

Aller's Paradox

Would using utility help reconciling typical choices by people with expected utility theory?

A: — 1M

B: $\frac{0.01}{0.89}$ 0M
 $\frac{0.1}{0.1}$ 1M
5M

C: $\frac{0.11}{0.89}$ 1M
0M

D: $\frac{0.1}{0.9}$ 5M
0

$$E[A] = 1M < E[B] = 0.01 \cdot 0 + 0.81 \cdot 1 + 0.1 \cdot 5 \\ = 0.81 + 0.5 = 1.31$$

$$E[C] = 0.11 \cdot 1 + 0.89 \cdot 0 = 0.11 < E[D] = 0.1 \cdot 5 + 0.9 \cdot 0 \\ = 0.5$$

According to expected monetary value:

$A < B$ and $C < D$

But in empirical studies many people prefer A over B.

Can utility explain this?

Let u be some utility function.
For simplicity, assume $u(0) = 0$.
(Generalise this in exercise sheet 4.)

$$E[u(A)] = u(1)$$

$$E[u(B)] = 0.89 \cdot u(1) + 0.1u(5)$$

$$E[u(C)] = 0.11 \cdot u(1)$$

$$E[u(D)] = 0.1 \cdot u(5)$$

$$E[u(A)] > E[u(B)]$$

$$\Leftrightarrow \underbrace{(1 - 0.89)}_{=0.11} \cdot u(1) > 0.1 u(5)$$

$$E[u(C)] < E[u(D)]$$

$$\Leftrightarrow 0.11 u(1) < 0.1 \cdot u(5) \quad \Downarrow$$