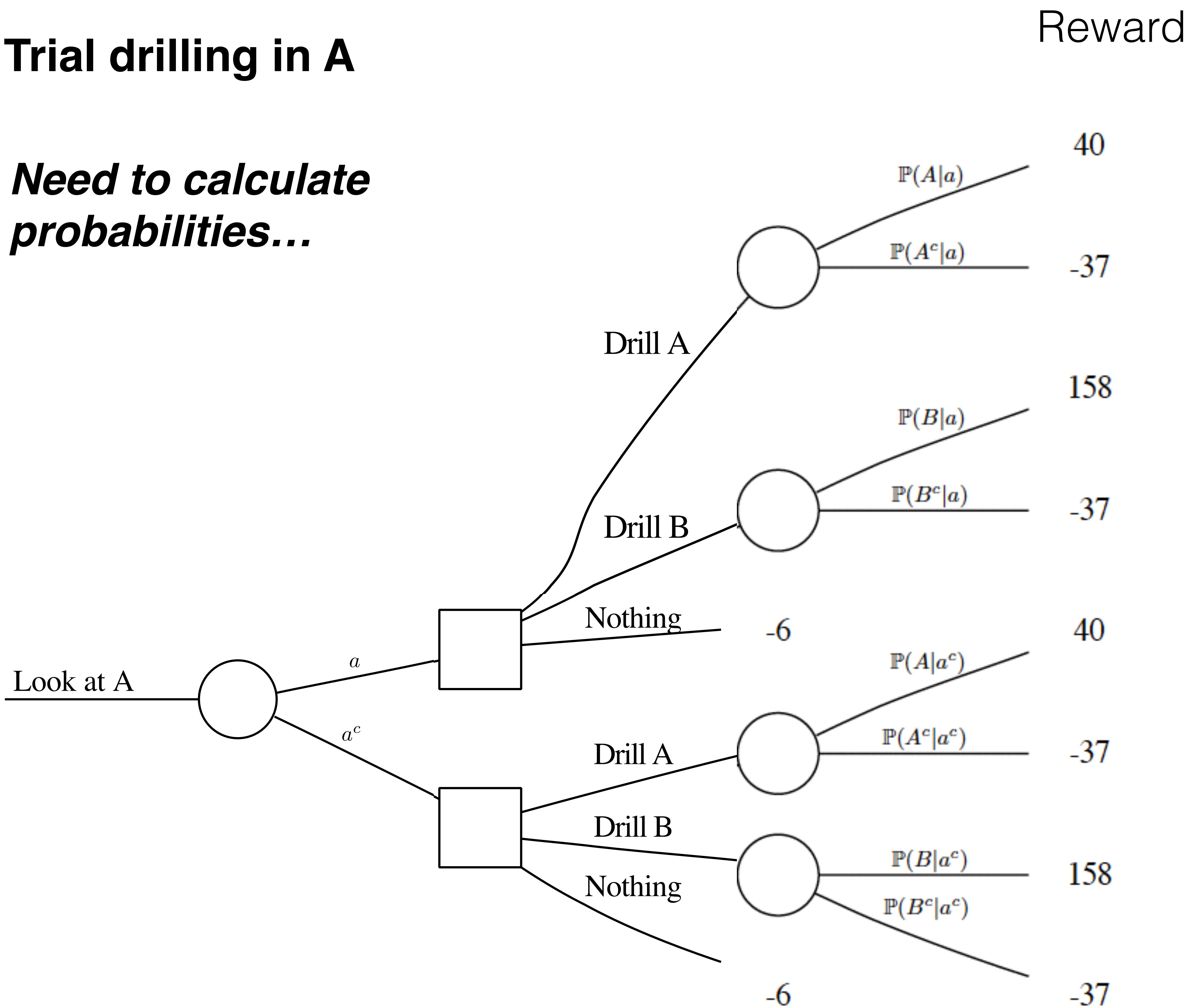


Trial drilling in A



Trial drilling in A

Need to calculate probabilities…



Total probability:

$$\mathbb{P}(a) = \mathbb{P}(a|A)\mathbb{P}(A) + \mathbb{P}(a|A^c)\mathbb{P}(A^c)$$
$$= 0.8 \times 0.4 + 0.2 \times 0.6 = 0.44$$

Bayes rule:

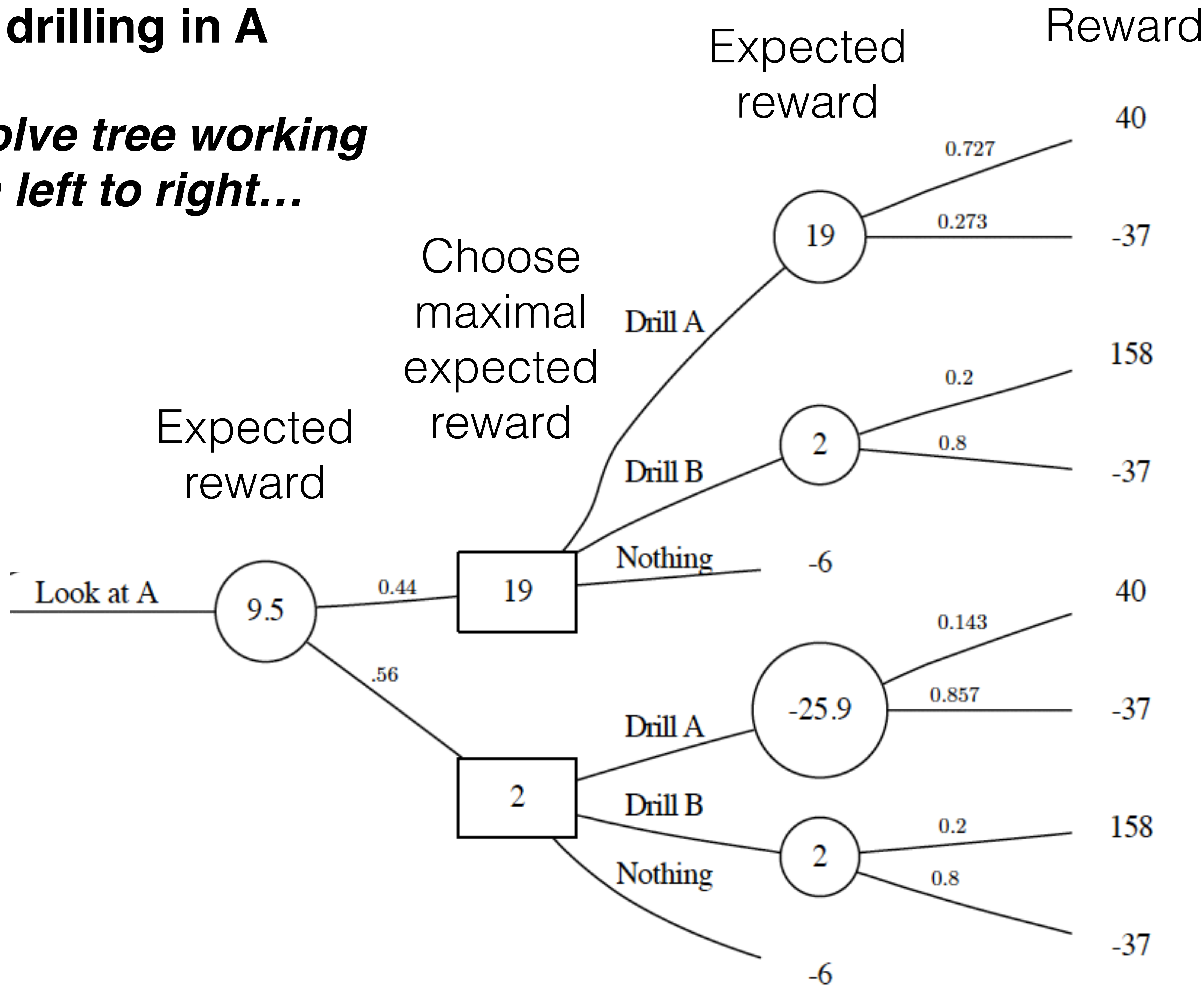
$$\mathbb{P}(A|a) = \frac{\mathbb{P}(a|A)\mathbb{P}(A)}{\mathbb{P}(a|A)\mathbb{P}(A) + \mathbb{P}(a|A^c)\mathbb{P}(A^c)}$$
$$= \frac{0.8 \times 0.4}{0.8 \times 0.4 + 0.2 \times 0.6} = 0.727$$

