# On Managerial Risk-taking Incentives When Compensation May be Hedged Against*

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We consider a continuous time principal-agent model where the principal/firm compensates an agent/manager who controls the output's exposure to risk and its expected return. Both the firm and the manager have exponential utility and can trade in a frictionless market. When the firm observes the manager's choice of effort and volatility, there is an optimal contract that induces the manager to not hedge. In a two factor specification of the model where an index and a bond are traded, the optimal contract is linear in output and the log return of the index. We also consider a manager who receives exogenous share or option compensation and illustrate how risk taking depends on the relative size of the systematic and firm-specific risk premia of the output and index. Whilst in most cases, options induce greater risk taking than shares, we find that there are also situations under which the hedging manager may take less risk than the non-hedging manager.

**Key words:** compensation, incentives, hedging, specific and systematic risk.

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# 1 Introduction

Although it is hard to track empirically the personal portfolio allocation of CEO's and top managers, there is evidence that managers engage in hedging their compensation. Bettis, Bizjak and Lemmon (2001) provide evidence that insiders use equity swaps and zero-cost collars to reduce their ownership of the firm. Similarly, Ofek and Yermack (2000) provide empirical evidence that managers sell previously owned company shares when they are granted with new stock based compensation. This evidence suggests that managers realize that their human capital is tied to the firm and respond by diversifying their wealth. These findings suggest that the theoretical assumption that managers cannot undo their compensation exposure is not always appropriate. Motivated by this evidence, we tackle the question of including the possibility of hedging into the firm's contracting problem with the manager with a focus on managerial risk taking. We also examine incentive effects of compensation on managerial risk taking in presence of hedging when compensation is exogenous and consists of shares or call options.

A number of other papers in recent years have studied the effect of managerial hedging on incentives. We mention here the works of Jin (2002), Garvey and Milbourn (2003), Acharya and Bisin (2009), Bisin, Gottardi and Rampini (2008) and Ozerturk (2006). Not unlike this paper, in those studies the firm and the manager have CARA preferences. However, unlike this article, most of those papers are set in a traditional principal/agent theory setting in which the agent controls *only the expected return*, and not the risk/volatility of the output, and consider only linear contracts. In such a framework, they focus on the dependence of the pay-per-performance sensitivity (PPS) on the underlying risks. We are, on the other hand, interested in how the possibility of hedging and the form of compensation contracts, possibly nonlinear, influence the manager's *choice of risk/volatility*.

We model the relationship between a principal (a firm) and an agent (a manager) where the principal proposes a compensation to an agent consisting of a lump sum payment at a fixed date in the future. The agent influences the output process by controlling in continuous time the exposure to the source of risk (i.e. by controlling the volatility) and possibly the expected return of the output process. Both the manager and the principal have access to a market of securities where they can trade in order to diversify their payoffs.

We find optimal contracts from the firm's point of view, in the first-best case of symmetric information where the principal can observe the agent's effort. Optimal contracts may depend on the returns of the assets available for hedging, belonging to the type of compensation usually referred to as relative performance evaluation, or RPE. That is, the manager is compensated partly relative to the performance of the exogenous assets, which she uses for hedging. Moreover, in the first-best framework of observable managerial actions, and if the manager has either CARA preferences or devotes zero initial capital to hedging (say, using futures or swaps, as in Ozerturk (2006) or zero-cost collars as in Bettis *et al* (2001)), we find that there is a contract which is optimal regardless of whether the manager can hedge or not, and which will, in fact, induce the manager not to hedge.

The RPE feature is partially due to the desire to remove the "systematic risk" component from the contract, a theoretical prediction that has been known since the work of Holmstrom (1982). Since our firm is risk averse, RPE is also motivated by hedging needs of the firm. Moreover, as mentioned above, the firm may want to offer a contract which preempts managerial hedging (by performing hedging on her account), and this also leads to the dependence of the payoff on the risk factors driving the hedging opportunities. This is in agreement with findings of Jin (2002) and Garvey and Milbourn (2003), who find that RPE should be used when the managers are constrained in their hedging opportunities, while there is no need for RPE if the managers have low cost of hedging. In relation to this, it should be mentioned that in our first-best world there may be more than one optimal contract, and we only consider the contracts which induce the manager not to hedge, which we call hedge-neutral contracts, which may then include the RPE component even when managerial hedging is costless.

In order to illustrate the impact of hedging on optimal contract in the symmetric information case, we consider a two-factor model where the firm's performance is driven by two sources of risk. One source of risk is interpreted as the market wide risk and the other source of risk is interpreted as the firm-specific (or specific) risk. The manager can possibly control the exposure to both sources of risk and she can also trade a risky security (interpreted as the index) and a risk free security. We find that the optimal hedge-neutral contract is linear in the output value and in the log-returns of the risky asset used for hedging, the latter being the RPE component. However, because of the linearity and costless hedging, there is actually no need for the RPE component: the firm is indifferent with respect to the size of the RPE in the contract.

The above optimal contract may not be feasible in practice, as the firm and the manager may not have CARA preferences, or the same (and known) hedging opportunities, and there may be limited liability constraints. For this reason, in the second part of the paper we utilize the two factor model to examine numerically the incentive effects of compensating a hedging CARA manager with contracts used in practice, that is, share and call options.[1]

Our first insight is that the manager's behavior depends on whether she controls the overall risk, or separately the specific and the systematic risk. Moreover, it depends on the size of the corresponding risk premia. In particular, when the output's risk premium is low, she will choose low total volatility when paid with shares, while choosing higher volatility when paid with calls. As the output's risk premium gets higher, she becomes less conservative when paid with shares. If she can influence only the systematic risk, when the systematic risk premium is equal to the hedging asset's risk premium the manager is indifferent with respect to the size of systematic risk when paid with shares, while calls encourage her to choose high systematic risk. When the manager can control only the specific risk, she may go for low or high risk, depending on the relative sizes of the risk premia.

Our findings suggest that the hedging manager is less conservative than the non-hedging manager, in general. However, if compensated with shares, the hedging manager may be more conservative than the non-hedging manager when modifying the total volatility or the systematic risk, if the corresponding output's risk premium is low. These results show that the intuition that the possibility of hedging will increase the manager's ability to bear risk is broadly correct, but that there can be situations where it is not the case.[2]

We describe the general setting in Section 2, analyze optimal contracts under symmetric information in Section 3, and discuss incentives of particular contracts in Section 4. Section 5 concludes, while technical proofs are provided in an Appendix.

# 2 Setting

Suppose a manager is promised a compensation with payoff $C_T$ at future time $T$. However, the manager also trades on her own account, with value $H_t$ at time $t$. She may be able to influence an output process (e.g., the stock price of a company, a portfolio value, and so on), by exerting effort and/or by choosing different "projects" (say, picking different stocks to invest in). Altogether, she is maximizing the value of

$$E[U_A(C_T - G_T + H_T)] \tag{2.1}$$

where $U_A$ is her utility function ($A$ for agent), and $G_T$ is a cost of exerting effort, if any. Notice that the manager can alter the utility either by changing the variables $C_T$ and $G_T$ by influencing the output process or by changing the variable $H_T$ by trading on a private account in the market for securities.

