For you (ST222@Warwick2017):

Which of the choices below (C or D) do you prefer?

In the first stage, there is a 75% chance to end the game without winning anything, and a 25% chance to move into the second stage. If you reach the second stage you have a choice:

C. a sure win of $30

D. 80% chance to win $45

Your choice must be made before the game starts.

Heuristics & biases: Framing of contingencies

Build mathematical models describing and predicting the observed behaviour of humans with respect to lotteries.

Paper TK'1981:
Amos Tversky; Daniel Kahneman, *The Framing of Decisions and the Psychology of Choice,*
Science, New Series, Vol. 211, No. 4481. (Jan. 30, 1981), pp. 453-458.

Problem 7 [N = 81]: Which of the following options do you prefer?

E. 25% chance to win $30 [42 percent]

P. 20% chance to win $45 [58 percent]

Multiply the probabilities with factor 4.
EUT: preferences should be the same (expectation is linear)

Problem 7 [N = 81]: Which of the following options do you prefer?

E. 25% chance to win $30 [42 percent] [Expectation: $7.5]

P. 20% chance to win $45 [58 percent] [Expectation: $9.0]

Problem 5 [N = 77]: Which of the following options do you prefer?

A. a sure win of $30 [78 percent] [Expectation: $30]

B. 80% chance to win $45 [22 percent] [Expectation: $36]

Preferences changed (reversed and more pronounced).

A version of Allais paradox.

Explanation: *Certainty effect*

Amos Tversky; Daniel Kahneman, *The Framing of Decisions and the Psychology of Choice,* Science, New Series, Vol. 211, No. 4481. (Jan. 30, 1981), pp. 453-458.

Data from TK'1981

Problem 7 [N = 81]: Which of the following options do you prefer?

E. 25% chance to win $30 [42 percent] [Expectation: $7.5]

P. 20% chance to win $45 [58 percent] [Expectation: $9.0]

Problem 5 [N = 77]: Which of the following options do you prefer?

A. a sure win of $30 [78 percent] [Expectation: $30]

B. 80% chance to win $45 [22 percent] [Expectation: $36]

Problem 6 [N = 85]: In the first stage, there is a 75% chance to end the game without winning anything, and a 25% chance to move into the second stage. If you reach the second stage you have a choice:

C. a sure win of $30 [???] [Expectation: $7.5]

D. 80% chance to win $45 [???] [Expectation: $9.0]

Your choice must be made before the game starts.

Problem 7 [N = 81]: Which of the following options do you prefer?

E. 25% chance to win $30 [42 percent] [Expectation: $7.5]

P. 20% chance to win $45 [58 percent] [Expectation: $9.0]

EUT

Problem 5 [N = 77]: Which of the following options do you prefer?

A. a sure win of $30 [78 percent] [Expectation: $30]

B. 80% chance to win $45 [22 percent] [Expectation: $36]

Certainty effect

Problem 6 [N = 85]: In the first stage, there is a 75% chance to end the game without winning anything, and a 25% chance to move into the second stage. If you reach the second stage you have a choice:

C. a sure win of $30 [74 percent] [Expectation: $7.5]

D. 80% chance to win $45 [26 percent] [Expectation: $9.0]

Your choice must be made before the game starts.

Ignoring first part of this compound bet (Isolation effect)

Interpretation

P5: Majority subject to certainty effect.

P6: While certainty is removed in compound bet, most people maintain same preferences.

P7: For the majority, preferences are reversed and follow EUT.

Problem 7 [N = 81]: Which of the following options do you prefer?

E. 25% chance to win $30 [42 percent] [Expectation: $7.5]

P. 20% chance to win $45 [58 percent] [Expectation: $9.0]

ST222'15@W:

23.7%

76.3%

Problem 5 [N = 77]: Which of the following options do you prefer?

A. a sure win of $30 [78 percent] [Expectation: $30] 59.5%

B. 80% chance to win $45 [22 percent] [Expectation: $36] 40.5%

Problem 6 [N = 85]: In the first stage, there is a 75% chance to end the game without winning anything, and a 25% chance to move into the second stage. If you reach the second stage you have a choice:

C. a sure win of $30 [74 percent] [Expectation: $7.5] 60.0%

D. 80% chance to win $45 [26 percent] [Expectation: $9.0] 40.0%

Your choice must be made before the game starts.

Interpretation:

ST222'15@Warwick has about the same preferences in P6 as in P5, and different from P7, all consistent with the preferences expressed by TK'1981 subjects.

In all problems, ST222'15@Warwick are more likely than TK'1981 to choose the option with the higher expectation, suggesting more of them are following EUT.

Discussion: Rationality concepts in K&T's work

- The work of Kahneman and Tversky (1960s to present) is largely based on the understanding that people would apply EUT.
- Alternative theories (e.g. regret theory, maximin, Gigerenzer's school etc) lead to other answers. People who apply them can still do so for *rational* reasons. They just have different priorities (e.g. in problem 5 ensuring to win at least something is more important than maximising the gain).
- However, K & T did *not* equate rationality with the application of EUT.
- The point they are making in e.g. TK'1981 is that people's behaviour is *inconsistent* with the respect to the question of application of EUT: While Problems 6 and 7 are mathematically equivalent, the subjects behaved differently. In Problem 7 a majority applied EUT, but in Problem 6 a majority did not do so. Instead, they answer as if this had been Problem 5, where behaviour is governed by the *certainty effect*. This is remarkable, because there is actually no certainty in Problem 6 if the compound bet is see as such. Instead, subjects seem to focus on just the second step of Problem 6 accepting that the first step is out of their control.

Examples: Empirically shown deviations from normative theory

- Gambler's fallacy, inverse gambler's fallacy, belief in hot hand
- Random sequences generation biases (starting value, runs)
- Clustering illusion
- Anchoring bias (with related and unrelated information)
- Framing effect
- Availability bias
- Certainty effect (Allais paradox)
- Disjunction effect
- Base rate neglect
- Relevant in finance, e.g. Disposition effect

Methodology: Normative theory versus descriptive theory

Normative theories of decision making:

- How idealised (rational) world behave when taking decisions
- Based on an idealised form of human being
- Methods: Mathematical axioms and optimisation