In short:

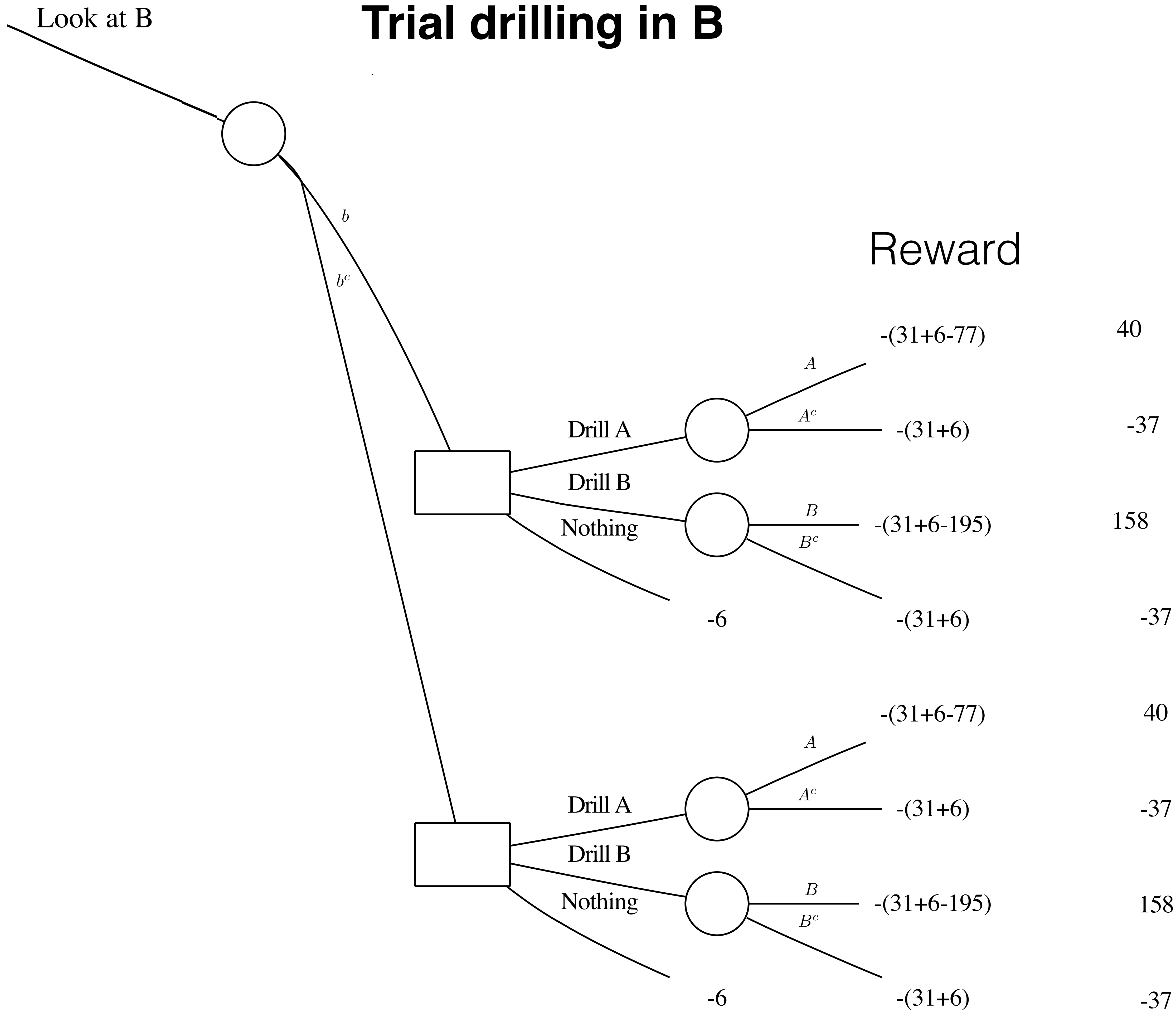
- $\mathbb{P}(A) = 0.4$
- $\mathbb{P}(B) = 0.2$
- $\mathbb{P}(a|A) = \mathbb{P}(b|B) = 0.8$
- $\mathbb{P}(a|A^c) = \mathbb{P}(b|B^c) = 0.2$

Trial drilling in A

Resolve tree working from left to right…



Trial drilling in B



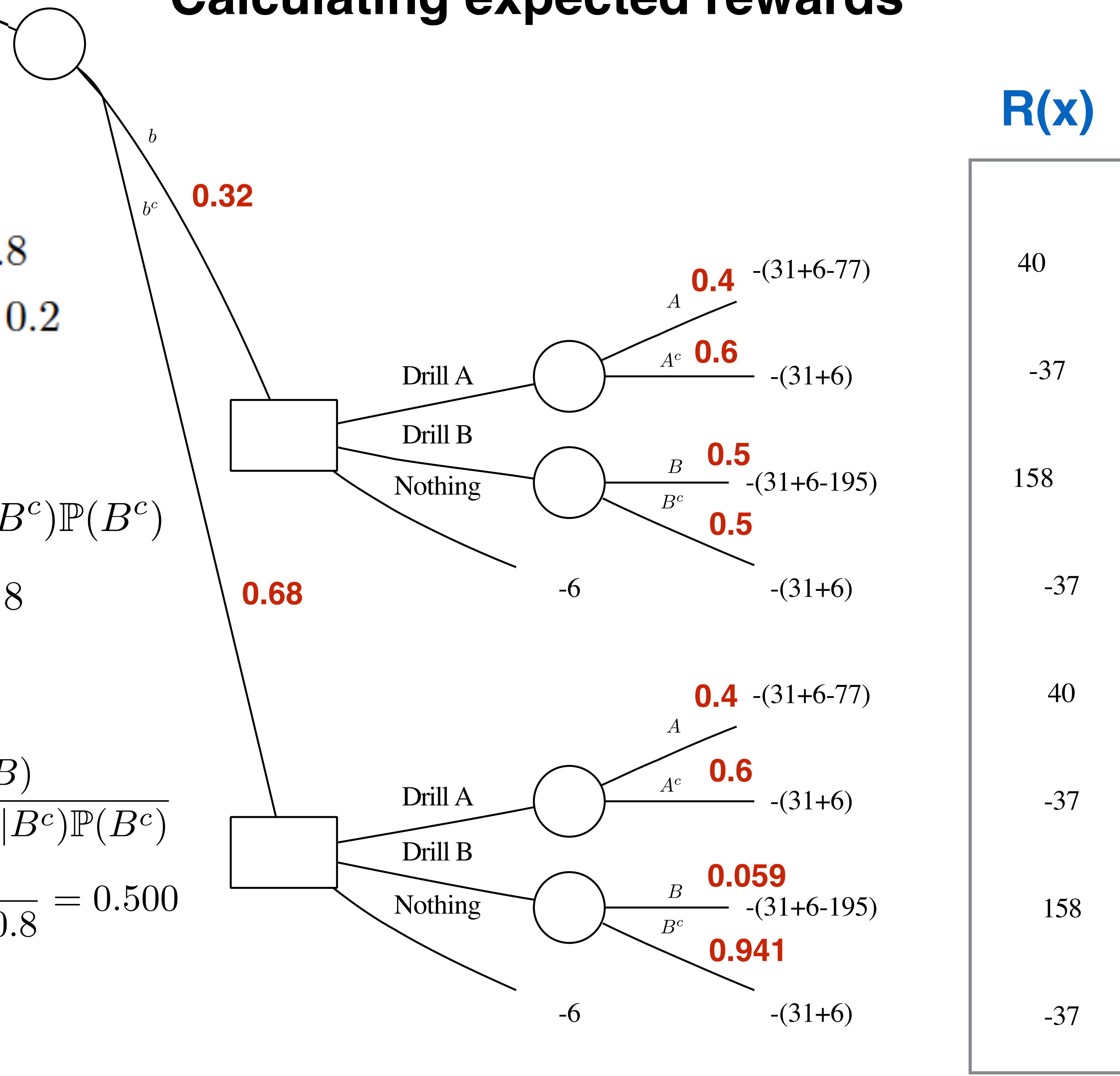
Trial drilling in B: Calculating expected rewards