Let us now fix the compensation $C_T$ and the cost $G_T$ (implicitly also fixing the actions of the manager related to the firm). Then, in the standard model, the optimal hedging strategy of the manager will result in the marginal utility of the manager being proportional to a stochastic discount factor (henceforth SDF) in the market in which she is trading, and which includes the claim $C_T - G_T$. More precisely, if we introduce the inverse of the marginal utility

$$I_A(x) = (U_A')^{-1} \tag{2.2}$$

the optimal hedging strategy results in

$$C_T - G_T + H_T = I_A(zZ_T^{C,G}) \tag{2.3}$$

where $Z_T^{C,G}$ is an appropriate SDF, and the constant $z$ is determined so that the budget constraint is satisfied:

$$H_0 = E[Z_T^{C,G} H_T] = E\left[Z_T^{C,G}\left(I_A(zZ_T^{C,G}) - C_T + G_T\right)\right] \tag{2.4}$$

Here, $H_0$ is the initial capital devoted to hedging. Thus, from (2.1) and (2.3), we see that the manager would like to choose her actions so as to maximize the value

$$E[U_A(I_A(zZ_T^{C,G}))] \tag{2.5}$$

under the budget constraint (2.4).

When the market is complete, the claim $C_T - G_T$ is attainable for any compensation package and effort cost. The manager can liquidate these cash flows and invest the proceeds optimally in the securities market. It is intuitive then that the manager should act to maximize the market value of the payoff $C_T - G_T$. More formally, when the market is complete, $Z_T$ is the unique SDF, independent of the contract $C_T$ and cost $G_T$. Then we conclude from (2.5) that the manager wants to minimize the value of $z$. From (2.4), we obtain the following "common wisdom" result:

**Proposition 1** *Suppose that the manager has access to a complete market with the SDF $Z_T$. In particular, suppose that for any feasible payoff $C_T$ and cost $G_T$ there is a hedging strategy which accomplishes* (2.3), *with* (2.4) *satisfied. Then the manager will choose her actions so as to maximize the no-arbitrage value $E[Z_T(C_T - G_T)]$ of her compensation minus the costs.*

Therefore, when it is possible to hedge the compensation risk by trading in a complete market, the risk averse manager acts as if she was risk neutral. For example, if $G_T = 0$

and if $C_T$ is a call option, and the manager's actions influence the volatility of the option's underlying, then the manager would choose the effort so as to increase the volatility as much as possible. This is in contrast with the case in which the manager cannot hedge at all. For example, Ross (2004) shows that a manager who cannot hedge, and has a DARA utility function, would not necessarily become less risk averse when call options are added to her compensation, and may, in fact, become more risk averse. Carpenter (2000) also pointed out that convex payments do not necessarily induce risk seeking behavior.[3] In our present setting, on the contrary, the possibility of perfect hedging always induces less risk aversion when compensation is in call options. However, this requires the assumption rarely satisfied in practice that no cost is incurred when modifying volatility.

If the market for hedging is incomplete, so that the manager can only partially hedge the risk of the compensation payoff, the situation is much more complicated. In that case there are many risk-neutral densities, hence many SDF's, and the marginal SDF for the manager's optimization problem may depend in a complex way on the payoff $C_T$ and cost $G_T$. In any case, the manager no longer maximizes solely the market price of $C_T - G_T$, and her risk aversion would come into the picture. The remainder of this paper concerns this more realistic situation. We consider some tractable cases in sections below.

# 3 Optimal Contracts with incomplete markets and symmetric Information

In this section we assume the first-best world of symmetric information, in which the firm can force the manager to apply actions of the firm's choosing. In that case, the firm is definitely not worse off if the manager can hedge, because it is easier for the firm to meet the manager's "participation constraint", that is, to meet her reservation wage, while there is no need to worry about incentives. Moreover, if we assume either CARA preferences for the manager, or that the manager starts hedging with zero initial capital, we show now that there is a contract which is optimal regardless of whether the manager can hedge or not. We start by studying a general context and then provide an example where the optimal contracts can be derived in closed form.

## 3.1 Optimal contract with CARA preferences

We take the following steps: (i) we find a representation for optimal contracts when the manager does not hedge; (ii) we show that given such a contract the manager would not hedge even if she could, but would deposit all the hedging capital into the risk-free account – we call such contracts "hedge neutral"; (iii) we show that even when the manager can hedge, the search for optimal contracts can be restricted to the family of hedge-neutral contracts.

**Step (i): a representation of optimal contracts.** Denote by $X_T$ the output influenced by (observable) manager's actions and by $G_T$ the cumulative cost of manager's actions. With symmetric information and in absence of managerial hedging, but allowing the firm to hedge, the firm would be maximizing, for some constant weight $L$ depending on the reservation wage, the expression

$$E[U_P(X_T - C_T + F_T)] + LE[U_A(C_T - G_T)] \tag{3.6}$$

over the contract payoff $C_T$, the actions influencing $X_T$ and the firm's hedging strategy with $T-$value $F_T$. Here $U_P$ is the utility function of the firm ($P$ for principal). We assume that the firm has access to a market of assets for the purpose of hedging, and we consider the family $\mathcal{Z}$ of SDFs in that market, in which the information structure is given by the random factors driving the traded assets, plus the random factors driving the manager's output process. Using standard martingale methods it follows that under a wide range of conditions the optimal hedging strategy will be such that

$$U_P'(X_T - C_T + F_T)] = z' Z_T \tag{3.7}$$

for some SDF $Z \in \mathcal{Z}$ and some constant $z'$. Then, taking a derivative inside the expectation in (3.6) with respect to $C_T$, we see that the first-order condition is

$$C_T = G_T + I_A(zZ_T) \tag{3.8}$$

for $z = z'/L$. If this is also a sufficient condition, the first-best contract will be of this form. We now argue that given such a contract, the manager will not hedge, but will deposit all her hedging money into the bank account.

**Step (ii): hedge-neutral contracts.** Assume the manager can hedge, starting with $H_0$ in initial capital, and resulting in the hedging portfolio value of $H_T$ at time $T$. For fixed $C_T, G_T$, introduce the value function of the manager:

$$V(H_0) = \max E[U_A(C_T + H_T - G_T)] \tag{3.9}$$

where the maximum is taken over hedging strategies, and it is assumed to be attained. We have the following result:

**Proposition 2** *Suppose either that the manager has CARA preferences, or that her initial hedging capital $H_0$ is zero. Also assume that the interest rate is deterministic.*[4] *If a contract $C_T$ is of the form*

$$C_T = G_T + I_A(zZ_T) \tag{3.10}$$

*where $Z_T$ is an SDF in $\mathcal{Z}$, then $C_T$ is hedge-neutral. Conversely, if the dual problem* (5.50) *in Appendix, with value $\bar{V}(H_0)$, has a solution for a given contract $C_T$ and if we have $V(H_0) = \bar{V}(H_0)$,*[5] *then, in order for $C_T$ to be hedge-neutral it has to be of the form* (3.10).

The proposition tells us that a hedge-neutral contract is such that the manager's marginal utility is proportional to an SDF. This is because that is exactly what the manager would like to accomplish by hedging.