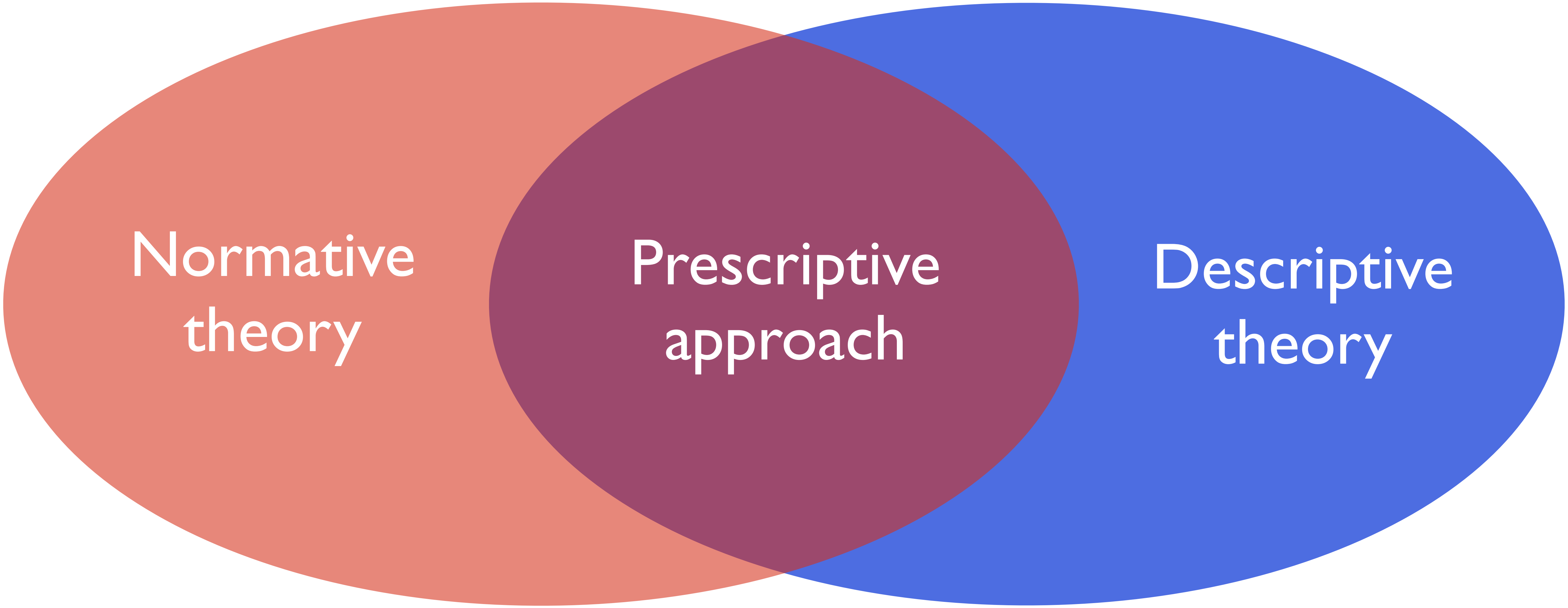
Descriptive theories of decision making

- How *people actually make* decisions
- Based on observation (empirical studies)
- Methods: empirical studies, revised models

Why is normative theory not enough?

Empirical studies have demonstrated that people do not always follow the axioms of probability (biases, fallacies, heuristics).

Methodology: Complementary theories of decision making



Why is normative theory not enough?

Empirical studies have demonstrated that people do not always follow the axioms of probability (biases, fallacies, heuristics).

Modelling:
Mathematics as bridge between theory & application

"The instrument that mediates between theory and practice, between thought and observation, is mathematics; it builds the connecting bridge and makes it stronger and stronger. Thus it happens that our entire present-day culture, insofar as it rests on intellectual insight into and harnessing of nature, is founded on mathematics."

David Hilbert

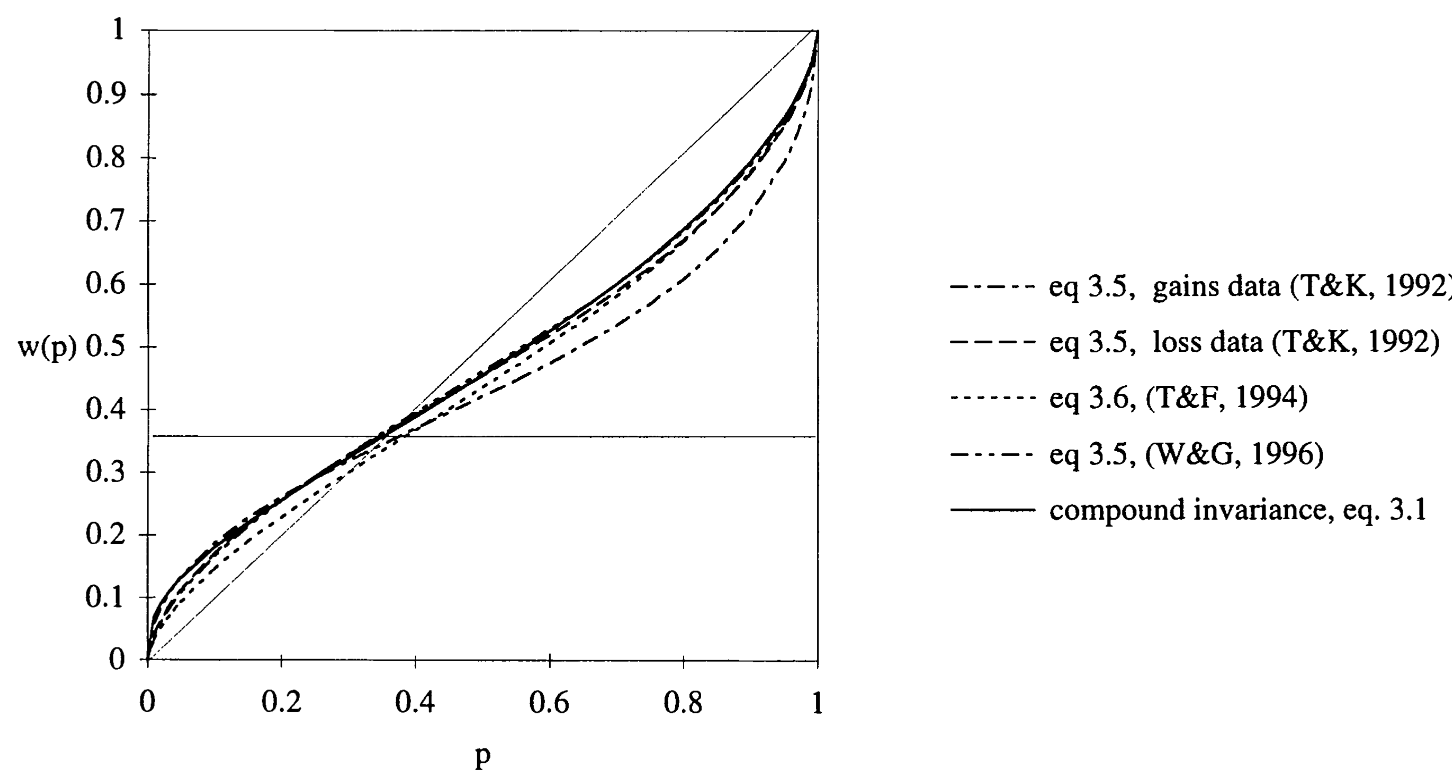
In Königsberg on 8 September 1930, David Hilbert addressed the yearly meeting of the Society of German Natural Scientists and Physicians (Gesellschaft der Deutschen Naturforscher und Ärzte). Generally regarded as the world's leading mathematician at the time, Hilbert was born and educated in Königsberg and spent the early years of his career there.
Full text of the speech in English and German at url below, including audio file:

http://math.sfsu.edu/smith/Documents/HilbertRadio/HilbertRadio.pdf

Prospect theory: Probability weighting

Suggested model by T&K: Four-fold pattern of risk attitudes

regressive—intersecting the diagonal from above,
asymmetric—with fixed point at about 1/3,
s-shaped—concave on an initial interval and convex beyond that,
reflective—assigning equal weight to a given loss-probability as to a given gain-probability.



The compound invariant form (solid line) and several empirical probability weighting functions. Estimates of the one-parameter equation (3.5) are taken from Tversky and Kahneman (1992) and Wu and Gonzalez (1996a); estimates of the two-parameter equation (3.6) are taken from Tversky and Fox (1994).