- $\mathbb{P}(A) = 0.4$
- $\mathbb{P}(B) = 0.2$
- $\mathbb{P}(a|A) = \mathbb{P}(b|B) = 0.8$
- $\mathbb{P}(a|A^c) = \mathbb{P}(b|B^c) = 0.2$

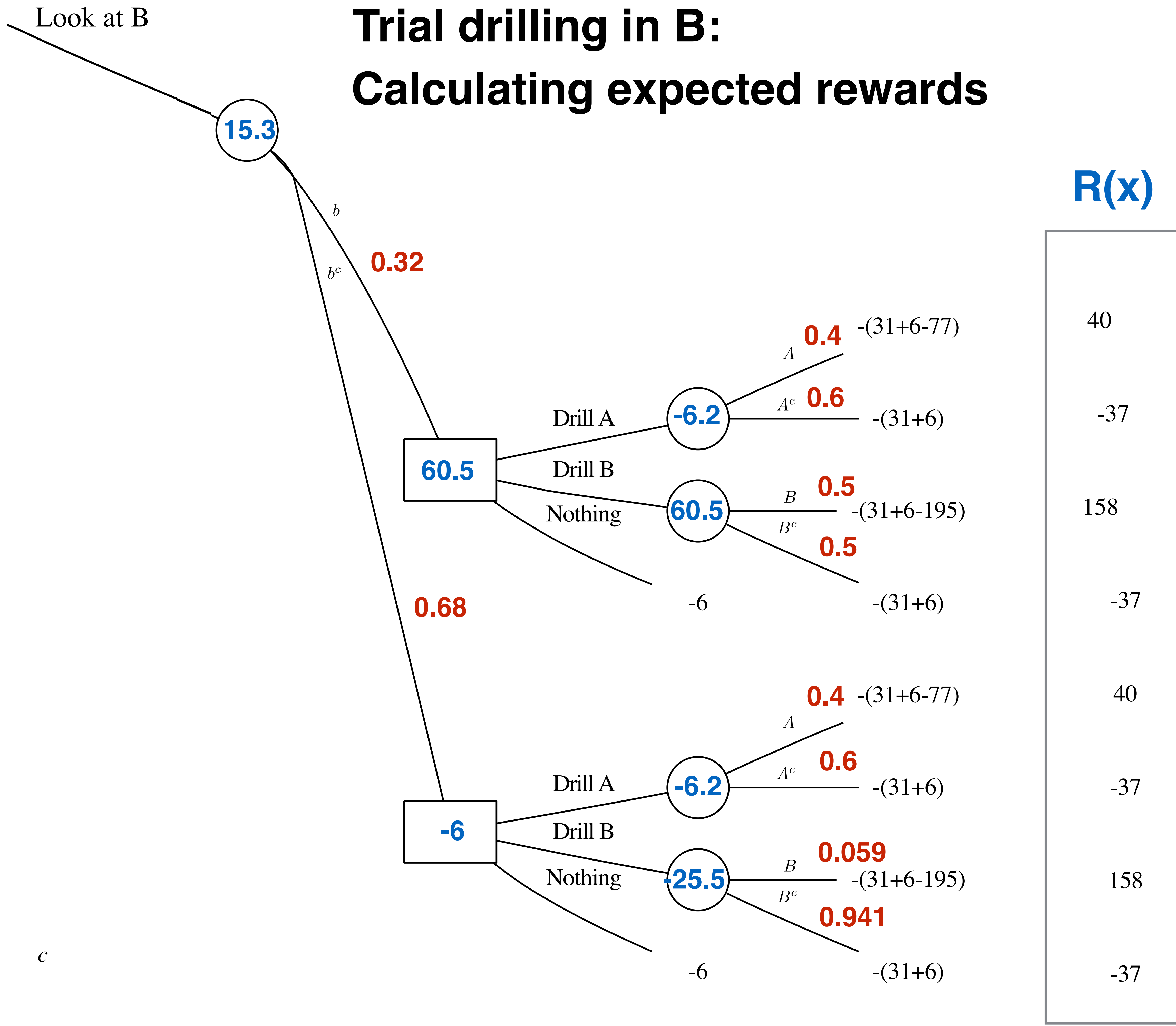
$$\begin{aligned}\mathbb{P}(b) =& \mathbb{P}(b|B)\mathbb{P}(B) + \mathbb{P}(b|B^c)\mathbb{P}(B^c)\\ =& 0.8 \times 0.2 + 0.2 \times 0.8\\ =& 0.16 + 0.16 = 0.32\end{aligned}$$