We provide a proof of the proposition in Appendix, while giving here an argument in a simple single-period market, with a risk-free rate $r$ and a risky asset $S$ whose possible values at time $T$ are denoted $S_T^i$, possible values of the SDF $Z$ are denoted $Z_T^i$, and the corresponding probabilities are $p_i$: The manager needs to maximize over the number of shares $\delta$

$$\sum_i p_i U_A \left( I_A(zZ_T^i) + \delta S_T^i + (H_0 - \delta S_0)(1+r) \right) \tag{3.11}$$

Under our assumptions, the maximizer of this expression does not change if we delete the term $H_0(1+r)$. The first-order condition is then

$$0 = \sum_i p_i U_A' \left( I_A(zZ_T^i) + \delta(S_T^i - S_0(1+r)) \right) (S_T^i - S_0(1+r)) \tag{3.12}$$

When $\delta = 0$, the right hand side of equation (3.12) becomes

$$\sum_i p_i U_A' \left( I_A(zZ_T^i) \right) (S_T^i - S_0(1+r)).$$

Since $U_A'(I_A(x)) = x$, and since $Z_T^i$ is an SDF, the first order condition (3.12) is satisfied when $\delta = 0$:

$$\sum_i p_i U_A' \left( I_A(zZ_T^i) \right) (S_T^i - S_0(1+r)) = z \sum_i p_i Z_T^i (S_T^i - S_0(1+r)) = 0.$$

**- Step (iii): it is enough for the firm to consider the hedge-neutral contracts.** Indeed, suppose that the firm gives a contract $C_T$ which induces the manager to hedge optimally in a way that results in the final amount of $H_T$, starting with initial capital $H_0$. Instead, the firm could borrow $H_0$ at the risk-free rate, invest it in the same way the manager would, and thus have $H_T$ at time $T$ as a result of that investment, and pay $C_T + H_T - H_0 B_T$ to the manager at time $T$, where $B_t$ is the value process of the risk-free account. This would result in the same utility for the firm as the contract with payoff $C_T$, because the firm would be able to return the debt of $H_0 B_T$, while it would forward the remaining profit/loss $H_T - H_0 B_T$ to the manager. It would also result in the manager's utility of $E[U_A(C_T + H_T - G_T)]$, if she invests all of the hedging capital $H_0$ in the risk-free asset. This is exactly what is optimal for the manager. Thus, to recap, the manager would not hedge with this contract, and both the firm and the manager are equally well-off as with the original contract.

Combining all of the above, we get the conclusion announced in the first paragraph of this section, which we now state as

**Theorem 1** *Under the above conditions, in particular assuming the company can observe the manager's actions, if the manager has CARA preferences or starts hedging with zero initial capital, there is a hedge-neutral contract which is optimal both in the presence and in the absence of hedging opportunities.*

Note that the contracts of the form (3.10) will not, in general, be based solely on the random factors driving the manager's performance, but also on those driving the assets in the hedging market, leading to a relative performance evaluation (RPE) component of the contract. We illustrate this fact with an example in the following subsection. The RPE component is present for the purpose of removing the non-specific risk from the manager's payoff (as in Holmstrom 1982), as well as from the firm's payoff as the firm is also risk averse. At the same time RPE also preempts the managerial need to hedge.

In general, the optimal contract we found requires full monitoring of the manager's actions by the firm. However, we show in the following section that there may exist an equivalent contract which requires no such monitoring.

## 3.2 A Two-Factor Model and CARA preferences

In Cadenillas, Cvitanić and Zapatero (2007) it was shown, in the absence of hedging and with zero cost function, that when the output is of the "portfolio value" form $dX_t = \alpha_t v_t dt + v_t dm_t$ for some martingale $m$, where $v$ can be modified freely so that all possible random outcomes can be replicated by $X$ ("complete market"), there exists a contract of the form $f(X_T)$ which is optimal and does not require that the firm monitor $v_t$, even with non-CARA preferences. We present here a model of similar type, but assuming CARA preferences and incomplete markets. It turns out that there are optimal contracts which are linear in the manager's output and in the log-value of the risky asset she is hedging with, and the firm is indifferent with respect to whether to include the hedging asset into the compensation package.

Consider the model

$$dX_t = [\alpha_x x_t + \alpha_y y_t]dt + x_t dW_t + y_t dM_t \tag{3.13}$$

$$dS_t/S_t = \mu dt + \sigma dW_t \tag{3.14}$$

where $W$ and $M$ are independent Brownian motions, $\alpha_i, \mu, \sigma$ are constants, and $x_t$, $y_t$ are adapted processes interpreted as the systematic risk and the specific risk. Then, $\alpha_x$ is interpreted as the systematic risk premium, and $\alpha_y$ is the specific risk premium.

Both the manager and the firm can hedge by trading in the risky asset $S$ and a risk-free asset. For simplicity, we set the risk-free rate to zero. Introduce the process

$$dZ_t^\theta = -Z_t^\theta[\lambda dW_t + \theta_t dM_t] \tag{3.15}$$

where $\lambda = \mu/\sigma$ is the risk premium of $S$, and $\theta_t$ is an arbitrary adapted process. Then, processes $Z^\theta$ represent SDFs for the incomplete market in which only $S$ is traded, but the information structure includes, in addition, the process $M$.

Suppose there is no cost of effort, so $G_T = 0$. We assume that the preferences are of CARA type:

$$U_i(x) = -\frac{1}{\gamma_i}e^{-\gamma_i x} \tag{3.16}$$

Then,

$$I_i(y) = -\frac{1}{\gamma_i}\log(y)\ , \quad U_i(I_i(y)) = -\frac{1}{\gamma_i}y \tag{3.17}$$

Thus, $E[U_A(I_A(zZ_T^\theta))] = -z/\gamma_A$, and $z$ is fixed by the manager's reservation value. We have shown above that an optimal contract for the firm is of the form $C_T = I_A(zZ_T^\theta)$. Given such contract, the firm is maximizing the value $E[U_P(X_T - I_A(zZ_T^\theta) + F_T)]$, where $F_T$ is the value of the firm's hedging portfolio. That value can also be written as

$$E\left[U_P\left(X_T + F_T - \frac{1}{\gamma_A}[\frac{1}{2}\int_0^T (\lambda^2 + \theta_t^2)dt + \int_0^T \lambda dW_t + \int_0^T \theta_t dM_t]\right)\right] \tag{3.18}$$

Straightforward computations (provided in an Appendix) now lead to the following conclusions:

- (i) Assume that the variables $x_t, y_t$ are taken as given by the manager and the firm and that they are deterministic. The optimal choice for the parameter process $\theta_t$ is

$$\theta_t \equiv y_t \frac{\gamma_A \gamma_P}{\gamma_A + \gamma_P} \tag{3.19}$$

The optimal amount $\pi$ invested in $S$ by the firm satisfies

$$\sigma \pi_t + x_t \equiv \lambda \frac{\gamma_A + \gamma_P}{\gamma_A \gamma_P} \tag{3.20}$$

- (ii) Suppose the specific risk $y_t$ can be modified by the manager, while $x_t \equiv x$ is constant and fixed. Then, it is optimal for the firm to have

$$y_t \equiv \alpha_y(\frac{1}{\gamma_A} + \frac{1}{\gamma_P}) \tag{3.21}$$

In other words, the specific risk will match the specific risk premium adjusted by the sum of inverted risk aversions. If either the firm or the manager are almost risk-neutral, the optimal specific risk becomes large. More interestingly, it can also be easily checked that the contract is ex-post linear, [6]

$$C_T = aX_T + b\log(S_T) + c \tag{3.22}$$

where we have

$$a = \frac{\gamma_P}{\gamma_A + \gamma_P} \ , \quad b = \frac{\lambda}{\sigma\gamma_A} - \frac{a}{\sigma}x \tag{3.23}$$

Actually, the firm is indifferent with respect to which $b$ it uses, due to the possibility of hedging with asset $S$; different choice of $b$ will induce a different choice of $c$, and the above choice of $b$ is the one with which the manager would not hedge. Moreover, if, instead of being promised $C_T = I_A(zZ_T^\theta)$, the manager is offered the contract in the form $C_T = aX_T + \tilde{b}\log(S_T) + c$ ex-ante, for $a$ as above and any $\tilde{b}$, she will use the above value of $y_t$, even without being dictated by the firm to do so, and thus, such linear contracts are optimal. When there are no hedging possibilities, it is known that a contract of the same linear form is optimal, with the same $a$, and with $b = 0$ (e.g., Cadenillas, Cvitanić and Zapatero (2007)).

