BOOTSTRAP PERCOLATION & ITS APPLICATIONS

April 14–19, 2024
Banff International Research Station (BIRS) [24w5300]
`https://users.ox.ac.uk/~stat0411/birs_bp.html`

Sunday, April 14

16:00–17:30	Check-in starts 4pm (Front Desk, open 24 hours)
17:30–19:30	Dinner (Vistas Dining Room)
19:30–21:00	Informal gathering (TCPL Foyer)

Monday, April 15

07:00–08:45	Breakfast (Vistas Dining Room)
08:45–09:00	Introduction and welcome by BIRS staff (TCPL 201)
09:00–09:50	**Rob Morris:** *Mini-course 1/3* (TCPL 201)
10:00–10:30	Coffee break (TCPL Foyer)
10:30–11:20	**Ivailo Hartarsky:** *Mini-course 2/3* (TCPL 201)
11:30–13:00	Lunch (Vistas Dining Room)
14:30–15:00	**Réka Szabó:** *Stability results for random monotone cellular automata* (TCPL 201)
15:00–15:30	Coffee break (TCPL Foyer)
15:30–16:20	**David Sivakoff:** *Supercritical neighborhood growth with one-dimensional nucleation* (TCPL 201)
17:30–19:30	Dinner (Vistas Dining Room)
20:00–20:45	*Open problems 1/2* (TCPL 201)

Tuesday, April 16

07:00–08:45	Breakfast (Vistas Dining Room)
09:00–09:50	**Ivailo Hartarsky:** *Mini-course 3/3* (TCPL 201)
10:00–10:30	Coffee break (TCPL Foyer)
10:30–11:20	**Gábor Pete:** *Volatility in dynamical bootstrap percolation on regular trees* (TCPL 201)
11:20–11:30	Group photo (TCPL Foyer)
11:30–13:00	Lunch (Vistas Dining Room)
13:15–14:05	**Tibor Szabó:** *Slow subgraph bootstrap percolation* (TCPL 201)
14:15–15:00	*Open problems 2/2* (TCPL 201)
15:00–15:30	Coffee break (TCPL Foyer)
15:30–16:30	*Group work*
17:30–19:30	Dinner (Vistas Dining Room)

Wednesday, April 17

07:00–08:45	Breakfast (Vistas Dining Room)
09:00–09:50	**Alexander Holroyd:** *Polluted bootstrap percolation* (TCPL 201)
10:00–10:30	Coffee break (TCPL Foyer)
10:30–11:20	**Augusto Teixeira:** *Two-neighbor bootstrap percolation is local* (TCPL 201)
11:30–13:00	Lunch (Vistas Dining Room)
13:00–17:30	Free afternoon
17:30–19:30	Dinner (Vistas Dining Room)

THURSDAY, APRIL 18

07:00–08:45 Breakfast (Vistas Dining Room)
09:00–09:50 **Paul Balister:** *Uncomputability in bootstrap percolation* (TCPL 201)
10:00–10:30 Coffee break (TCPL Foyer)
10:30–11:00 **Gal Kronenberg:** *Independent sets in random subgraphs of the hypercube* (TCPL 201)
11:10–11:40 **Zsolt Bartha:** *Critical thresholds in graph bootstrap percolation* (TCPL 201)
11:40–13:00 Lunch (Vistas Dining Room)
14:10–15:00 *Simulations & applications* (TCPL 201)
15:00–15:30 Coffee break (TCPL Foyer)
15:30–16:20 **Mihyun Kang:** *Majority bootstrap percolation on high-dimensional geometric graphs* (TCPL 201)
17:30–19:30 Dinner (Vistas Dining Room)
20:00–21:00 *Group activity* (TCPL 201)

FRIDAY, APRIL 19

07:00–08:45 Breakfast (Vistas Dining Room)
09:00–10:00 *Lightning talks* (TCPL 201)
10:00–10:30 Coffee break (TCPL Foyer)
10:30–11:00 Checkout by 11am
11:30–13:30 Lunch (Vistas Dining Room)

ABSTACTS
(IN ORDER OF APPEARANCE)

Rob Morris: *Mini-course 1/3*

In this lecture we provide a very broad overview of the area of bootstrap percolation. This includes motivation from related fields, sharp results on classical models, as well as the universality theory.