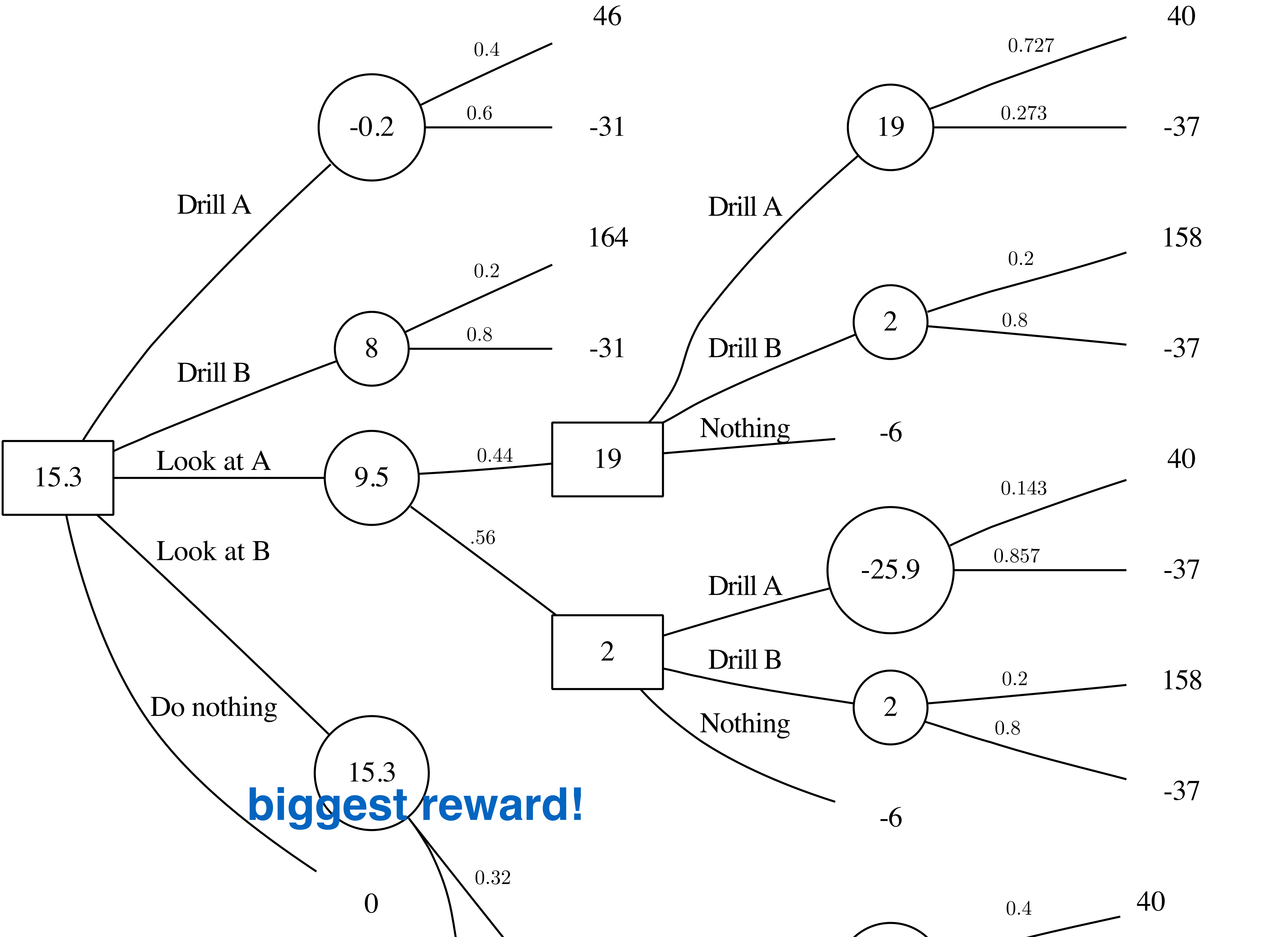
$$\begin{aligned}\mathbb{P}(B|b) =& \frac{\mathbb{P}(b|B)\mathbb{P}(B)}{\mathbb{P}(b|B)\mathbb{P}(B) + \mathbb{P}(b|B^c)\mathbb{P}(B^c)}\\ =& \frac{0.8 \times 0.2}{0.8 \times 0.2 + 0.2 \times 0.8} = 0.500\end{aligned}$$

similarly for b^c

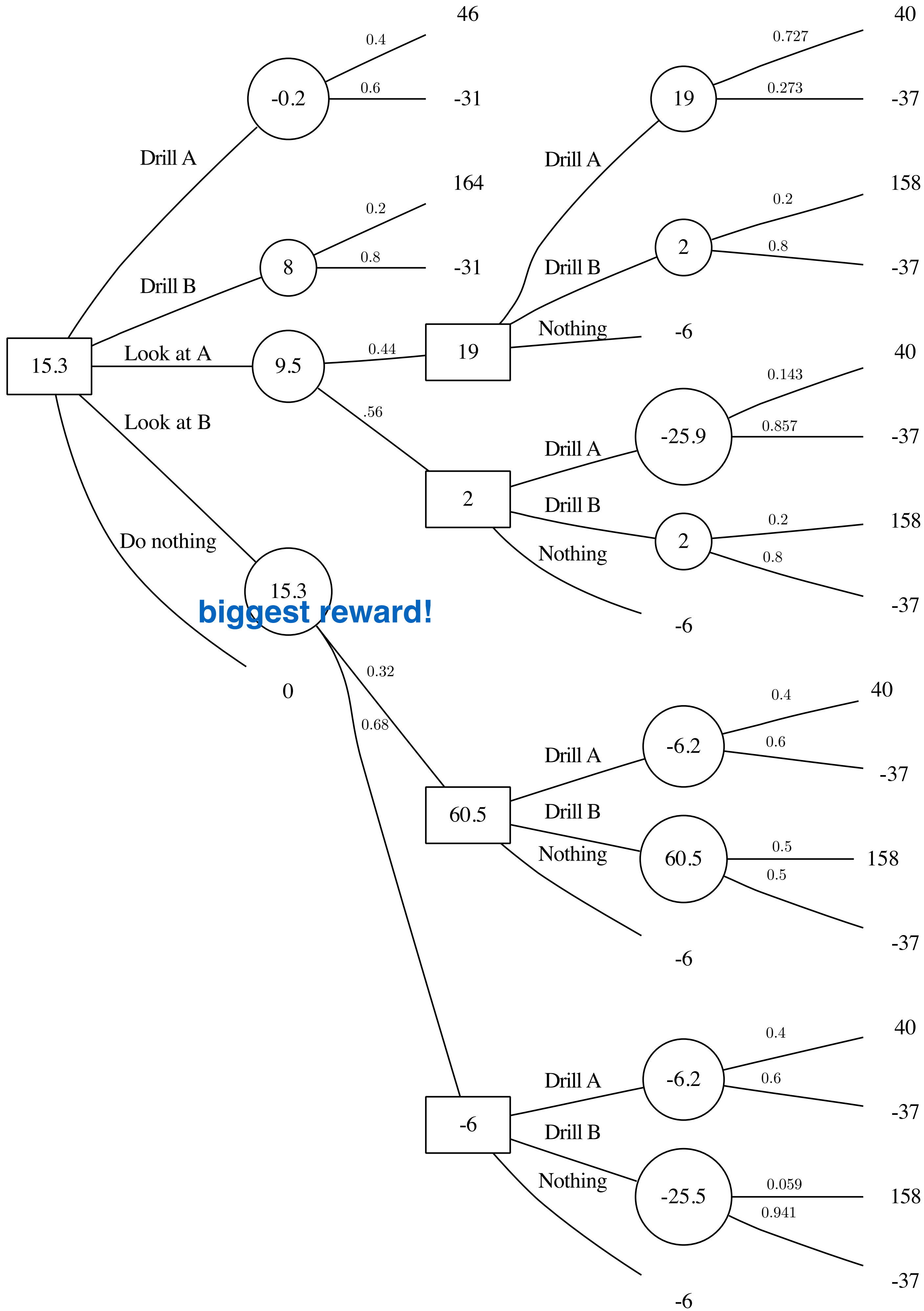


Trial drilling in B:
Calculating expected rewards





15.3
Drill A
Drill B
Look at A
Look at B
Do nothing
-0.2
0.4
46
0.6
-31
8
0.2
164
0.8
-31
9.5
0.44
19
Drill A
19
0.727
40
0.273
-37
Drill B
2
0.2
158
0.8
-37
Nothing
-6
.56
2
Drill A
-25.9
0.143
40
0.857
-37
Drill B
2
0.2
158
0.8
-37
Nothing
-6
15.3
biggest reward!
0.32
0
0.4
40



