Comparing Risks by Acceptance and Rejection

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Papers

- Dean Foster and Sergiu Hart
  "An Operational Measure of Riskiness"
  Journal of Political Economy (2009)
  www.ma.huji.ac.il/hart/abs/risk.html

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  "An Operational Measure of Riskiness"
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- Dean Foster and Sergiu Hart
  "A Reserve-Based Axiomatization of the Measure of Riskiness" (2008)
  www.ma.huji.ac.il/hart/abs/risk-ax.html

Papers (continued)

- Sergiu Hart
  "A Simple Riskiness Order Leading to the Aumann–Serrano Index of Riskiness" (2008)
  www.ma.huji.ac.il/hart/abs/risk-as.html

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- Sergiu Hart
"A Simple Riskiness Order Leading to the Aumann–Serrano Index of Riskiness" (2008)
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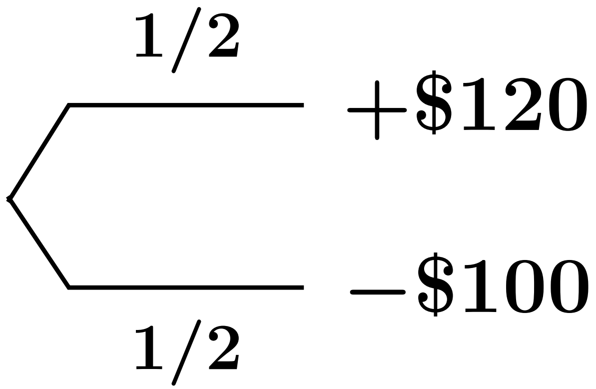
- Sergiu Hart
"Comparing Risks by Acceptance and Rejection" (2009)
`www.ma.huji.ac.il/hart/abs/risk-u.html`

Gamble (“Risky Asset”)

$g =$



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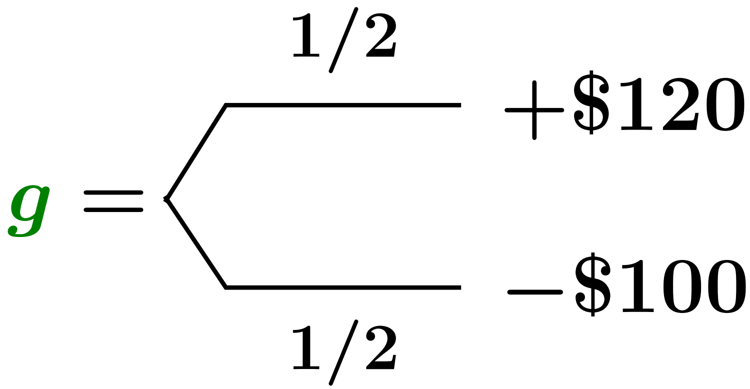
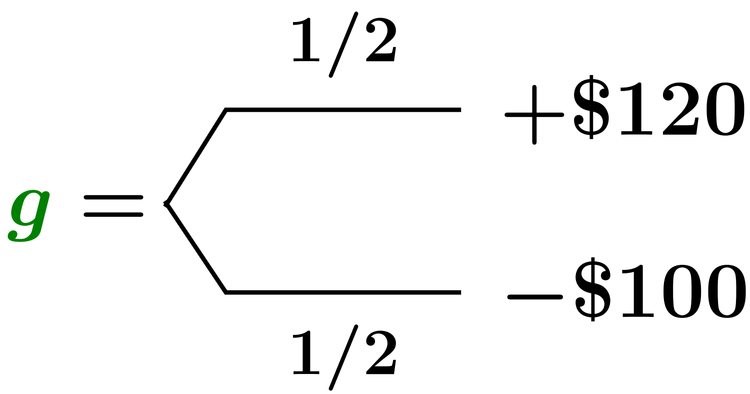


- *Net* gains and losses

Gamble ("Risky Asset")