Variations

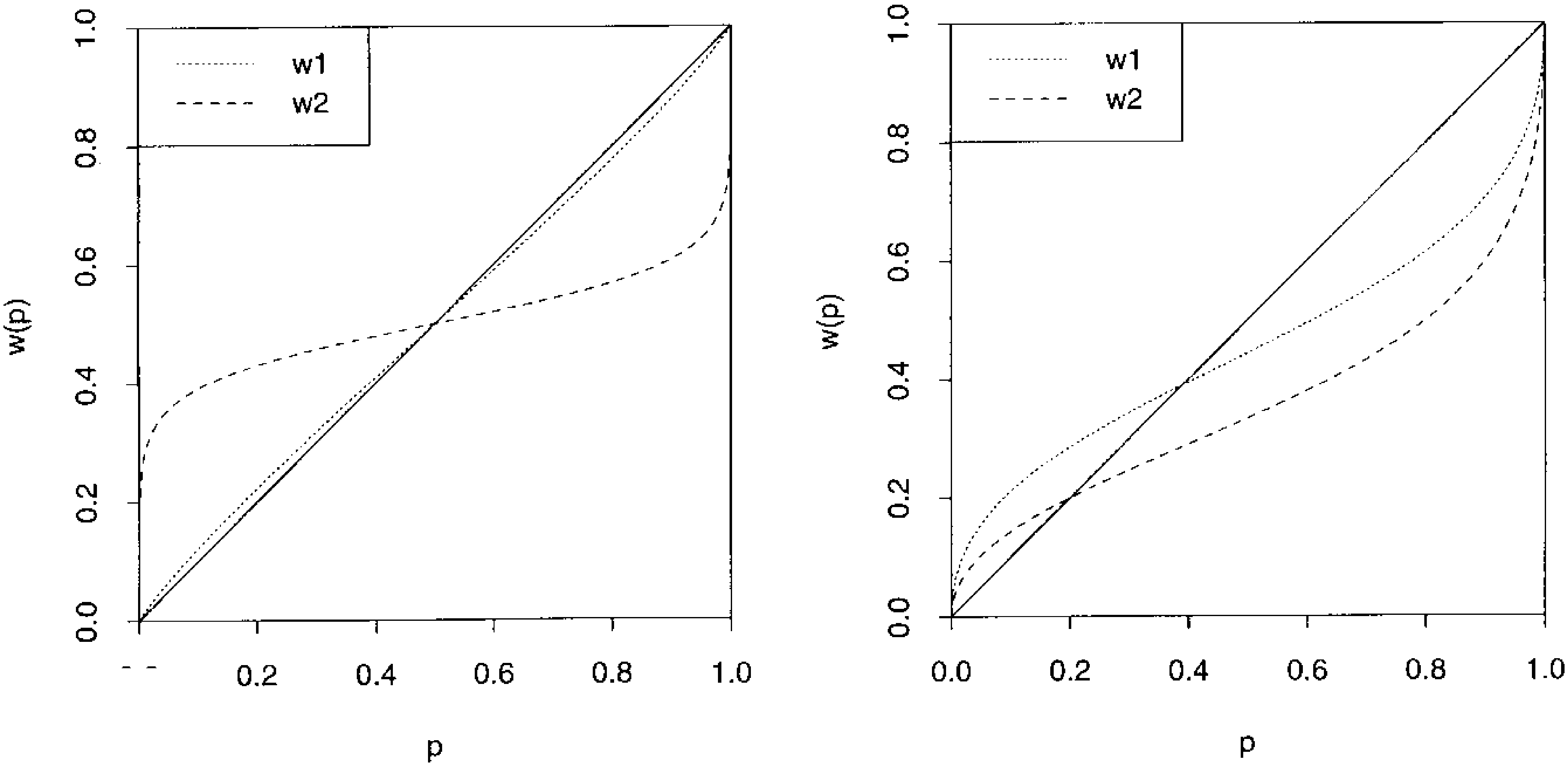


FIG. 3. (Left) Two weighting functions that differ primarily in curvature—w_1 is relatively linear and w_2 is almost a step function. (Right) Two weighting functions that differ primarily in elevation—w_1 overweights relative to w_2.

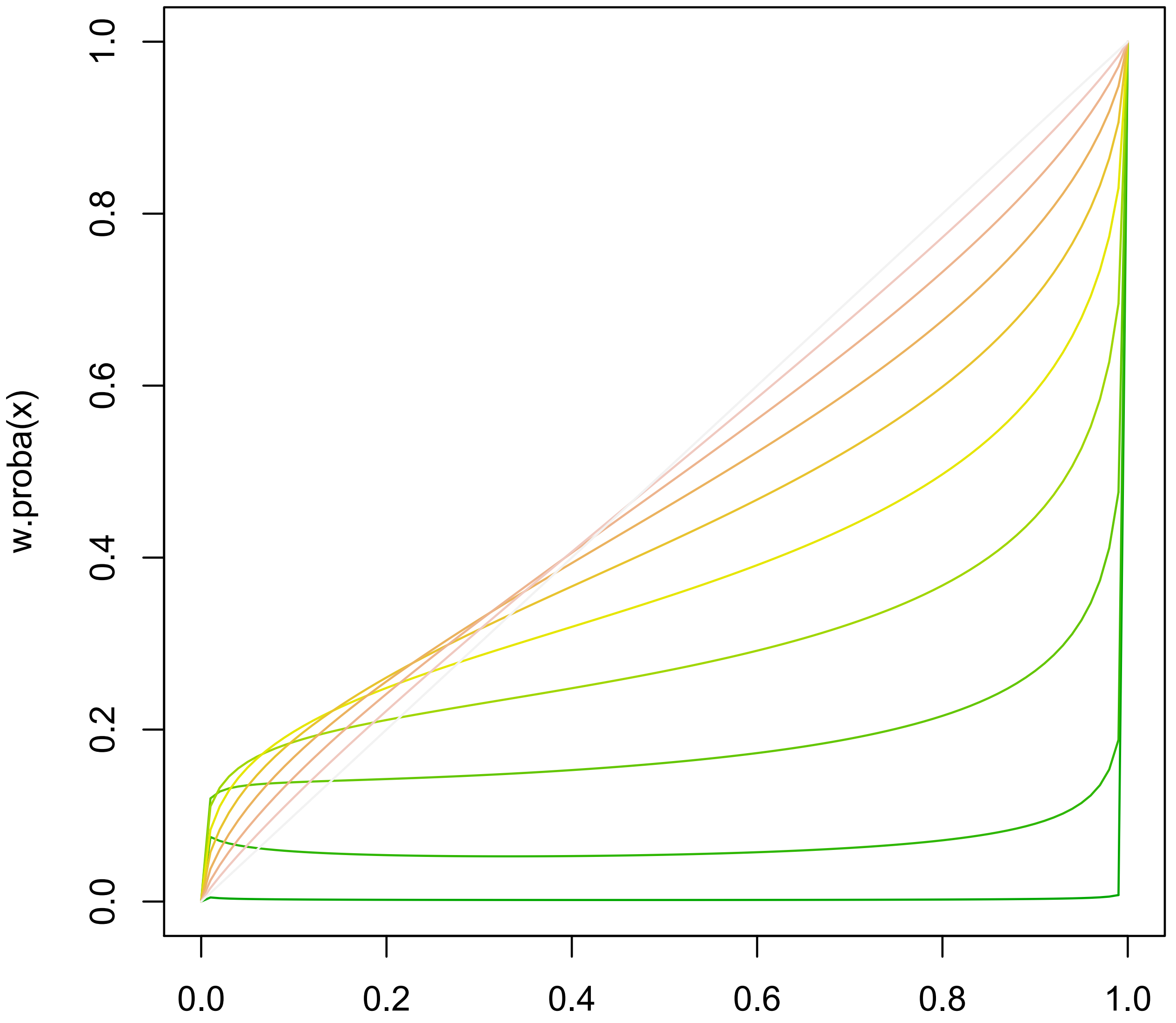
Definition: probability weighting function

Definition *By a probability weighting function we mean a strictly increasing function* $w : [0,1] \overset{onto}{\longrightarrow} [0,1]$.