Resulting EMV Strategy:

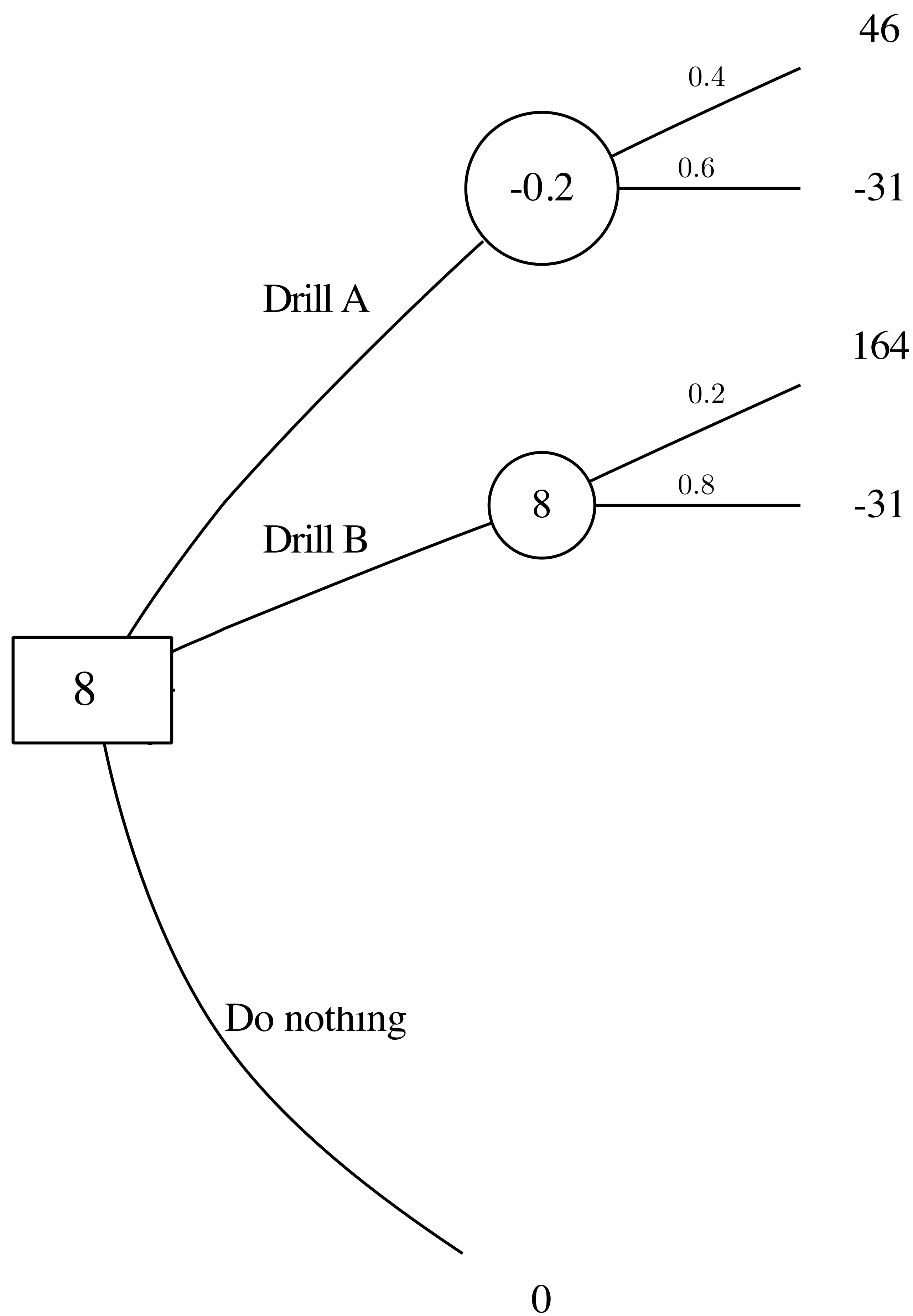
Look at B and
if b: drill in B
if not b: do nothing

EVPI and EVII

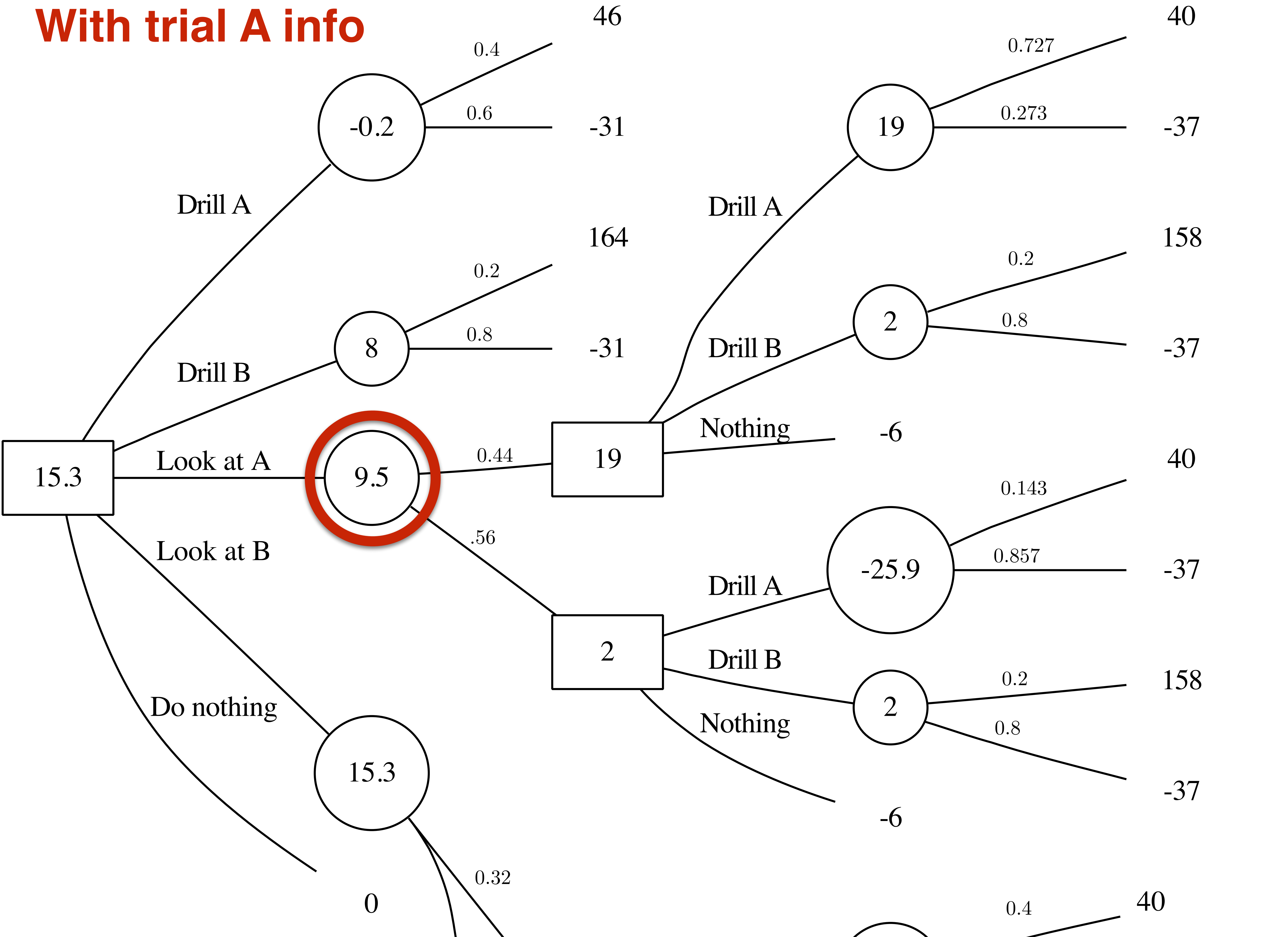
Expected Value of Perfect Information (EVPI):
Difference in expected value of a decision problem in which decisions are made with full knowledge of the outcome of chance events and the corresponding decision problem in which no additional knowledge is available.