- *Net* gains and losses
- Positive expectation

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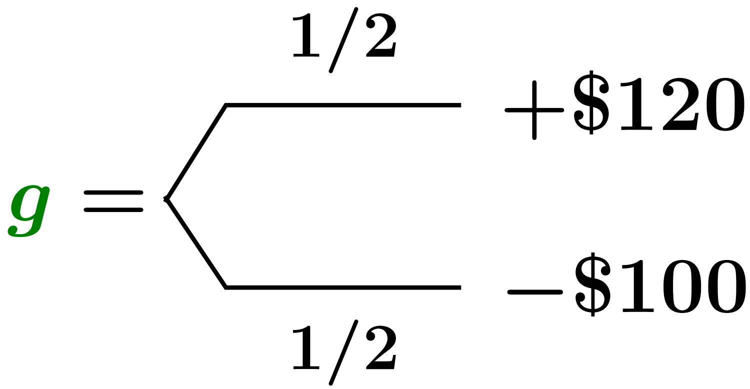

- *Net* gains and losses
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- Some losses

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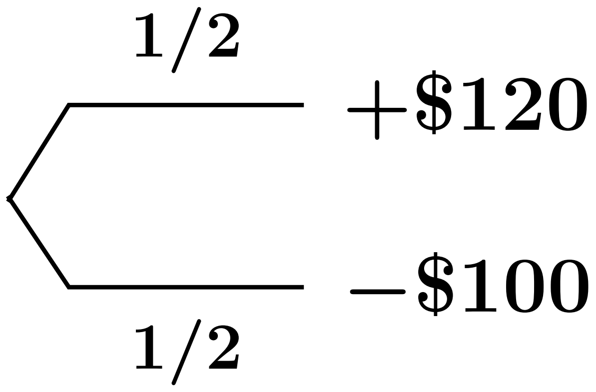
$g =$ 

- *Net* gains and losses
- Positive expectation
- Some losses
- Pure risk (known probabilities)

Comparing Risks

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Let g and h be gambles

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Question:

Comparing Risks

Let g and h be gambles

Question:

When is g **LESS RISKY THAN** h ?

Comparing Risks

Let g and h be gambles

Question:

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Answer:

Comparing Risks

Let g and h be gambles

Question:

When is g LESS RISKY THAN h ?

Answer:

When RISK-AVERSE decision-makers are LESS AVERSE to g than to h !

Comparing Risks

"risk-averse decision-makers are **LESS AVERSE** to g than to h" $= ?$

Comparing Risks: Take 1

"risk-averse decision-makers are **LESS AVERSE** to g than to h" =

g is **PREFERRED** to h

Comparing Risks: Take 1

"risk-averse decision-makers are
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$$\mathbf{E}\left[u(w+g)\right] \geq \mathbf{E}\left[u(w+h)\right]$$

for every (concave) utility u and wealth w

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g **STOCHASTICALLY DOMINATES** h (2nd-degree)

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Stochastic Dominance

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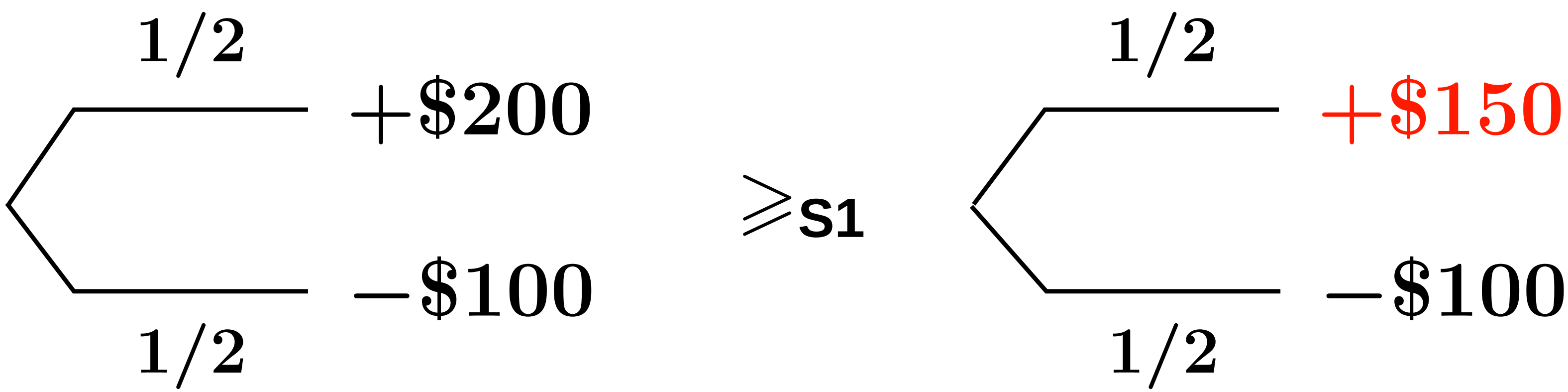