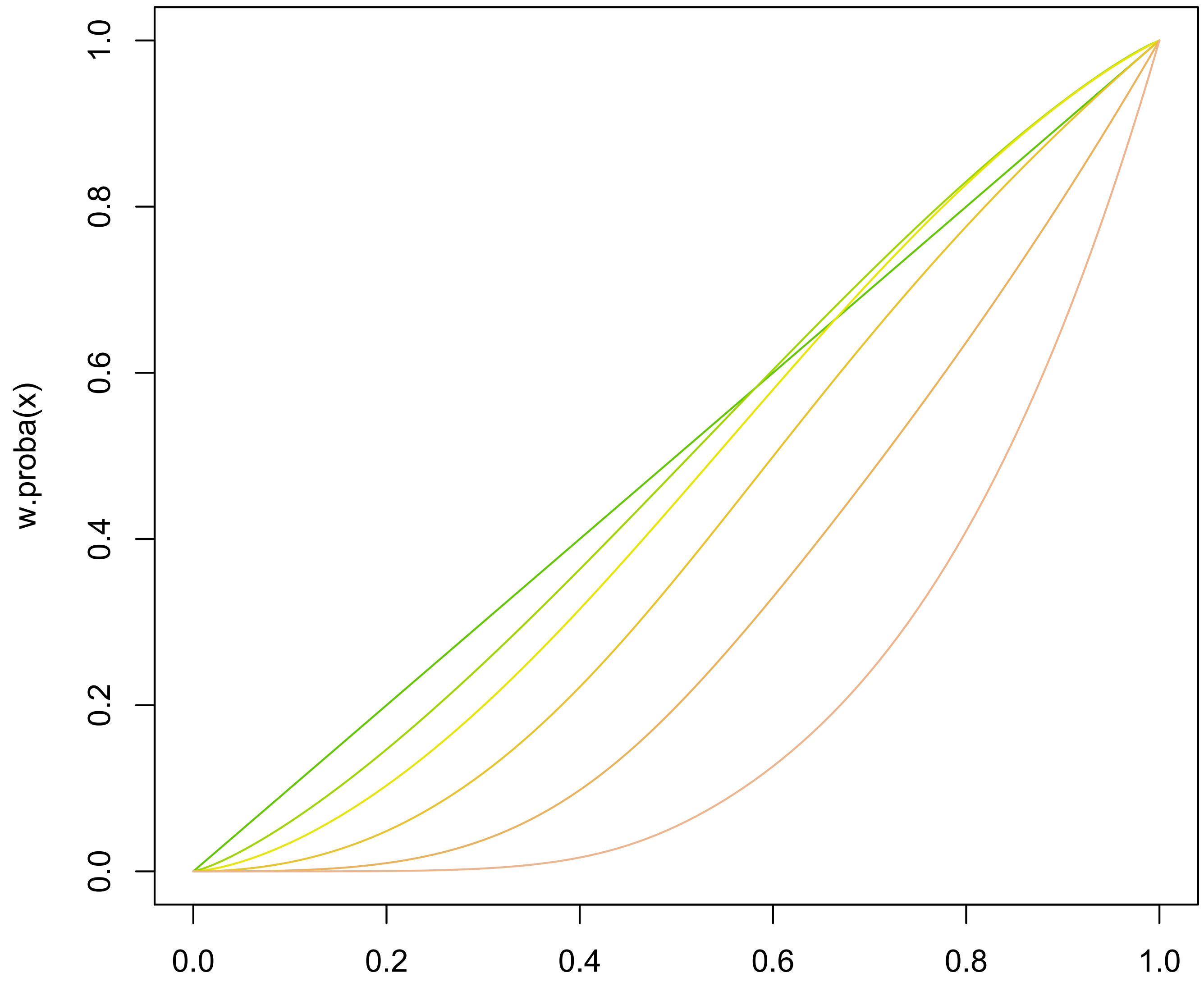
Note that a probability weighting function, w, has a unique inverse, $w^{-1} : [0,1] \overset{onto}{\longrightarrow} [0,1]$ and that w^{-1} is strictly increasing. Hence, w^{-1} is also a probability weighting function. Furthermore, it follows that w and w^{-1} are continuous and must satisfy $w(0) = w^{-1}(0) = 0$ and $w(1) = w^{-1}(1) = 1$.

Probability weighting: T&K's probability weighting function family

$$w(p) = \frac{p^{\gamma}}{\left[p^{\gamma} + (1-p)^{\gamma}\right]^{\frac{1}{\gamma}}},\ 0.5 \leq \gamma < 1,\ 0 \leq p \leq 1$$



gamma = seq(0.1,1,0.1) (<0.5 not used!)



gamma = 1, 1.25, 1.5, 2, 3, 5

Alternative probability weighting: Prelec function

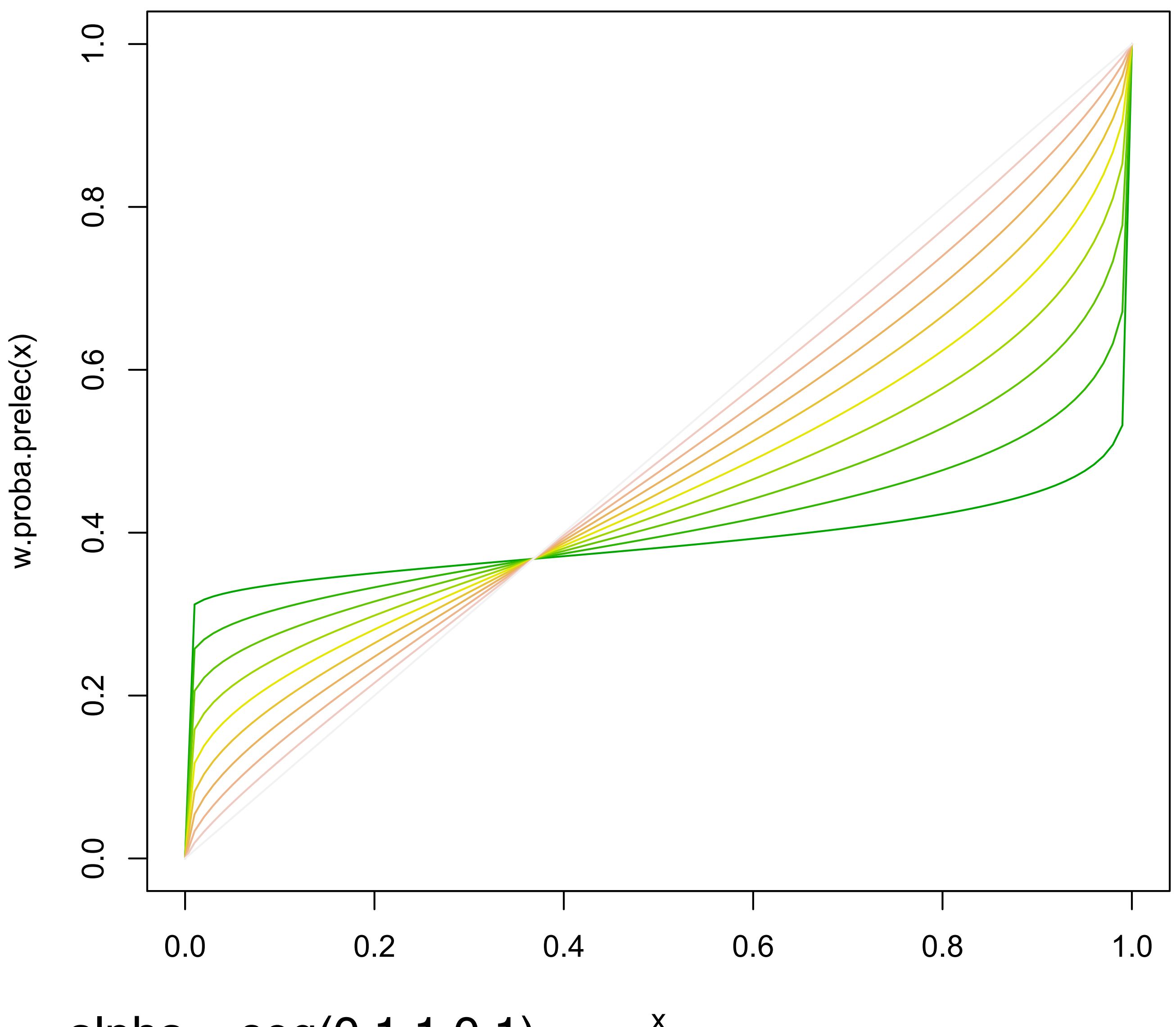
Definition 4 *: (Prelec, 1998). By the Prelec function we mean the probability weighting function* $w : [0,1] \xrightarrow{onto} [0,1]$ *given by*