Expected Value of Imperfect Information (EVII):
Difference in expected value of a decision problem in which decisions are made with an imperfect source of information about the outcome of chance events and the corresponding decision problem in which no additional knowledge is available.

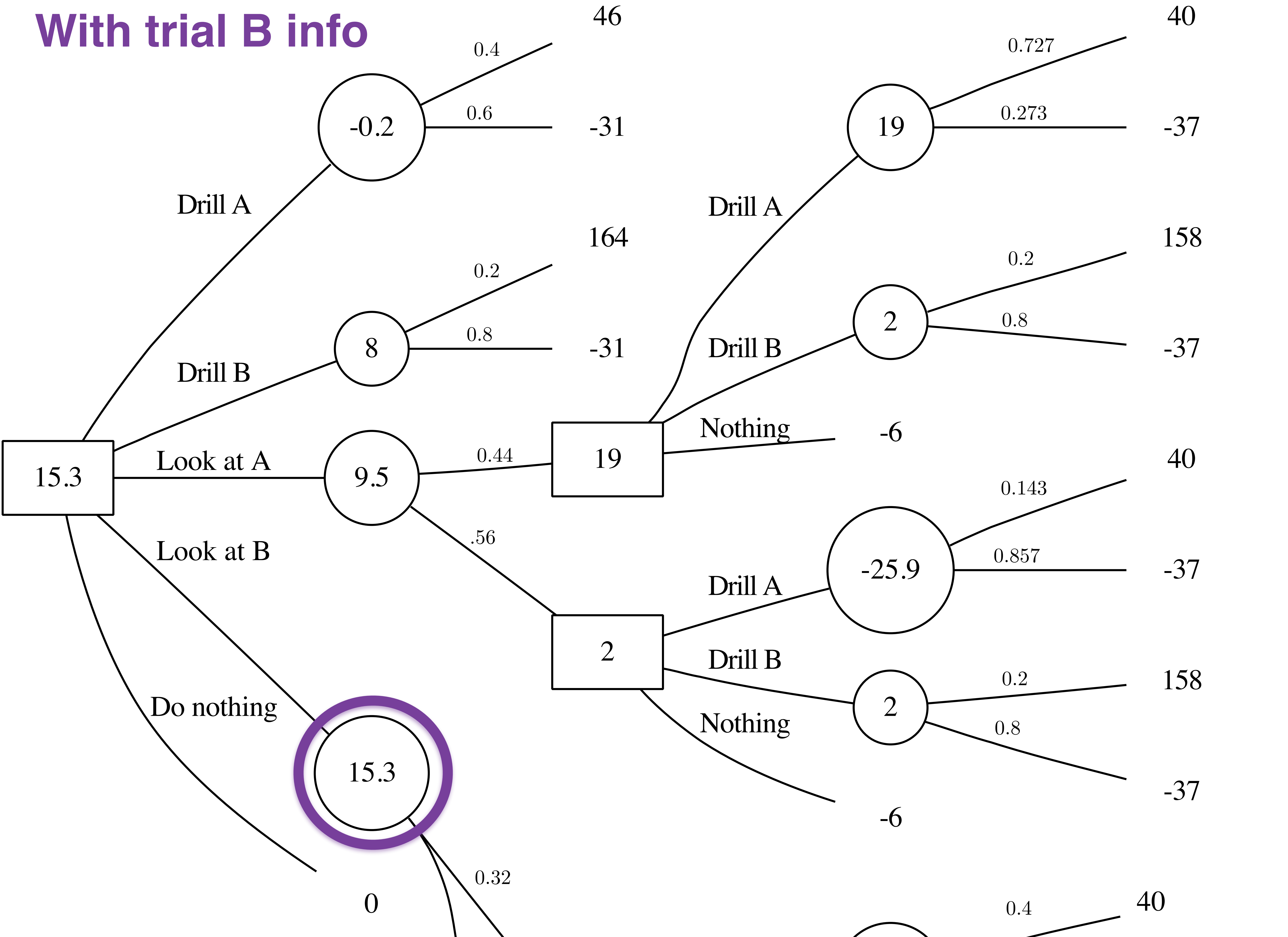
Decision problems without additional knowledge



With trial A info



With trial B info



Expected Value of Imperfect Information (EVII):
Difference in expected value of a decision problem in which decisions are made with an imperfect source of information about the outcome of chance events and the corresponding decision problem in which no additional knowledge is available.

With trial A info	EVII(trail A) = 9.5 - 8 = 1.5 > 0
With trial B info	EVII(trail B) = 15.3 - 8 = 7.3 > 0

Question: Is this additive?
No, because you can only drill in one place.
(In other situations, there may be other reason, e.g. correlation.)

Expected Value of Perfect Information (EVPI)

How much would you pay for full knowledge of the outcome?

For the sake of the drilling decision, full knowledge means you know which of the subsets of the following partition you are facing:

$$\{A \cap B, A \cap B^c, A^c \cap B, A^c \cap B^c\}$$

Then you could choose best strategy for each case (in bold):

$R(d,x)$	$A \cap B$	$A \cap B^c$	$A^c \cap B$	$A^c \cap B^c$
Drill A	46	**46**	-31	-31
Drill B	**164**	-31	**164**	-31
Do Nothing	0	0	0	**0**
$\mathbb{P}$	0.08	0.32	0.12	0.48