Ivailo Hartarsky: *Mini-course 2/3*

This lecture focuses on upper bounds on the infection time (or critical parameter) for a simplified variant of the two-neighbour model. We start by proving Holroyd's sharp threshold bound. We then move on to more precise results and their connection to the bootstrap percolation paradox.

Réka Szabó: *Stability results for random monotone cellular automata*

In a monotone cellular automaton, each site in the d-dimensional integer lattice can at each integer time take the values zero or one. The value of a site at a given time is a monotone function of the values of the site and finitely many of its neighbours at the previous time. Toom's stability theorem gives necessary and sufficient conditions for the all one state to be stable under small random perturbations. We review Toom's Peierls argument and extend it to random cellular automata, in which the functions that determine the value at a given space-time point are random and i.i.d. We are especially interested in the case where with positive probability, the identity map is applied, that just copies the value of a site at the previous time. We derive sufficient conditions for the stability of such random cellular automata. Joint work with Cristina Toninelli and Jan Swart.

David Sivakoff: *Supercritical neighborhood growth with one-dimensional nucleation*

Supercritical neighborhood growth (or $\mathscr{U}$-bootstrap percolation) rules in two dimensions are classified by the existence of finite configurations that lead to unbounded growth. Bollobás, Smith and Uzzell proved that when the initially occupied set is random with density p, the first occupation time of the origin is $p^{-\Theta(1)}$ as $p \to 0$. We take a closer look at supercritical growth rules with $+$-shaped neighborhoods. These rules exhibit one-dimensional nucleation in the sense that lines begin to grow before forming two-dimensional nuclei. In many cases, we show that the first occupation time is $p^{-\gamma+o(1)}$ for an explicit constant γ depending on the rule. In one case, we establish a logarithmic correction to the polynomial passage time due to a growth trajectory that resembles a branching process, while in other cases the first occupation time is of pure polynomial order $p^{-\gamma}$.

Based on joint work with Daniel Blanquicett, Janko Gravner and Luke Wilson.

Ivailo Hartarsky: *Mini-course 3/3*

This lecture is dedicated to lower bounds. We start with the rectangles process and the resulting Aizenman–Lebowitz bound. We then aim to cover the scheme of Holroyd's proof.

Gábor Pete: *Volatility in dynamical bootstrap percolation on regular trees*

Consider 2-neighbour bootstrap percolation on the 4-regular infinite tree. The critical density of initial occupation is $1/9$, at which point there are already vacant 3-regular subtrees a.s., stopping the bootstrap process from completely occupying the tree. Now let the initial status of every site be resampled according to an independent Poisson process, keeping the density critical. Are there exceptional times in this dynamics when all the vacant 3-regular subtrees get destroyed, hence the entire tree gets bootstrap-occupied?

By a Baire category argument, the existence of exceptional times is equivalent to the set of times at which the root is bootstrap-occupied being everywhere dense a.s. We don't know this, but show semi-volatility: in any time interval $[0,\varepsilon)$, starting from the root being bootstrap-vacant, the bootstrapped status of the root changes infinitely many times with probability at least $c\sqrt{\varepsilon}$.

Joint work with Marek Biskup and Àbel Farkas.

Tibor Szabó: *Slow subgraph bootstrap percolation*

We study subgraph bootstrap percolation, introduced by Bollobás, where the process is governed by copies of a fixed graph H in the complete graph. We are interesed in the extremal question of the maximum running time, over all possible choices of a starting graph on n vertices. We initiate a systematic study of this parameter, denoted $M_H(n)$, and its dependence on properties of the graph H. In a series of works we determine the precise maximum running time for several graph classes. In general, we study necessary and sufficient conditions on H for fast, i.e. sublinear, or linear H-bootstrap percolation, and in particular explore the relationship between running time and minimum vertex degree and connectivity. Furthermore we investigate the superlinear regime, obtain the maximum running time of the process for typical H and discover several graphs exhibiting surprising behaviour. The talk represents joint work with David Fabian and Patrick Morris.