$$w(p) = e^{-\beta(-\ln p)^{\alpha}}, \alpha > 0, \beta > 0 \tag{2.1}$$

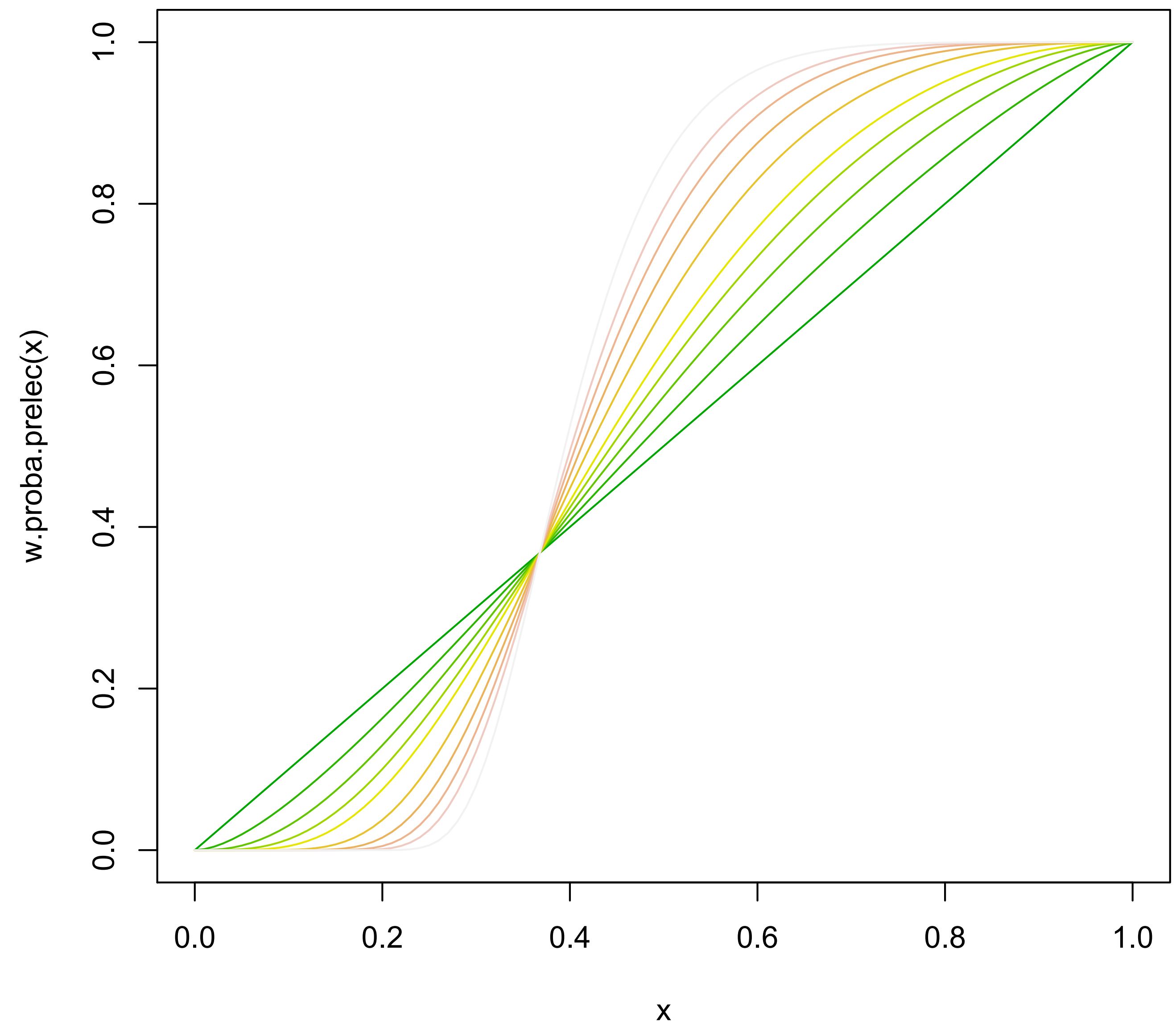
$$w(p) = e^{-\beta(-\ln p)^{\alpha}}, \alpha > 0, \beta > 0$$

Ali al-Nowaihi, Sanjit Dhami, *A simple derivation of Prelec's probability weighting function*, June 2005

Plots: Prelec functions for various alphas (beta=1)



alpha = seq(0.1,1,0.1)



alpha = 1, 1.25, 1.5, 1.75, 2, 2.5, 3, 3.5, 4, 5)

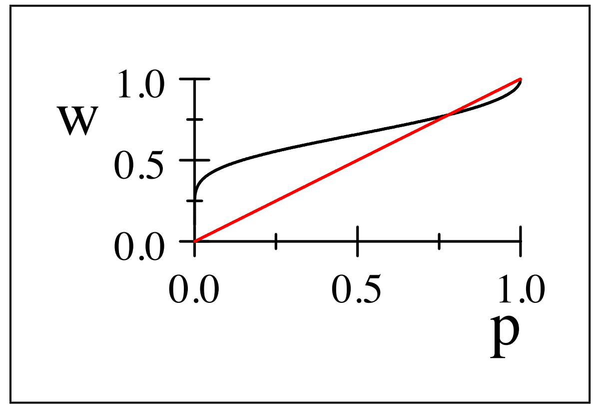
$$w\left(p\right) = e^{-\beta(-\ln p)^{\alpha}}, \alpha > 0, \beta > 0$$

α curvature: convexity/concavity

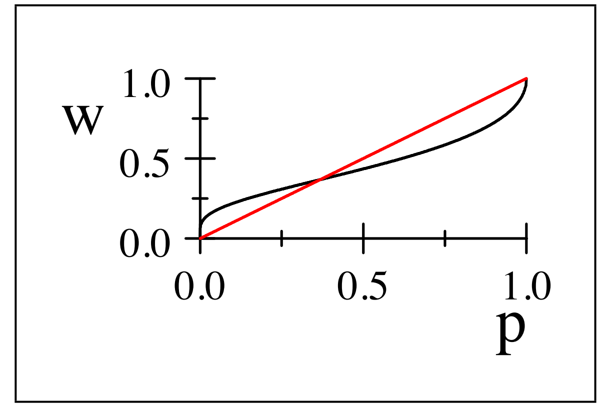
β elevation: inflection point at exp(-1) for beta=1

varying beta

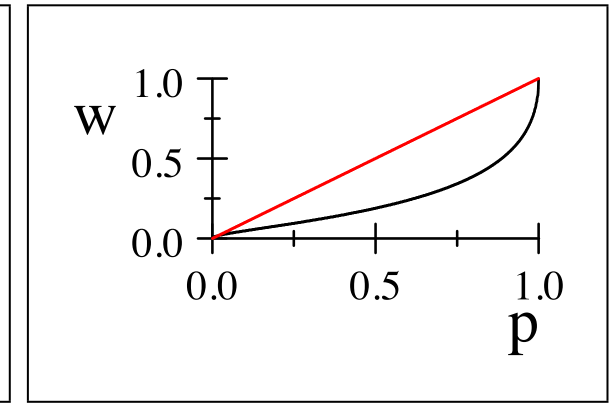
$\beta = \frac{1}{2}$ | $\beta = 1$ | $\beta = 2$