Select one from each column

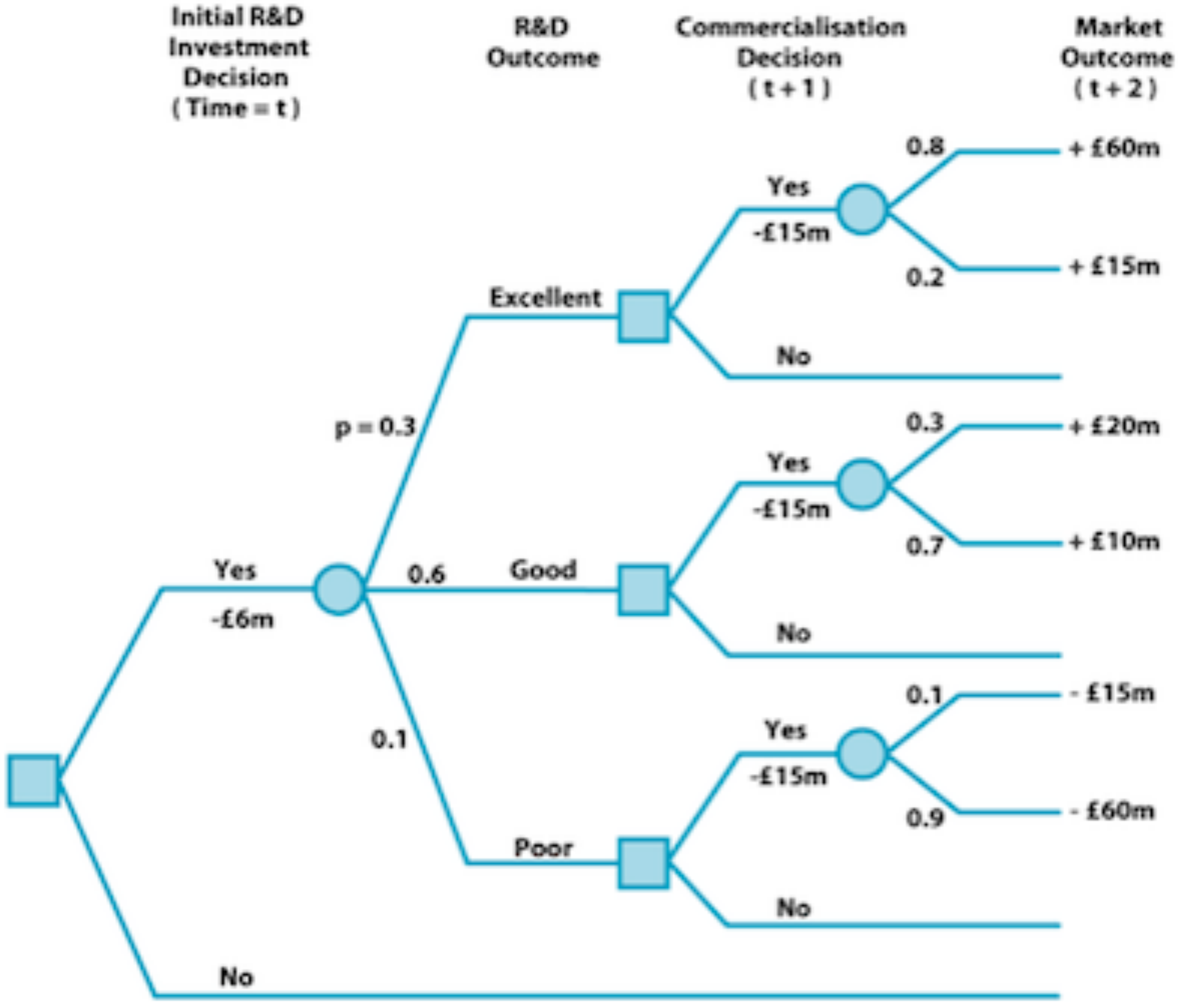
Now multiply each of the scenarios with its likelihood to occur, i.e. calculate the expected reward given full knowledge:

$$(0.08 + 0.12) \times \pounds 164M + 0.32 \times \pounds 46M + 0.48 \times \pounds 0M = \pounds 47.52M$$