- (iii) Suppose the systematic risk $x_t$ can be modified, while $y_t$ is deterministic and fixed. It now has to be the case that the corresponding risk premia are equal, $\lambda = \alpha_x$, otherwise there is arbitrage for the firm by appropriate choice of $x$ and $\pi$. Assuming $\lambda = \alpha_x$, it is optimal for the firm to have

$$x_t \equiv \alpha_x\left(\frac{1}{\gamma_A} + \frac{1}{\gamma_P}\right) \tag{3.24}$$

The manager would not hedge if given a contract $C_T = aX_T + c$, with $a$ as above. Moreover, if the manager was given the contract $C_T = aX_T + \tilde{b}\log S_T + c$ for any $\tilde{b}$, instead of being promised $C_T = I_A(zZ_T^\theta)$, she would be indifferent between various choices for the systematic risk $x$. Intuitively, the manager being able to modify systematic risk and having the ability to hedge are substitutes - hence the manager adjusts to his desired level of systematic risk via choice of $x_t$ and the firm does not need to provide a contract with RPE component.

- (iv) Suppose both the systematic risk $x_t$ and the specific risk $y_t$ can be controlled. We again need to have $\lambda = \alpha_x$. Then, it is optimal for the firm to have the same $x_t$ as in (iii), and the same $y_t$ as in (ii). We again have that $C_T = aX_T + c$ induces the manager not to hedge, with the same $a$ as in (iii), and a similar discussion applies, with the following modification: if the manager was offered $C_T = aX_T + \tilde{b}\log(S_T) + c$ for any $\tilde{b}$, she would be indifferent between various choices for the systematic risk $x$, and she would choose the specific risk $y_t$ which is optimal for the firm, too.

- (v) A manager who cannot hedge and is offered $C_T = aX_T + \tilde{b}\log(S_T) + c$ would choose optimally the values also optimal for the firm.

- (vi) Assume now that only the total volatility $v$ can be controlled, with the model for $X$ being

$$dX_t = \lambda v_t dt + \rho v_t dW_t + \sqrt{1-\rho^2} v_t dM_t \tag{3.25}$$

It can be computed that the optimal volatility for the firm is

$$v = \frac{\gamma_A + \gamma_P}{\gamma_A \gamma_P} \frac{(1-\rho)\lambda}{1-\rho^2}$$

If the manager was offered $C_T = aX_T + \tilde{b}\log(S_T) + c$ for any $\tilde{b}$, she would choose the same $v_t$ optimal for the firm, too.

The main message of this discussion is that with CARA preferences, simple linear contracts are optimal even in the presence of hedging opportunities, and they do not require the firm to monitor the manager's choice of risk. Furthermore, the firm is indifferent with respect to the contract's level of dependence on the value of hedging assets (the RPE level).

# 4 Performance Based Contracts in a Two-factor Model

The above-discussed optimality of linear contracts for CARA managers hinges on several assumptions: the firm also has CARA preferences; the firm and the manager have the same hedging opportunities and that fact is known by the firm; and there are no limited liability constraints (the contract payoff may take negative values). In reality, these assumptions may not be satisfied, and the firm may offer nonlinear contracts, and, if the hedging opportunities of the manager are not known, the contracts may depend only on the manager's output. Motivated by this, we study now the effect on managerial actions of compensation in call options as compared to compensation in shares. We consider a two-factor model that is analogous to the ones in the previous sections. Given two Brownian Motions $W$ and $\tilde{W}$

with correlation $\rho$, the manager can trade in a risky asset which satisfies

$$dS_t/S_t = \mu_t dt + \sigma_t dW_t \tag{4.26}$$

and the bank account satisfying

$$dB_t = B_t r_t dt \ , \quad B_0 = 1 \tag{4.27}$$

She can influence the output process given by

$$dX_t = [r_t X_t + \alpha_t v_t]dt + v_t d\tilde{W}_t \tag{4.28}$$

by modifying the values of $v_t$. For example, this includes the case when $X$ is a value of a managed portfolio when investing in a risky asset with risk-premium $\alpha_t$ and a risk-free asset with interest rate $r_t$, according to a portfolio strategy $v_t$.

Denote by $\tilde{F} = \{\tilde{\mathcal{F}}_t\}$ the filtration generated by $\tilde{W}$ and introduce the risk-premium process

$$\lambda_t = \frac{\mu_t - r_t}{\sigma_t} \tag{4.29}$$

We assume that the parameters $v, \alpha, r, \lambda$ are $\tilde{F}$-adapted, and that the interest rate $r$ is constant. In this section we set $G_T \equiv 0$ and we consider only the contract payoffs $C_T$ which are measurable with respect to $\tilde{\mathcal{F}}_T$. For example, $C_T$ can be a functional of the output process $X$, such as an option written on that process. However, it cannot depend on the hedging asset $S$. We recall now a result by Henderson (2002) and extended by Tehranchi (2004).

**Proposition 3** *The optimal value of the expected utility for the CARA manager given payment $C_T$, is*

$$V_A := E[U_A(C_T + H_T)] = -\frac{1}{\gamma_A} e^{-\gamma_A H_0 e^{rT}} \left( E^Q e^{-\gamma_A (1-\rho^2) C_T - \frac{1-\rho^2}{2} \int_0^T \lambda_s^2 ds} \right)^{\frac{1}{1-\rho^2}} \tag{4.30}$$

*where $Q$ is the probability measure under which the processes*

$$W_t^Q = \tilde{W}_t + \rho \int_0^t \lambda_s ds \tag{4.31}$$

*and $W$ are Brownian motions.*[7]

Recall that the agent maximizes the expected exponential utility by altering the compensation $C_T$ through the choice of the volatility process $v_t$ and also by altering the hedging portfolio through trading the stock and the bond. Proposition 3 shows that the agent's can solve the optimization problem by ignoring the hedging decision and maximizing a state dependent utility function with a modified risk aversion and a modified subjective beliefs. Specifically, the agent chooses the volatility process by maximizing

$$E^Q\left[\xi(\omega)e^{-\gamma_A(1-\rho^2)C_T}\right]$$

where $\xi(\omega) = e^{-\frac{1-\rho^2}{2}\int_0^T \lambda_s^2 ds}$. When the hedging asset's risk premium $\lambda$ is independent of the contract compensation $C_T$ under measure $Q$, the variable $\xi(\omega)$ can be taken out of the expectation and *the CARA manager will choose her actions as would the manager who cannot hedge, but who has risk aversion reduced by a factor of $1-\rho^2$, and whose subjective probability is $Q$.* The reduction of risk aversion would make the hedging manager more aggressive, but the change of the subjective probability has ambiguous effects. Effectively, the latter reduces the output's drift by the amount $\rho\lambda\mathbf{v}$, as in (4.32) below, and thus may lead to the manager becoming more conservative. The combination of the two effects will determine whether the manager is less or more aggressive when she can hedge and thus whether the ability to hedge increases (or not) the manager's ability to bear risk.