$w\left(p\right) = e^{-\frac{1}{2}(-\ln p)^{\frac{1}{2}}}$

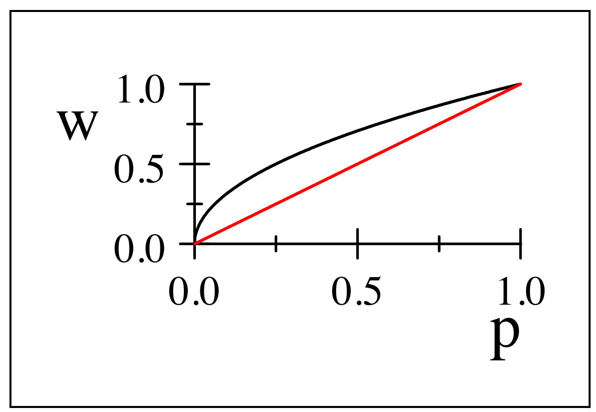


$w\left(p\right) = e^{-(-\ln p)^{\frac{1}{2}}}.$

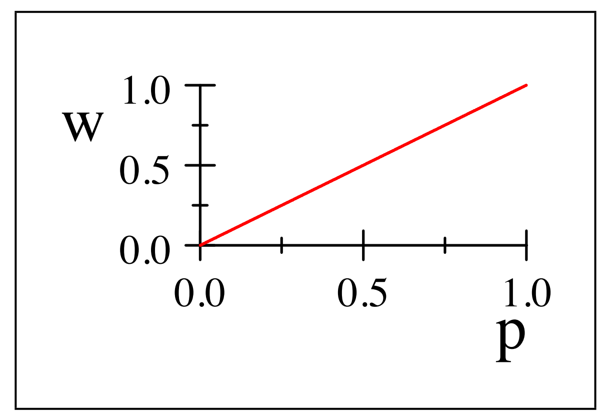


$w\left(p\right) = e^{-2(-\ln p)^{\frac{1}{2}}}$

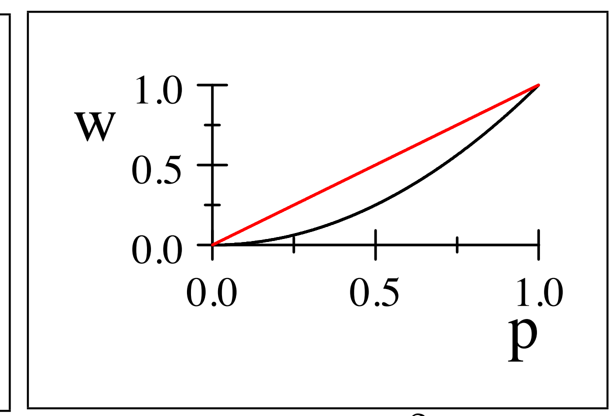
$\alpha = \frac{1}{2}$



$w\left(p\right) = p^{\frac{1}{2}}$

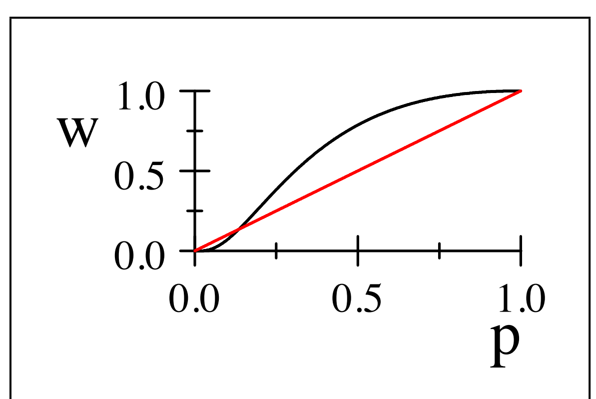


$w\left(p\right) = p$

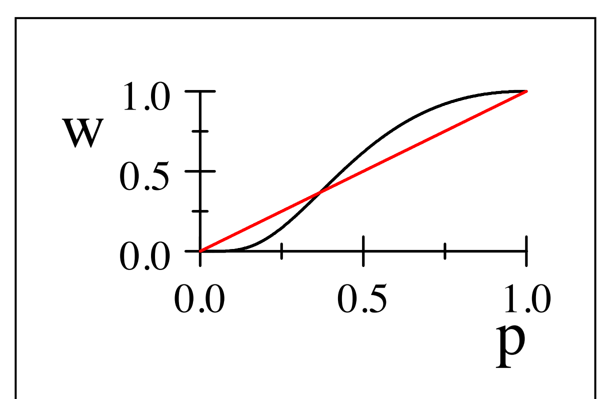


$w\left(p\right) = p^2$

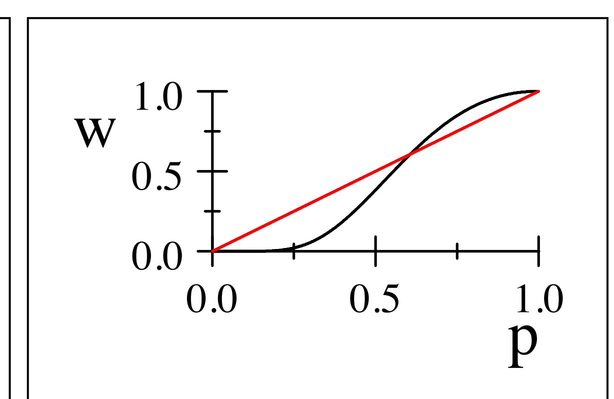
$\alpha = 1$



$w\left(p\right) = e^{-\frac{1}{2}(-\ln p)^2}.$



$w\left(p\right) = e^{-(-\ln p)^2}.$



$w\left(p\right) = e^{-2(-\ln p)^2}$

$\alpha = 2$

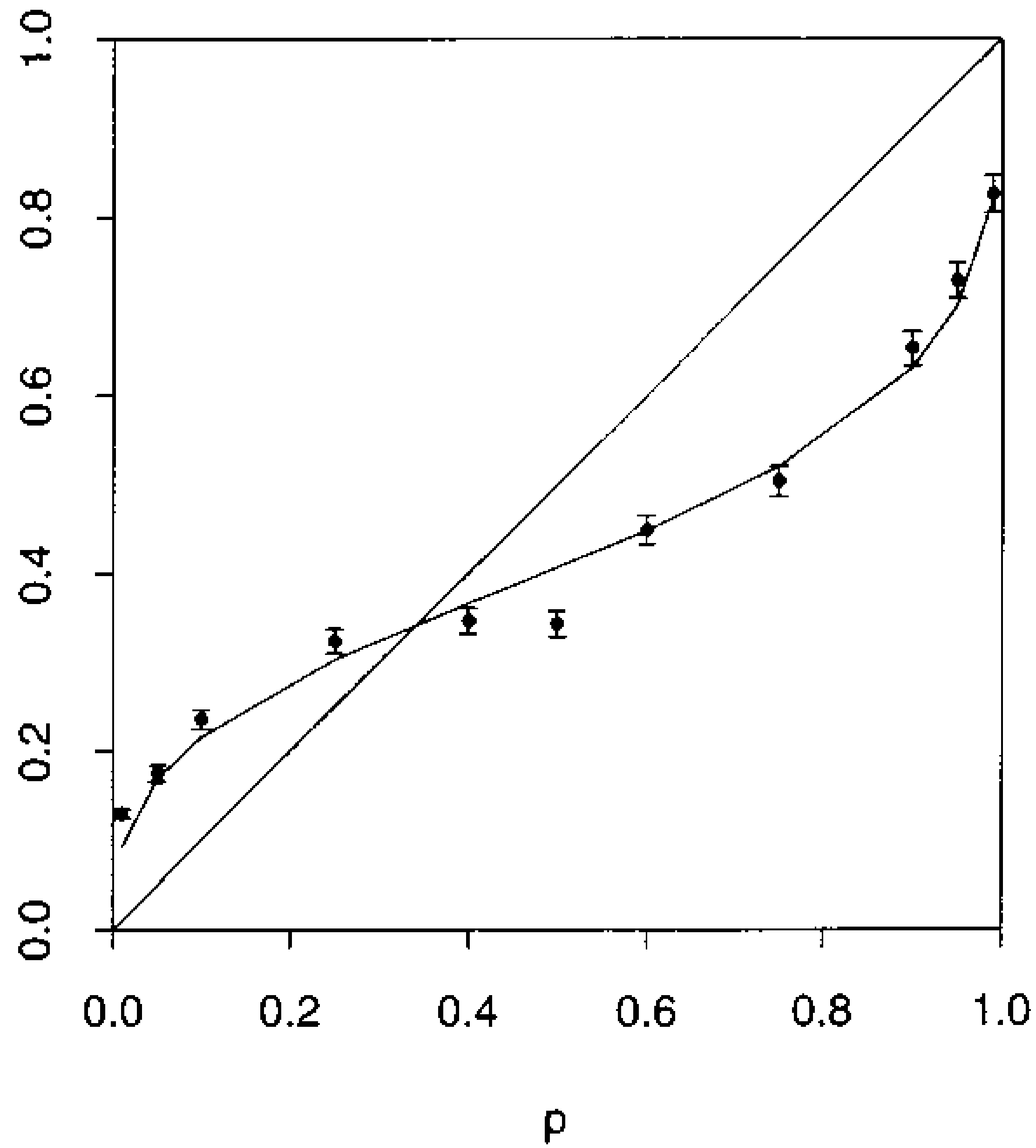
Empirical evidence: Probability weighting function

Gonzalez & Wu establish both a rationale and evidence for the shape of w:

- overweigh small probabilities (if larger than 0)
- underweigh large probabilities (if smaller than 1)