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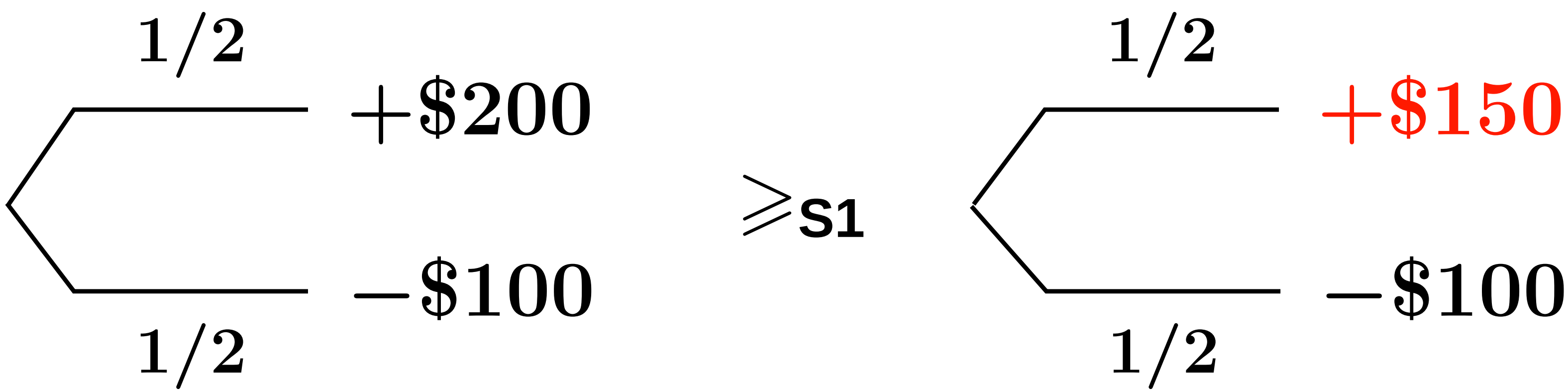

- $g' \geqslant h'$
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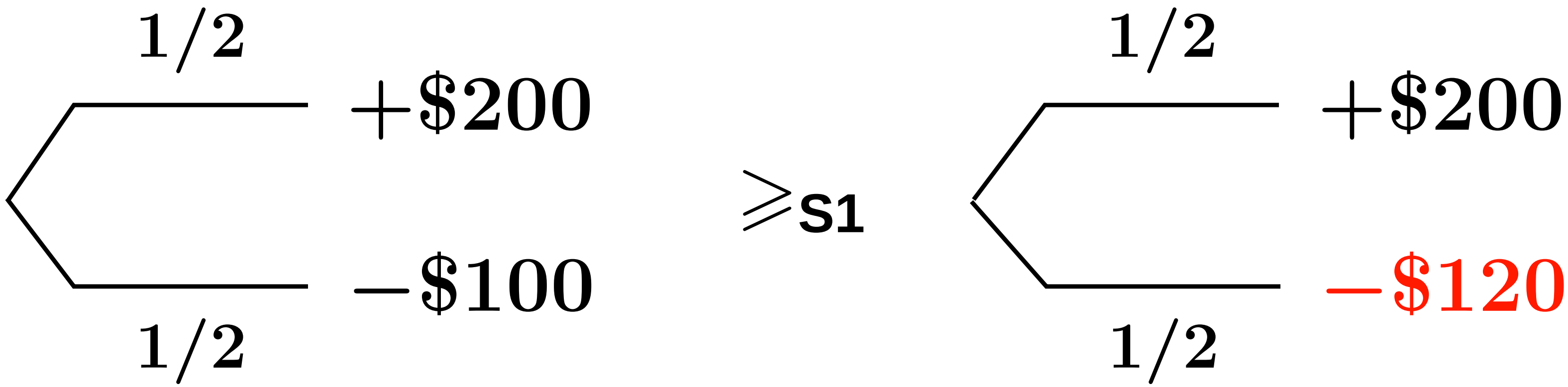