The above result will enable us to compute such incentive effects when offering to the manager various types of contracts. We specialize the model to the case where the process

$v_t$ is assume to be constant, *i.e.* $v_t = v$,

$$dX_t = (\alpha - \lambda\rho)vdt + vdW_t^Q \tag{4.32}$$

where we assume that all the parameters are constant. For the computations, it will be useful to denote

$$b(v, \rho) = X_0 + vT(\alpha - \rho\lambda) \tag{4.33}$$

$$c(\rho) = n\gamma_A(1 - \rho^2) \tag{4.34}$$

where $n$ is the number of contracts issued. We also normalize $H_0$ to be zero.

We will consider two settings, one in which the manager can control the total volatility $v$, and another in which she may possibly be able to control separately the specific risk and the systematic risk, as above.

### 4.0.1 Share contract

Assume the manager is given $n$ shares,

$$C_T = nX_T \tag{4.35}$$

Then, it is straightforward from (4.30) to compute the manager's utility

$$V_A = -\frac{1}{\gamma_A} e^{-T\lambda^2/2 - \gamma_A n b(v,\rho)) + \frac{1}{2}T\gamma_A^2 n^2(1-\rho^2)v^2} \tag{4.36}$$

Suppose the manager can control the total volatility $v$. Recall that we showed above that in the case of hedging in a complete market the manager compensated by shares is indifferent with respect to which volatility to choose, as the market value of the shares is fixed. This is not the case here, where the optimal level of volatility is

$$v(\rho) = \frac{1}{n\gamma_A} \frac{\alpha - \lambda\rho}{1 - \rho^2} \tag{4.37}$$

With $\rho = 0$ the manager cannot hedge the exposure to the compensation risk. In that case the optimal total volatility is $v(0) = \frac{\alpha}{n\gamma_A}$. The difference is

$$v(\rho) - v(0) = \frac{\rho(\alpha\rho - \lambda)}{(1 - \rho^2)n\gamma_A} \tag{4.38}$$

Thus, we have the following conclusions:

- If the correlation is positive and the output's risk premium adjusted by correlation, $\alpha\rho$ is larger than the hedging asset's risk premium $\lambda$, then the hedging manager prefers higher volatility than the non-hedging manager. And vice versa when $\alpha\rho < \lambda$. The difference in preferred volatilities becomes small as the number of shares or the risk aversion become large. It can become huge as the correlation gets closer to 1 or -1.

- It may very well happen that the manager who hedges would prefer lower volatility even in absolute value, than the manager who does not hedge. For example, for fixed positive $\rho$, because of diversification the hedging manager wants volatility that is decreasing in the hedging asset's risk premium $\lambda$, and for the values of $\lambda$ close to $\alpha/\rho$ she will want volatility close to zero.

- Similar results would hold in the case of CRRA utility functions, if $X$ was a geometric Brownian motion process instead of arithmetic Brownian motion.

Suppose now we fix $v$. The value of the manager's utility depends on the value of $\rho$ in a quadratic fashion (up to a monotone transformation), and it is highest for $\rho = -1$, if $\lambda v > 0$. When $\rho = -1$, the highest utility is attained for the highest value of $v$, if $\alpha > \rho\lambda$. In other words, if $\alpha > \rho\lambda$, if the manager can find an asset fully negatively correlated with the stock compensation, she will choose highest volatility. On the other hand, in the case $\alpha < \lambda\rho$, $\rho = -1$, the manager wants as low volatility as possible. In the boundary case when $\alpha = \rho\lambda$, the manager wants to make the specific risk term $(1 - \rho^2)v^2$ small, and doesn't care about the systematic risk term $\rho^2 v^2$.

We next consider the case when the manager can control the systematic risk $x$ and the specific risk $y$ separately, so that the output process is given by

$$dX_t = \alpha\sqrt{x^2 + y^2}dt + xdW + ydM \tag{4.39}$$

where $M$ represents an idiosyncratic Brownian motion which is independent from $W$. We also assume that the systematic risk premium $\alpha_x$ and the specific risk premium $\alpha_y$ are constant so that the price of risk has the form

$$\alpha = \rho\alpha_x + \sqrt{1 - \rho^2}\alpha_y \tag{4.40}$$

for some constants $\alpha_x$ and $\alpha_y$. Notice that $x$ and $y$ can be mapped to $v$ through

$$x = v\rho\ , y = v\sqrt{1 - \rho^2} \tag{4.41}$$

We get the following conclusions, consistent with a previous section:

- If the risky asset's risk premium is equal to the corresponding systematic risk premium of the output, that is, if $\lambda = \alpha_x$, the manager is indifferent between various choices for the systematic risk $x$.

- Note that if the output $X$ can be traded in the market, then we necessarily have $\lambda = \alpha_x$, in the absence of arbitrage. For completeness, we state the other two cases as well. That is, if $\alpha_x < \lambda$, then the manager likes as small systematic risk as possible, while she likes it to be high when $\alpha_x > \lambda$.

- The optimal value for the specific risk $y$ is

$$y = \frac{1}{n\gamma_A}\alpha_y \tag{4.42}$$

That is, if the specific risk premium $\alpha_y$ is positive, the manager would like to choose lower specific risk with higher number of shares paid, and higher risk aversion.

A manager who cannot hedge would choose optimally the value $x_t \equiv \alpha_x/(n\gamma_A)$, and, the same as the hedging manager, the value $y_t \equiv \alpha_y/(n\gamma_A)$.

### 4.0.2 Call options contracts

Assume now that the manager is compensated with a stock option with exercise price $K$:

$$C_T = n(X_T - K)^+ \tag{4.43}$$

Using the fact that process $v$ is simply a constant, standard Gaussian truncated moments calculations show that the manager's utility is given by

$$-\gamma_A V_A = e^{-T\lambda^2/2}(I_1 + I_2)^{\frac{1}{1-\rho^2}} \tag{4.44}$$

where

$$I_2 = Q(X_T < K) = N\left(\frac{1}{|v|\sqrt{T}}\left[K - b(v,\rho)\right]\right) \tag{4.45}$$

and

$$\begin{aligned} I_1 &= e^{c(\rho)[K-b(v,\rho)]}E^Q\left[e^{-c(\rho)vW_T^Q}\mathbf{1}\{X_T > K\}\right] \\ &= e^{c(\rho)[K-b(v,\rho)]+\frac{1}{2}c^2(\rho)v^2T}\hat{Q}\left(b(v,\rho) - c(\rho)v^2T + v\hat{W}_T > K\right) \end{aligned} \tag{4.46}$$

where $\hat{Q}$ is the measure under which $\hat{W}_t := W_t^Q + c(\rho)vt$ is a Brownian motion. Thus, we can compute

$$I_1 = e^{c(\rho)[K-b(v,\rho)]+\frac{1}{2}c^2(\rho)v^2T}N\left(\frac{1}{|v|\sqrt{T}}\left[b(v,\rho) - c(\rho)v^2T - K\right]\right) \tag{4.47}$$

## 4.1 Comparative Statics

We now present comparative statics results obtained numerically in Excel using the above expressions.[8] While we only report here a benchmark case, our results are qualitatively robust with respect to the choice of parameters, unless otherwise noted.