- nonparametric estimation procedure for assessing the probability weighting function and value function at the level of the individual subject
- interpretation: one parameter measures how the decision maker discriminates probabilities, and the other parameter measures how attractive the decision maker views gambling

Richard Gonzalez and George Wu, *On the Shape of the Probability Weighting Function.* Cognitive Psychology **38,** 129–166 (1999)

Data from one of the subjects in the empirical study by Gonzales & Wu justifying the S-shaped probability weighting function



More subjects' estimated probability weighting functions in column 2 and 4

Column 1 and 3 related to experiments estimating the utility function v which was also part of this study.

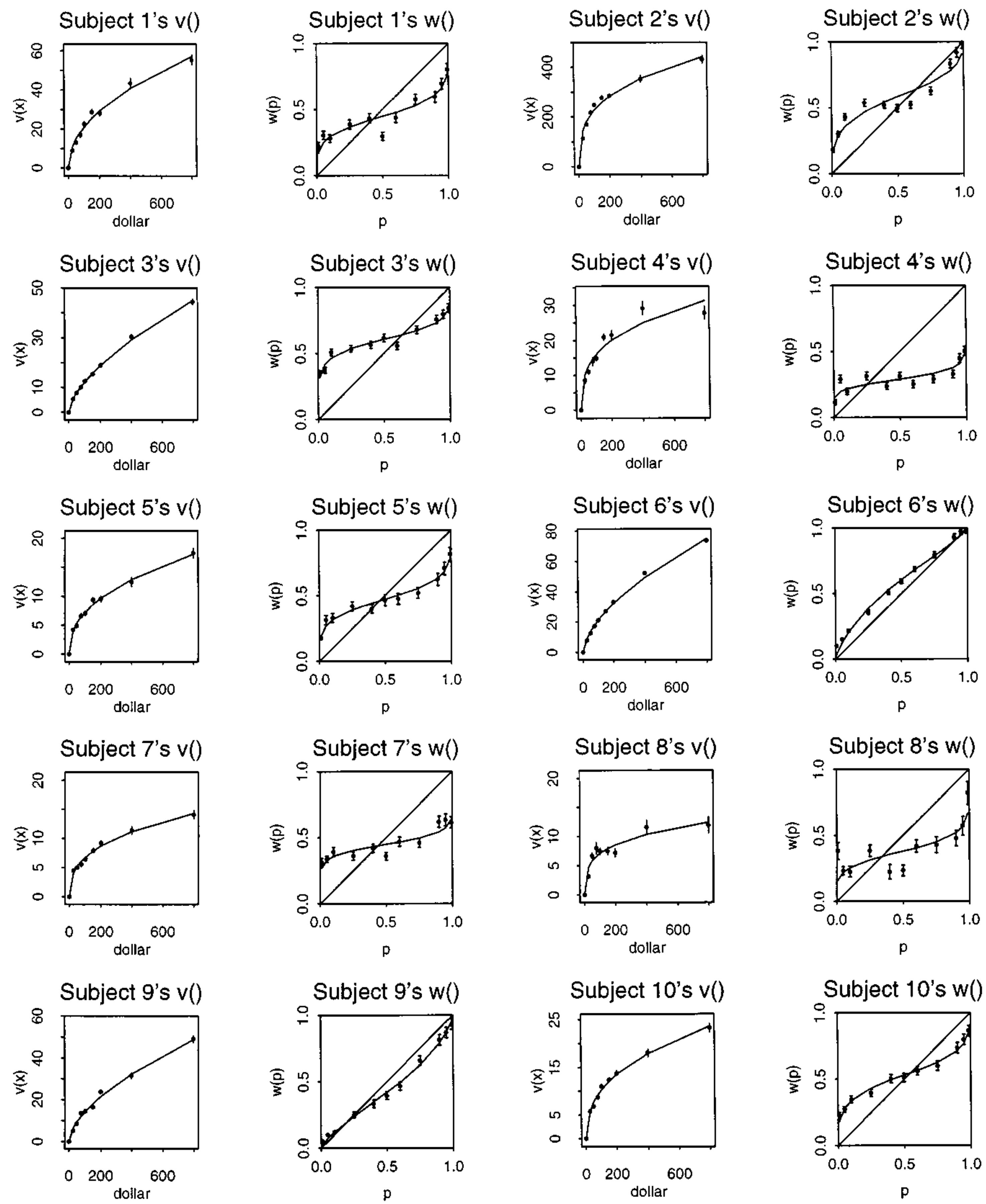


FIG. 8. Same as Fig. 6 with best fitting power function for *v* and best fitting linear in log odds *w* overlays. Note that value plots are scaled to each participant's own *v*.

