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CORRIGENDUM TO "EXACT SIMULATION OF THE WRIGHT-FISHER DIFFUSION"

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This correction refers to the version of this paper appearing in *Annals of Applied Probability* (Jenkins and Spanò, 2017). In the proof of Lemma 6.1, case $l \leq \lfloor mz \rfloor$, the second inequality uses an incorrect upper bound for a function

$$g(m) := \frac{m+1}{m+1-mz} \cdot \frac{\theta+m}{\theta_2+m-mz}$$

by assuming that it is increasing. We can bound this function correctly by following an argument presented by García-Pareja, Hult and Koski (2019): Write $g(m) = a(m)b(m)$ and note that

$$a(m) := \frac{m+1}{1+m(1-z)}$$

is increasing in m so that $a(m) \leq a(\infty) = (1-z)^{-1}$; and, for $m \geq 1$,

$$b(m) := \frac{\theta+m}{\theta_2+m-mz} \leq \frac{\theta+m}{m(1-z)} = \frac{1+\frac{\theta}{m}}{1-z} \leq \frac{1+\theta}{1-z}.$$

Hence $g(m) \leq g(0) \vee a(\infty)\frac{1+\theta}{1-z} = \frac{\theta}{\theta_2} \vee \frac{1+\theta}{(1-z)^2}$.

The argument for the case $l \geq \lfloor mz \rfloor$ needs an upper bound for

$$h(l,m) := \frac{m+1}{l+1} \cdot \frac{\theta+m}{\theta_1+l}.$$

We consider the cases (i) $mz > 1$ and (ii) $mz \leq 1$. For (i) we have

$$\begin{aligned} h(l,m) \leq h(mz-1,m) &= \frac{m+1}{mz} \cdot \frac{\theta+m}{\theta_1+mz-1} \\ &\leq \left(1+\frac{1}{z}\right) \cdot \frac{\theta+m}{\theta_1+mz-1} =: c(m). \end{aligned}$$

It is straightforward to verify that the sign of $c'(m)$ is independent of m, and thus

$$h(l,m) \leq c(z^{-1}) \vee c(\infty) = \left(1+\frac{1}{z}\right)\left[\frac{z\theta+1}{z\theta_1} \vee \frac{1}{z}\right].$$

For case (ii), we have

$$h(l,m) \le h(0,m) = (m+1) \cdot \frac{\theta + m}{\theta_1} \le \left(1 + \frac{1}{z}\right) \cdot \frac{z\theta + 1}{z\theta_1}.$$

Thus, in both cases we have

$$h(l,m) \le \left(1 + \frac{1}{z}\right) \left[\frac{z\theta + 1}{z\theta_1} \vee \frac{1}{z}\right].$$

Lemma 6.1 is therefore correct provided we instead define the stated constant $K^{(x,z)}$ as

$$K^{(x,z)} := \left(\frac{\theta}{\theta_2}(1-z) \vee \frac{1+\theta}{1-z}\right)(1-x) + \left(1 + \frac{1}{z}\right)\left[\frac{z\theta + 1}{\theta_1} \vee 1\right] x,$$

or the slightly simpler (using $0 \le x \le 1$, $0 \le z \le 1$), but less tight,

$$K^{(x,z)} := \left(\frac{\theta}{\theta_2} \vee \frac{1+\theta}{1-z}\right) + \left(1 + \frac{1}{z}\right) \frac{\theta + 1}{\theta_1}.$$

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References.

García-Pareja, C., Hult, H. and Koski, T. (2019). Exact simulation of coupled Wright-Fisher diffusions. arXiv:1909.11626.

Jenkins, P. A. and Spanò, D. (2017). Exact simulation of the Wright-Fisher diffusion. *Annals of Applied Probability* **27** 1478–1509.

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