[8]The Excel spreadsheet is available at http://www.hss.caltech.edu/ cvitanic/PAPERS/Excelhedge.xls

We use the following parameters for the benchmark case:

$$r = 0 \ , \mu = 0.1 \ , \sigma = 0.4 \ , X_0 = 1 \ , K = 1 \ , T = 5 \ . \qquad (4.48)$$

Moreover, we choose $\gamma_A$ so that $\lambda/(\gamma_A \sigma) = 0.7nX_0$. This means that the manager who maximizes her utility from investing in $S$ and the risk-free account, disregarding the hedging of the compensation package, and starts with $nX_0$, would invest 70% of that amount in $S$. In addition, we use $n = n_L = 0.001$ for the linear shares contract, and $n_C = 10n$ for the call contract, in order to make the manager's expected utility of the same order of magnitude across the two contracts.

**(i) Changing total volatility v.** We only consider positive values for $v$. We set the correlation to $\rho = 0.5$, but the results are robust to this choice.

- For low values of the output's risk premium, $0 < \alpha < \lambda\rho$, the hedging manager's utility with the shares contract is decreasing in volatility on its positive domain, while with the call contract it is increasing. However, as the strike price gets lower, the call utility becomes decreasing. For high values of the output's risk premium, $\alpha > \lambda\rho$, with both the shares and call contracts it has a maximum, attained at a higher value for the calls then with the shares. Calls make the utility increase more steeply when their downside protection is more useful, that is, when $\alpha$ is low.

Shares utility of the manager who does not hedge always has a maximum and thus she is less conservative than the hedging manager for low values of $\alpha$. Call option utilities are increasing, but less steeply than for the hedging manager, for all values of $\alpha$.

**(ii) Changing correlation $\rho$.**

- With both the shares and call compensation the utility is maximized at $\rho = -1$.

**(iii) Changing systematic risk $x = \rho v$.** We set $y = 0.1$, but the results are robust to its value. Other parameters are the same as before. The range of $x$ we consider is $[-0.25, 0.25]$.

- As noted above, in the case the output $X$ is traded, absence of arbitrage implies $\alpha_x = \lambda$.

In that case the shares utility for the hedging manager is constant in $x$, while call utility is symmetric around zero and has a minimum at zero. The manager who does not hedge and receives shares likes the value $x = \alpha_x/(n\gamma_A)$, while with options compensation, with low values of $\alpha_y$ she likes high systematic risk in its positive domain. The call utility is no longer decreasing in the negative domain of $x$, but increasing. Thus, the non-hedging manager may again be somewhat more conservative with options. Moreover, with high values of $\alpha_y$ the call utility has a maximum close to zero.

For completeness, we state also the comparative statics in the case when $\alpha_x$ is different from $\lambda$:

- For the values in the low range of the systematic risk premium, $\alpha_x < \lambda$, shares utility for the hedging manager is decreasing in $x$. For call, if $\alpha_y$ is not very large, the maximum is attained at the lowest negative value of $x$. However, if $x$ is restricted to positive values, that is, the systematic risk cannot be shorted, call utility is the highest at the highest value of $x$. On the other hand, if $\alpha_y$ is very large, the call utility becomes decreasing in systematic risk.

Compared to this, the manager who does not hedge, when compensated with shares, is less conservative in her choice of positive systematic risk (a maximum is attained at $x = \alpha_x/(n\gamma_A)$). With call options, if the specific risk premium $\alpha_y$ is low, she still likes as high as possible systematic risk in its positive domain, but if $\alpha_y$ is moderately large, she likes it close to zero. The behavior can become completely different in the absence of hedging: for example, with $\alpha_x = 0.1$ and $\alpha_y = 0.5$, the call utility with hedging has a minimum close to zero, while without hedging it has a maximum close to zero.

- In the range $\alpha_x > \lambda$, shares utility for the hedging manager is increasing in $x$, while call utility has a minimum, and its maximum is attained at the highest value of $x$. Thus, both types of compensation induce high systematic risk taking in the positive domain. For the non-hedging manager, the shares utility has again the same maximum point $x = \alpha_x/n\gamma_A$. With low $\alpha_y$ the call utility is increasing, less steeply than when hedging, while with high $\alpha_y$ it has a maximum close to zero. Thus, in all cases the non-hedging manager is more conservative.

**(iii) Changing specific risk $y$.** We set $x = 0.1$, and we comment below on other values. Other parameters are the same as before. The range of $y$ we consider is $[0, 0.5]$.

- Case A: For most values of systematic risk premium $\alpha_x$ and specific risk premium $\alpha_y$, both the shares and the call utility for the hedging manager have a maximum.

- Case B: As the values of $\alpha_y$ get sufficiently low relative to $\alpha_x$, the call utility becomes decreasing in $y$. However, when systematic risk $x$ is low enough, the corresponding behavior remains the same as in Case A.

For the non-hedging manager, the shares utility has the same maximum point as with hedging. In the analogue of Case A the call utility also behaves similarly as with hedging.

To summarize, in the presence of hedging opportunities, a CARA manager would have the following incentives:

- In terms of total volatility, when the output's risk premium is low, she is conservative when paid with shares, and aggressive when paid with calls. When the output's risk premium is high, she becomes less conservative when paid with shares. She is more conservative than the non-hedging manager when paid with shares if the output's risk premium is low, and less conservative when it is high. She is always less conservative than the non-hedging manager when paid in options.

- In terms of systematic risk, when the systematic risk premium is equal to the hedging asset's risk premium, shares provide no incentives, while calls make the manager choose large systematic risk (in absolute value). As to the comparison to the non-hedging manager, she is either less conservative or indifferent with respect to the choice of the systematic risk.

- In terms of specific risk, for most cases both the shares and the calls make the manager somewhat aggressive. However, when the specific risk premium is sufficiently low relative to the systematic risk premium, the incentives change in the opposite direction. Her level of aggressiveness is the same or similar as for the non-hedging manager.

Overall, in most cases calls will induce higher risk taking than shares, and both will do that more so than in the absence of hedging, except when the output's risk premium is low.

## 5 Conclusion

We study risk-taking incentives when the manager can hedge her compensation payoff. When the manager assigns zero initial capital to hedging or has CARA preferences, there is a contract which is optimal regardless of whether the manager can hedge or not. The payoff of that contract may include the returns on the assets available for hedging, in order to preclude the manager from using them. We also compute incentive effects of compensating a CARA manager with shares and call options. We find that options do, indeed, increase the appetite for risk in most cases. Moreover, the hedging manager is typically less conservative than the non-hedging manager, except when compensated with shares and the output's risk premium is low. This is broadly consistent with the intuition that the possibility of hedging increases the manager's ability to bear risk.

In our analysis, we have assumed that the volatility choices of the manager are observable by the firm, or that we have CARA preferences and linear contracts, in which case observability of the volatility choices is irrelevant. It would be of interest, but not easy, to extend that framework to the moral hazard case of unobserved risk taking actions, with non-CARA preferences and nonlinear contracts.[9] Moreover, as option compensation may be partly motivated by a need to distinguish between managers of varying abilities, one would like to see the incentive effects studied when the type of the manager is unknown, namely, the case of adverse selection. Similarly, the firm might not know what assets the manager has available for hedging, and in particular, it might not know the correlation between those assets and the firm's output.