Similar estimates across the literature

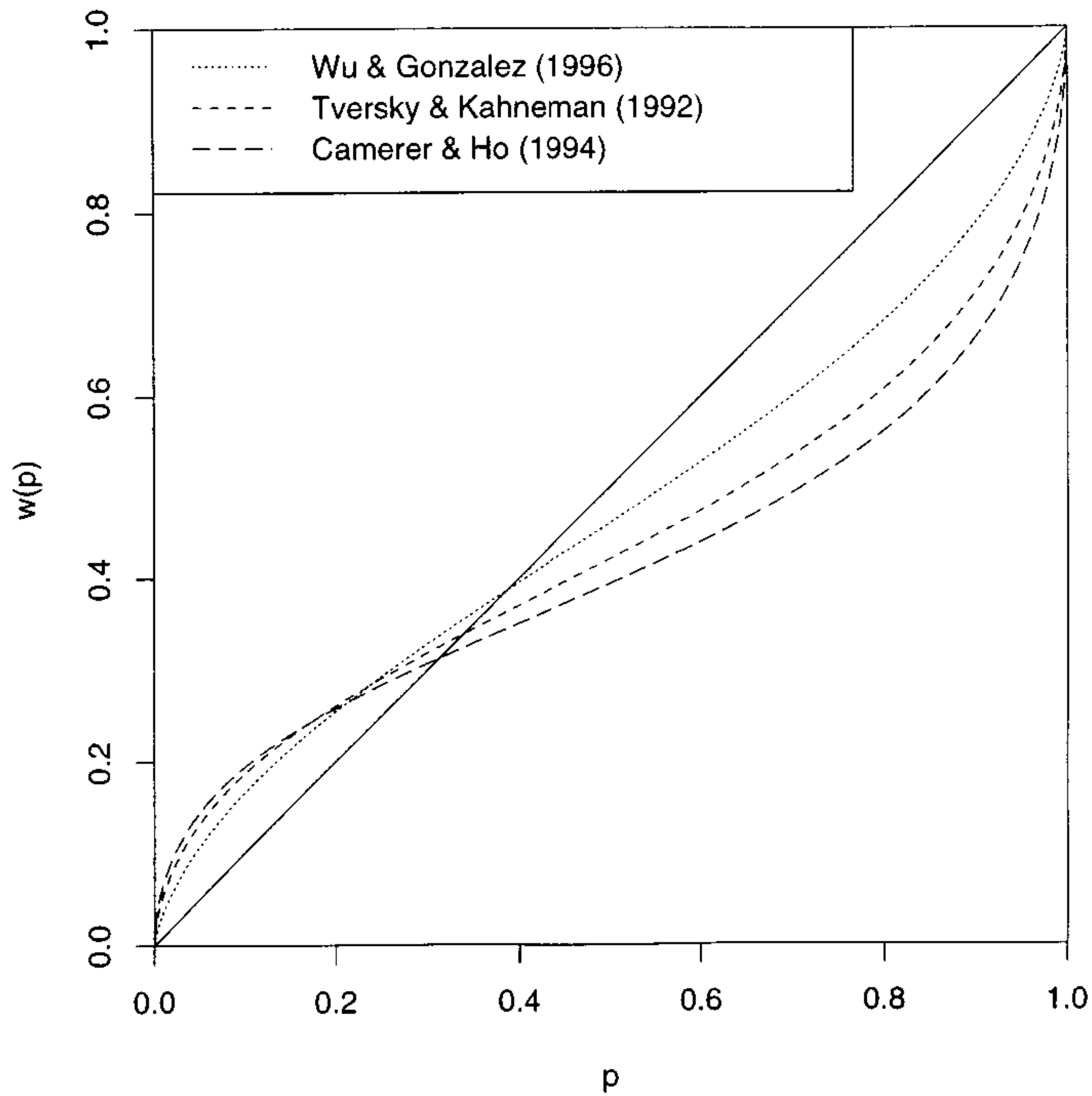


FIG. 2. One-parameter weighting functions estimated by Camerer and Ho (1994), Tversky and Kahneman (1992), and Wu and Gonzalez (1996) using $w(p) = (p^\beta/(p^\beta + (1-p)^\beta)^{1/\beta})$. The parameter estimates were .56, .61, and .71, respectively.

Concepts: Prospect theory's utility function

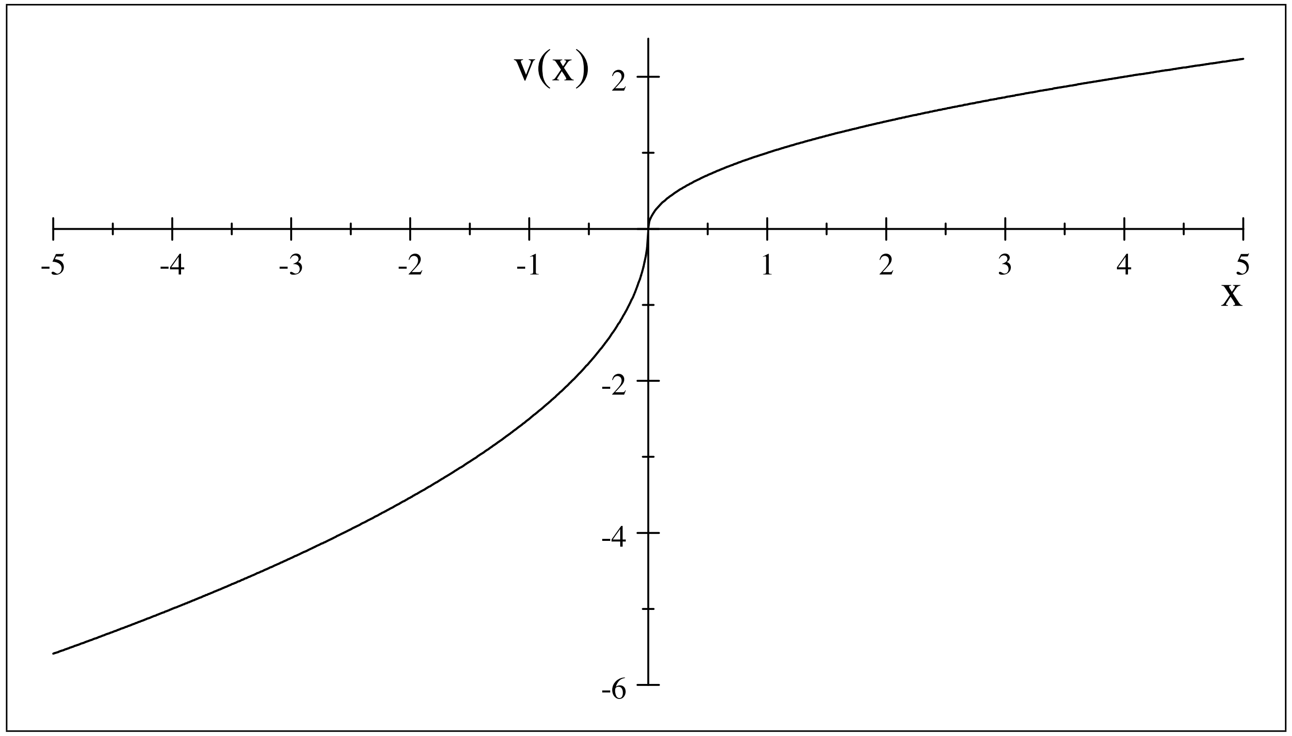
Definition *(Tversky and Kahneman, 1979). A utility function, $v(x)$, is a continuous, strictly increasing, mapping $v : \mathbb{R} \to \mathbb{R}$ that satisfies:*

1. $v(0) = 0$ (reference dependence). **(*)**

2. $v(x)$ is concave for $x \geq 0$ (declining sensitivity for gains).

3. $v(x)$ is convex for $x \leq 0$ (declining sensitivity for losses).

4. $-v(-x) > v(x)$ for $x > 0$ (loss aversion).



Parametric family for utility function often used by T&K:

$$v(x) = \begin{cases} x^{\alpha} & x \geq 0 \\ -\lambda(-x)^{\beta} & x < 0 \end{cases}$$

$\alpha > 0$: degree of risk aversion in gains

$\beta > 0$: degree of risk seeking in losses

$\lambda > 0$: degree of loss aversion

(*) Note: reference point depends on context, so may be changed

Plots: T & K's utility function for various parameters

alpha =
`0.25, 0.5,`
`1, 2, 4`

`beta = 1`