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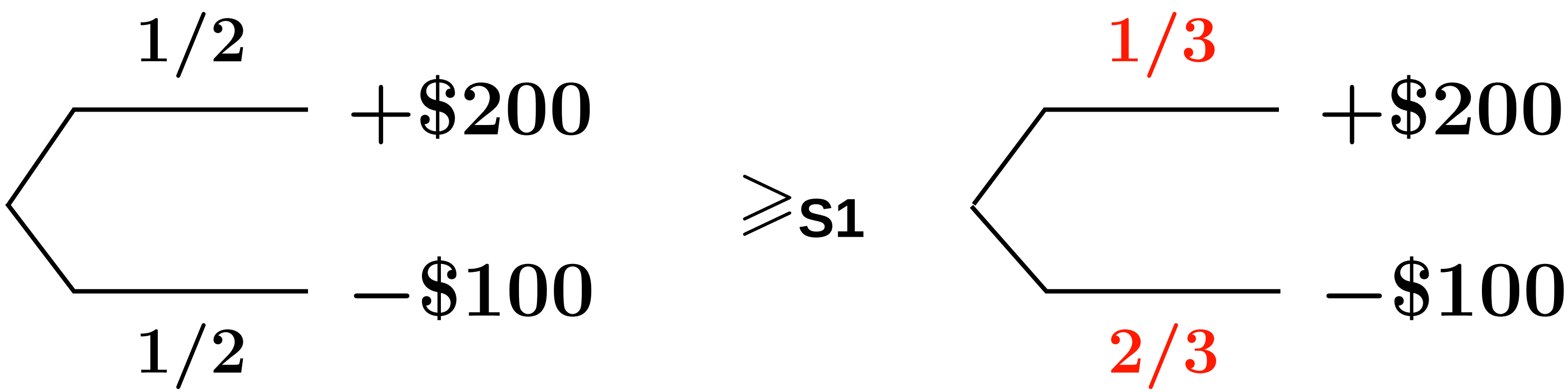