It would also be of interest to test empirically implications of our analysis. In particular, our results predict that, when employed by firms with low systematic risk premia and compensated by shares, the hedging managers may be less aggressive than the non-hedging managers. Another prediction is that, typically, the increased ability to hedge will induce higher managerial risk-taking when compensated with options than when compensated with shares.

Finally, the hedging managers compensated with shares have a similar attitude towards the specific risk as the non-hedging managers, while they are indifferent with respect to the systematic risk level. This is to be contrasted with Acharya and Bisin (2009), who find, in a CAPM equilibrium framework, that the managers would like to substitute the hedgeable systematic risk for the un-hedgeable specific risk. The difference in the predictions is due to two reasons: first, in our model there is no cost in hedging the risk, which is why the manager is indifferent with respect to the systematic risk which she can hedge completely; second, we assume a partial equilibrium framework in which the systematic and the specific risk premia are exogenously fixed, independently of each other, which is why the hedging and the non-hedging manager choose the same value for the specific risk.

## Appendix

**Proof of Proposition 2:** The manager wants to maximize $E[U_A(C_T - G_T + H_T)]$, which, under our assumptions, is the same as maximizing $E[U_A(C_T - G_T + H_T - H_0 B_T)]$. The standard martingale/duality approach to portfolio selection (see, e.g., Karatzas and Shreve (1997) for diffusion models, or Kramkov and Schachermayer (1999) for general semi-martingale models) says that she will hedge so that

$$C_T - G_T + H_T - H_0 B_T = I_A(yY_T) \tag{5.49}$$

where $y, Y_T$ solves, over constant numbers $y$ and SDFs $Y_T \in \mathcal{Z}$, the dual problem

$$\bar{V}(H_0) := \min_{y,Y} E[U_A(I_A(yY_T)) - yY_T I_A(yY_T) + yY_T(C_T - G_T) + yY_T(H_T - H_0 B_T)] \tag{5.50}$$

if a solution exists, and if we have $V(H_0) = \bar{V}(H_0)$. The last term in (5.50) disappears, because $E[Y_T(H_T - H_0 B_T)] = 0$. Then, if (3.10) is satisfied, it is easily seen that the above is minimized for $yY_T = zZ_T$. From (5.49) this implies $H_T = H_0 B_T$. Conversely, if the optimal hedging strategy results in $H_T = H_0 B_T$, we see from (5.49) that $C_T$ has to be of the form (3.10).

■

**Computations for Section 3.2:** Consider the more general model where output is given by

$$dX_t = [\delta u_t + \alpha_x x_t + \alpha_y y_t]dt + x_t dW_t + y_t dM_t$$

and where the manager pays the cost $G_T = \int_0^T g(u_t)dt$ with $g(u) = \frac{u^2}{2k}$ for $k > 0$ when exerting the effort $u$. Under optimal behavior of the firm, the manager is given a contract of the form $C_T = aX_T + b\log(S_T) + c$. Denote by $\tilde{c}$ the certainty equivalent (CE) of $c$ and with $\tilde{R}$ the CE of the manager's reservation utility. Also denote by $\pi_A$ the amount of capital manager invest in $S$, and by $\pi_P$ the analogous hedging amount for the firm. Given that at the optimum $\pi_A, \pi_P$ and effort $u$ are constant in our framework, the manager's CE is then

$$\tilde{c}+a[X_0+\alpha_y yT+\delta uT]+[\alpha_x ax+\lambda\sigma\pi_A-g(u)]T+b[\log S_0+(\mu-\frac{\sigma^2}{2})T]-\frac{\gamma_A T}{2}[(ax+\pi_A\sigma+b\sigma)^2+a^2y^2] \quad (5.51)$$

Maximizing over $u, \pi_A$ and $y$ we get

$$ax + \sigma\pi_A + b\sigma = \frac{\lambda}{\gamma_A} \quad (5.52)$$

$$y = \frac{\alpha_y}{a\gamma_A} \quad (5.53)$$

$$g'(u) = a\delta \quad (5.54)$$

This means that the manager's CE is

$$\tilde{R} = \tilde{c}+a[X_0+(\frac{\alpha_y^2}{a\gamma_A}+\delta u)T]-g(u)T+(\alpha_x-\lambda)ax+\lambda[\frac{\lambda}{\gamma_A}-b\sigma]T+b[\log S_0+(\mu-\sigma^2/2)T] \quad (5.55)$$

$$-\frac{T}{2}\gamma_A[(\frac{\lambda}{\gamma_A})^2+(\frac{\alpha_y}{\gamma_A})^2] \quad (5.56)$$

The firm's CE is

$$-\tilde{c} + (1-a)[X_0 + (\frac{\alpha_y^2}{a\gamma_A} + \delta u)T] + [\alpha_x(1-a)x + \lambda\sigma\pi_P]T - b[\log S_0 + (\mu - \sigma^2/2)T] \quad (5.57)$$

$$-\frac{T}{2}\gamma_P[((1-a)x + \pi_P\sigma - b\sigma)^2 + (1-a)^2(\frac{\alpha_y}{a\gamma_A})^2] \quad (5.58)$$

Fixing $\tilde{R}$ and computing $\tilde{c}$ from the previous expression we get the firm's CE as

$$-\tilde{R} - g(u)T + (\alpha_x - \lambda)ax + \lambda[\frac{\lambda}{\gamma_A} - b\sigma]T - \frac{T}{2}\gamma_A[(\frac{\lambda}{\gamma_A})^2 + (\frac{\alpha_y}{\gamma_A})^2] + [X_0 + (\frac{\alpha_y^2}{a\gamma_A} + \delta u)T] \quad (5.59)$$

$$+[\alpha_x(1-a)x + \lambda\sigma\pi_P]T - \frac{T}{2}\gamma_P[((1-a)x + \pi_P\sigma - b\sigma)^2 + (1-a)^2(\frac{\alpha_y}{a\gamma_A})^2] \quad (5.60)$$

This means that the hedging portfolio $\pi_P$ is chosen so that

$$\gamma_P[(1-a)x + \pi_P\sigma - b\sigma] = \lambda \quad (5.61)$$

and hence the firm needs to maximize

$$\delta u - g(u) - \lambda b\sigma + \frac{\alpha_y^2}{a\gamma_A} + (\alpha_x - \lambda)x + \lambda\sigma b - \frac{\gamma_P}{2}(1-a)^2(\frac{\alpha_y}{a\gamma_A})^2 \quad (5.62)$$

We see that the principal is indifferent with respect to the choice of $b$. In case there is no effort $u$, $u = g(u) = 0$, we get

$$a = \frac{\gamma_P}{\gamma_A + \gamma_P} \quad (5.63)$$

In case $g(u) = u^2/(2k)$ is quadratic, taking into account that $g'(u) = \delta a$, we can check that the derivative of the firm's CE is

$$\frac{1}{a^3}[a^3(1-a)k\delta^2 - \frac{1}{\gamma_A}\alpha_y^2[a(1+\gamma_P/\gamma_A) - \gamma_P/\gamma_A]$$

This is positive for $a$ positive and close to zero and negative for $a$ close to one, thus there is exactly one maximizer $\hat{a} \in [0,1]$. Moreover, it is seen that increasing $\alpha_y^2$ also increases the derivative in absolute value. Note also that the firm's CE converges to $-\infty$ at $a = 0$, and has a higher value at $a = 1$ for higher $\alpha_y^2$. A combination of all these properties is possible only if the maximizer $\hat{a}$ moves to higher values when $\alpha_y^2$ is increased.