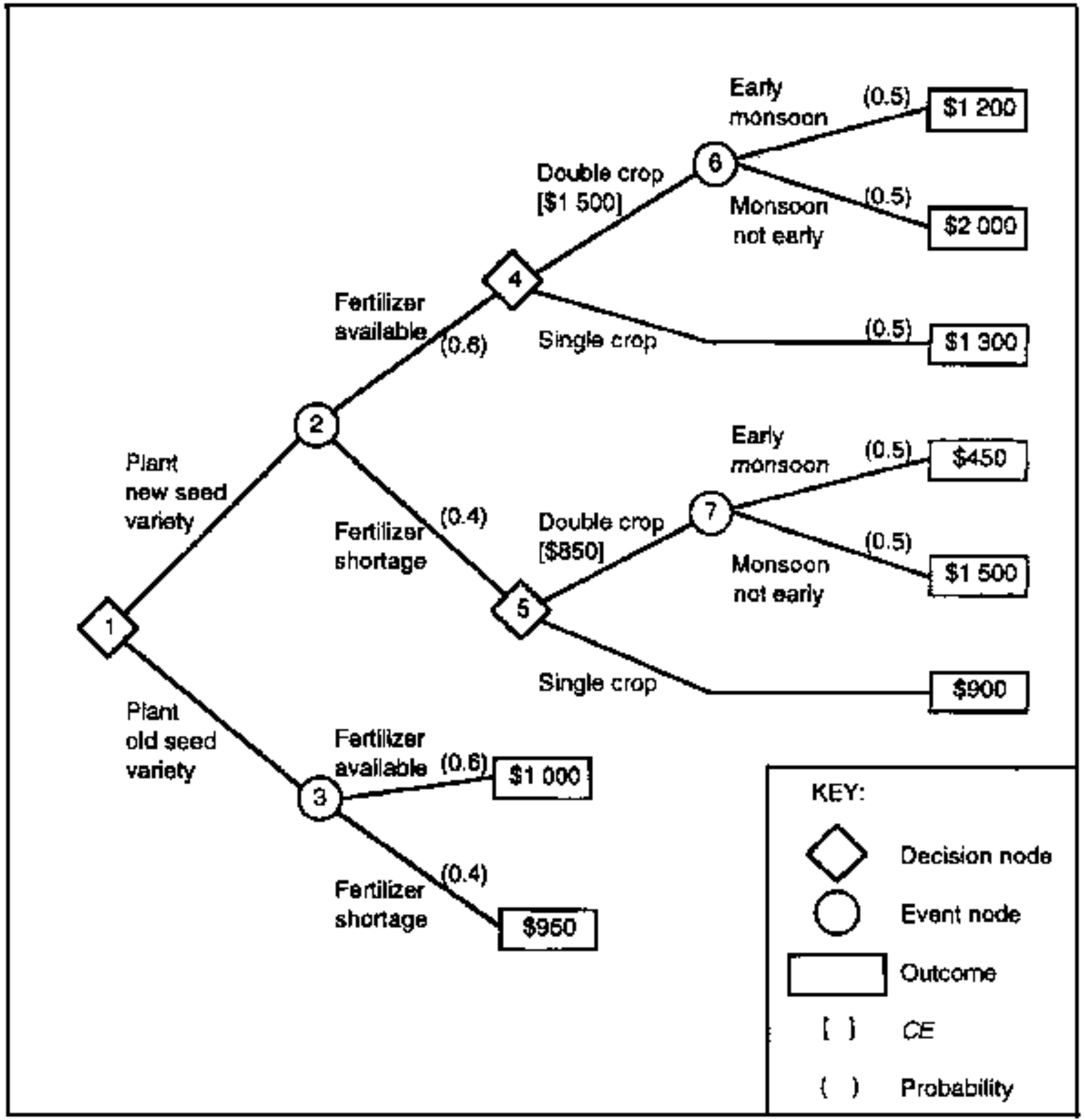
Hence, EVPI equals £47.52M-£8M=£39.52M

EXAMPLES FOR DECISION TREE MODELS IN INDUSTRY

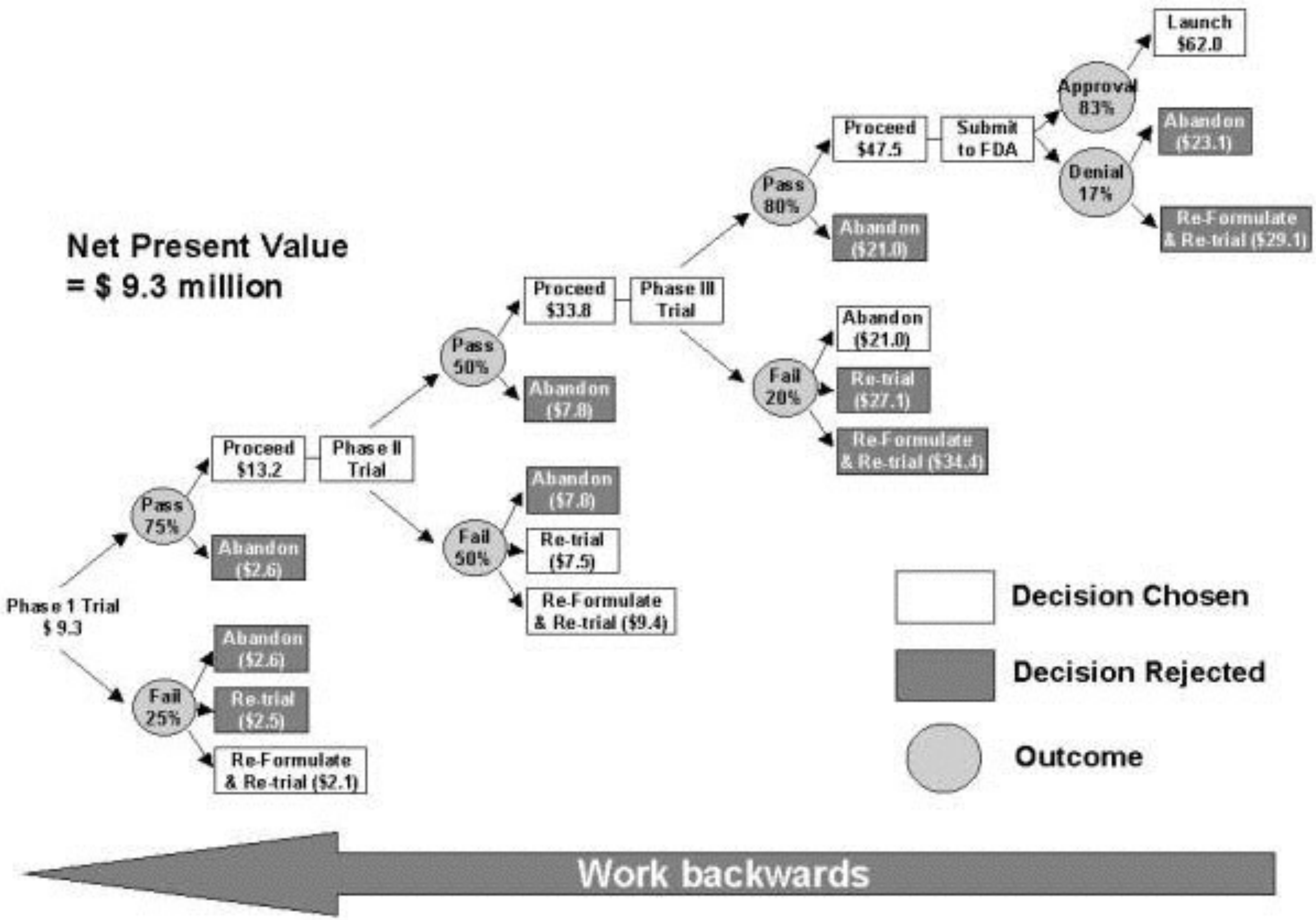
Product development



Agriculture

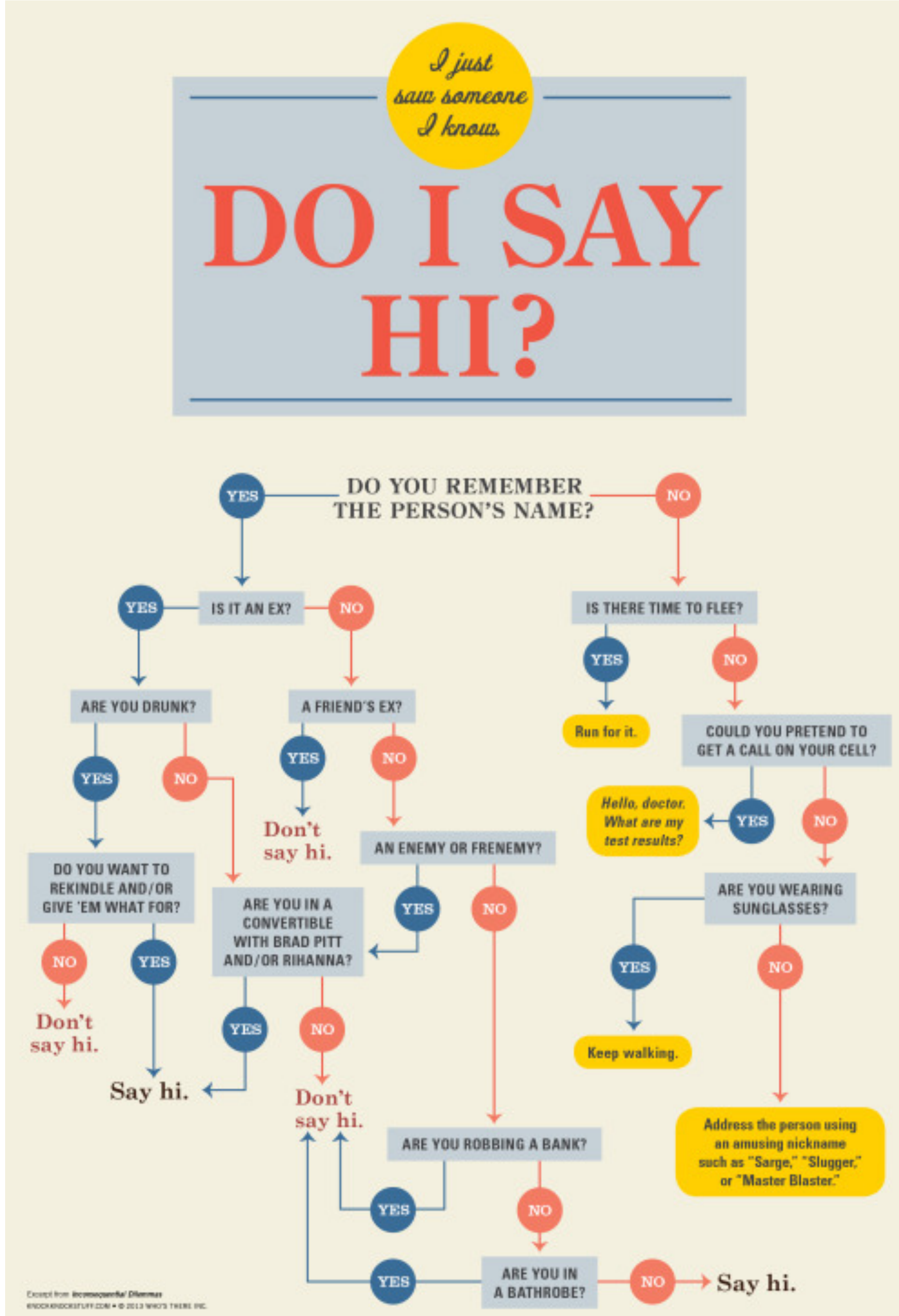


Drug development (clinical trial)



DECISIONS IN REAL LIFE

Source: "Inconsequential dilemmas", Knock Knock, Venice, California



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