Alexander Holroyd: *Polluted bootstrap percolation*

TBA

Augusto Teixeira: *Two-neighbour bootstrap percolation is local*

Metastability thresholds lie at the heart of bootstrap percolation theory. Yet proving precise lower bounds for these quantities is notoriously hard. In this talk we show that for two of the most classical models (two-neighbour and Froböse), the same methodology that is typically used to prove upper bounds can be used to provide lower bounds as well. This is done by linking the models to their local counterparts. As a consequence, we are able to establish the second order term for the infection time of these two models. We will also see how this locality viewpoint can be used to resolve the so-called bootstrap percolation paradox. More precisely, we will present an exact (deterministic) algorithm which exponentially outperforms previous Monte Carlo approaches. We expect this methodology to be applicable to a wider range of models and we finish our talk with a number of open problems.

This talk will be based on a joint work with Ivailo Hartarsky.

Paul Balister: *Uncomputability in Bootstrap Percolation*

It is well known that even very simple cellular automata, such as Conway's 'Game of Life' can express extremely complex behaviour, including the ability to emulate a universal Turing Machine. Hence certain aspects of the evolution can be uncomputable, as they can in some cases be equivalent to the Halting problem. Surprisingly, this also holds for generalized bootstrap percolation models in all dimensions

$d \geqslant 2$, despite the fact that such models are highly restricted by being required to be monotone. In particular, we show that when the initial set is given by a random i.i.d. infection with probability $p = p(n)$ in $(\mathbb{Z}/n\mathbb{Z})^d$, even the exponents in the threshold value of p for which percolation occurs can be uncomputable in general.

Joint work with Béla Bollobás, Robert Morris, and Paul Smith.

Gal Kronenberg: *Independent sets in random subgraphs of the hypercube*

Independent sets in bipartite regular graphs have been studied extensively in combinatorics, probability, computer science and more. The problem of counting independent sets is particularly interesting in the d-dimensional hypercube $\{0,1\}^d$, motivated by the lattice gas hardcore model from statistical physics. Independent sets also turn out to be very interesting in the context of random graphs.

The number of independent sets in the hypercube $\{0,1\}^d$ was estimated precisely by Korshunov and Sapozhenko in the 1980s and recently refined by Jenssen and Perkins.

In this talk, we will discuss some results on the number of independent sets in a random subgraph of the hypercube. The results extend to the hardcore model and rely on an analysis of the antiferromagnetic Ising model on the hypercube.

This talk is based on joint work with Yinon Spinka.

Zsolt Bartha: *Critical thresholds in graph bootstrap percolation*

Graph bootstrap percolation is a process introduced by Bollobás in 1968. Fixing a graph H, we start from an initial graph G_0 and iteratively add edges to it that complete copies of H. When G_0 is the Erdős–Rényi random graph $G(n,p)$, there is a critical threshold p_c above which this process will be likely to reach the complete graph K_n. We obtain a general asymptotic lower bound for p_c, which is sharp in some cases, and matches the upper bound given by Balogh, Bollobás and Morris up to poly-logarithmic factors for so-called balanced graphs H. We show that this class of graphs contains $G(k,1/2)$ with high probability, thus identifying p_c for uniformly random H.

Based on joint works with Brett Kolesnik and Gal Kronenberg.

Mihyun Kang: *Majority bootstrap percolation on high-dimensional geometric graphs*

Majority bootstrap percolation is a monotone cellular automata that can be thought of as a model of infection spreading in networks. Starting with an initially infected set, new vertices become infected once more than half their neighbours are infected. The average case behaviour of this process was studied on the n-dimensional hypercube by Balogh, Bollobás and Morris, who showed that there is a phase transition as the typical density of the initially infected set increases, for small enough densities the spread of infection is typically local, whereas for large enough densities typically the whole graph eventually becomes infected. They showed that the critical window in which this phase transition occurs is bounded away from $1/2$, and they gave good bounds on its width.

In this talk we consider the majority bootstrap percolation process on a class of high-dimensional geometric graphs which includes many of the graphs families on which percolation processes are typically considered, such as grids, tori and Hamming graphs, as well as other well-studied families of graphs such as (bipartite) Kneser graphs, including the odd graph and the middle layer graph, and the permutahedron. We show similar quantitative behaviour in terms of the location and width of the critical window for the majority bootstrap percolation process for this class of graphs.

Joint work with Maurício Collares, Joshua Erde, and Anna Geisler.