# References


[1] Acharya, V. and A. Bisin, 2009, Managerial hedging, equity ownership, and firm value, *RAND Journal of Economics*, 40, 1, 47–77.

[2] Basak, S., A. Pavlova and A. Shapiro, 2007, Optimal Asset Allocation and Risk Shifting in Money Management, *Review of Financial Studies*, 20, 5, 1583–1621.

[3] Basak, S., A. Shapiro, and L. Tepla, 2006, Risk Management with Benchmarking, *Management Science*, 54, 542–557.

[4] Bettis, J. C., Bizjak, J.M. and M. L. Lemmon, 2001, Managerial ownership, incentive contracting, and the use of zero-cost collars and equity swaps by corporate insiders, *Journal of Financial and Quantitative Analysis* 36, 345–370.

[5] Bisin, A., Gottardi, P. and A.A. Rampini, 2008, Managerial hedging and portfolio monitoring, *Journal of the European Economic Association*, 6, 1, 158-209.

[6] Cadenillas, A., Cvitanić and F. Zapatero, 2007, Optimal Risk-Sharing with Effort and Project Choice, *Journal of Economic Theory*, 133, 403–440.

[7] Carpenter, J. N., 2000, Does option compensation increase managerial risk appetite?, *The Journal of Finance* 55, 2311–2331.

[8] Cuoco, D., and R. Kaniel, 2011, Equilibrium Prices in the presence of Delegated Portfolio Management, *Journal of Financial Economics*, 101, 264–296.

[9] Garvey, G. and T. Milbourn, 2003, Incentive Compensation When Executives Can Hedge the Market: Evidence of Relative Performance Evaluation in the Cross Section, *The Journal of Finance*, 58, 1557–1581.

[10] Grinblatt M. and S. Titman, 1989, Adverse Risk Incentives and the Design of Performance Based Contracts, *Management Science*, 35, 7,807-822.

[11] Guo, M. and H. Ou-Yang, 2006, Incentives and performance in the presence of wealth effects and endogenous risk, *J. of Economic Theory* 129, 150-191.

[12] Henderson V., 2002, Valuation of Claims on Non-Traded Assets using Utility Maximization, *Mathematical Finance*, 12, 4, 351-373.

[13] Henderson V., 2005, The Impact of the Market Portfolio on the Valuation, Incentives and Optimality of Executive Stock Options, *Quantitative Finance*, 5(1), 1-13.

[14] Hodder J.E. and J.C. Jackwerth, 2007, Incentive Contracts and Hedge Fund Management, *Journal of Financial and Quantitative Analysis*, 42, 4, 811-826.

[15] Hodder J.E. and J.C. Jackwerth, 2011, Managerial Responses to Incentives: Control of Firm-Risk, Derivative Pricing Implications, and Outside Wealth Management, *Journal of Banking and Finance*, 35, 6, 1507–1518.

[16] Hugonnier, J. and R. Kaniel, 2010, Mutual Fund Portfolio Choice in the Presence of Dynamic Flows, *Mathematical Finance*, 20, 2, 187–227.

[17] Holmstrom, B., 1982, Moral hazard in teams, *Bell Journal of Economics*, 19, 324–340.

[18] Jin, L., 2002, CEO Compensation, Diversification and Incentives: Theory and Empirical Results, *J. of Financial Economics*, 66, 29–63.

[19] I. Karatzas and S. Shreve, 1998. *Methods of Mathematical Finance*, (Springer-Verlag, New York, NY).

[20] Kramkov,D. and W. Schachermayer, 1999, A condition on the asymptotic elasticity of utility functions and optimal investment in incomplete markets, *The Annals of Applied Probability* 9, 904–950.

[21] Ofek E. and D. Yermack, 2000, Taking Stock: Equity-Based Compensation and the Evolution of Managerial Ownership, *Journal of Finance*, 55, 1367-1384.

[22] Ozerturk, S., 2006, Financial innovations and managerial incentive contracting, *Canadian Journal of Economics*, 39, 434–454.

[23] Panageas, S. and M. M. Westerfield, 2009, High-Water Marks: High Risk Appetites? Convex Compensation, Long Horizons, and Portfolio Choice, *The Journal of Finance*, 64, 1, 1–36.

[24] Ross, S.A., 2004, Compensation, Incentives, and the Duality of Risk Aversion and Riskiness, *The Journal of Finance*, 59, 207–225.

[25] Tehranchi, M., 2004, Explicit solutions of some utility maximization problems in incomplete markets, *Stochastic Processes and Their Applications* 114, 109–125.

## Footnotes

[1] Other papers considering the effect of specific contracts on portfolio managers, typically without possibility of hedging, include Basak, Pavlova and Shapiro (2007), Basak, Shapiro and Tepla (2006), Cuoco and Kaniel (2011), Hodder and Jackwerth (2007), and Hugonnier and Kaniel (2010). Grinblatt and Titman (1989), Henderson (2005) and Hodder and Jackwerth (2011) include hedging possibilities, again only in frameworks with specific contracts.

[2]Whilst Hodder and Jackwerth (2011) obtain numerical results showing that the manager prefers low total volatility when compensated with shares, and higher volatility when compensated with calls, they do not consider separately controlling systematic and firm-specific risks, and do not delineate any dependence on corresponding risk premia. Henderson (2005) does consider separately systematic and firm-specific risk but only in the case where the output $X$ is traded and $\alpha_x = \lambda$.

[3] Panageas and Westerfield (2009) show that even the risk-neutral managers need not behave aggressively when paid with high water mark contracts, if the time-horizon of their compensation is not fixed.

[4]Alternatively, we could assume that the firm and the manager enjoy utility from discounted values.

[5]These technical conditions are typically either satisfied, or they are satisfied if instead of $\mathcal{Z}$ we consider its closure in an appropriate topology. In particular, the condition $V(H_0) = \bar{V}(H_0)$ means that there is no duality gap between the primal problem of portfolio optimization and the dual problem of finding the optimal dual state-price density. Economically, it means that the standard marginal utility expression holds for the agent's problem at

the optimum: $U'_A(P_T) = yY_T$ for the agent's final total wealth $P_T$, for some constant $y$ and some SDF $Y_T$ (or $Y_T$ in the closure of the set $\{\mathcal{Z}\}$ of SDF's). For details, see the references mentioned in the proof.

[6]If we modeled $S$ as a Brownian motion with drift rather than a geometric Brownian motion, the optimal contract would be linear in $S$, not in $\log(S)$, in the case of CARA preferences.

[7]Alternatively, we could assume that the market trades a risk-free asset with the interest rate independent of all the risk-neutral densities in the market.

[8]In order to interpret this, write now

$$\tilde{W} = \rho W + \sqrt{1-\rho^2} M \tag{5.64}$$

for a Brownian Motion $M$ independent of $W$. Note that we have

$$dS/S = [r+\sigma\lambda]dt + \sigma/\rho d\tilde{W}_t - \sigma\sqrt{1-\rho^2}/\rho dM_t = rdt + \sigma/\rho dW^Q - \sigma\sqrt{1-\rho^2}/\rho dM_t \tag{5.65}$$

Thus, $Q$ is a risk-neutral measure for $S$. It is actually the projection on $\tilde{F}$ of the measure which would be the risk-neutral measure if $S$ was the only traded asset.

[9]Guo and Ou-Yang (2006) consider the case of non-CARA preferences with no hedging, and with linear contracts.