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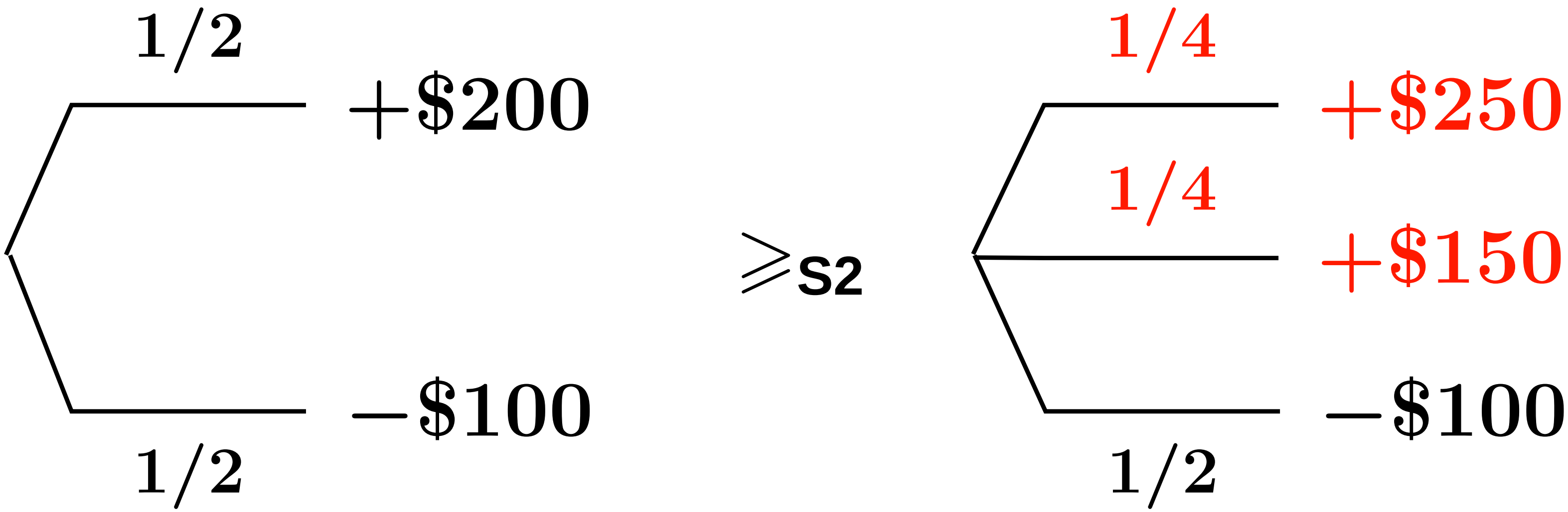

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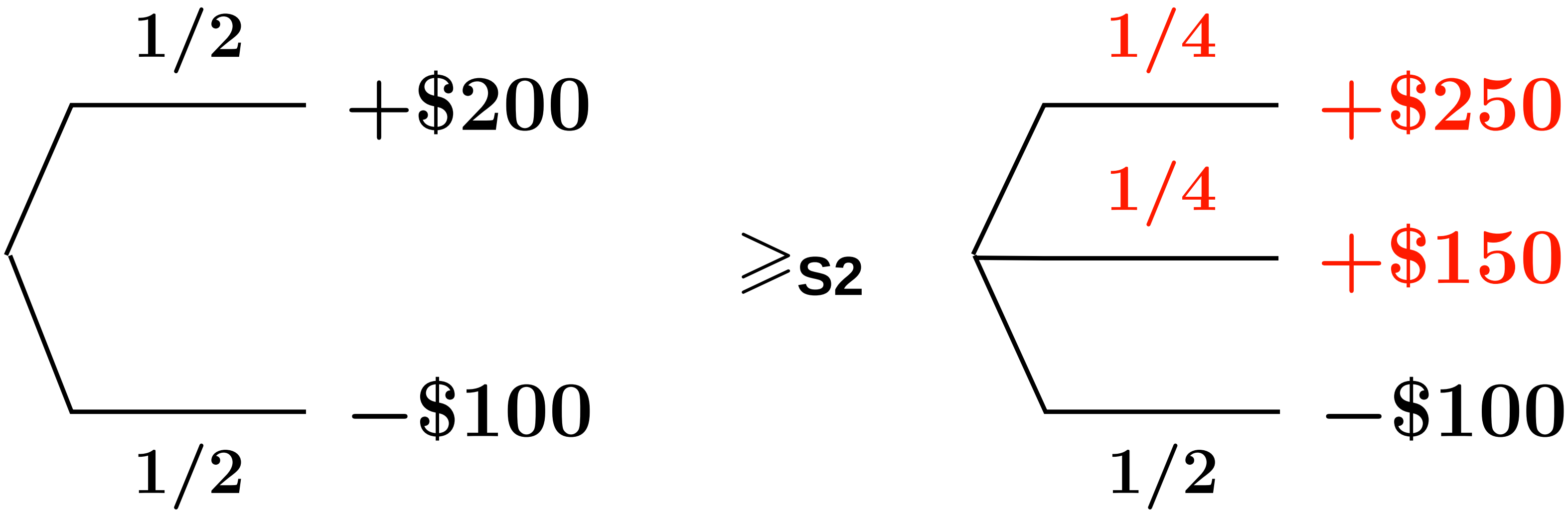

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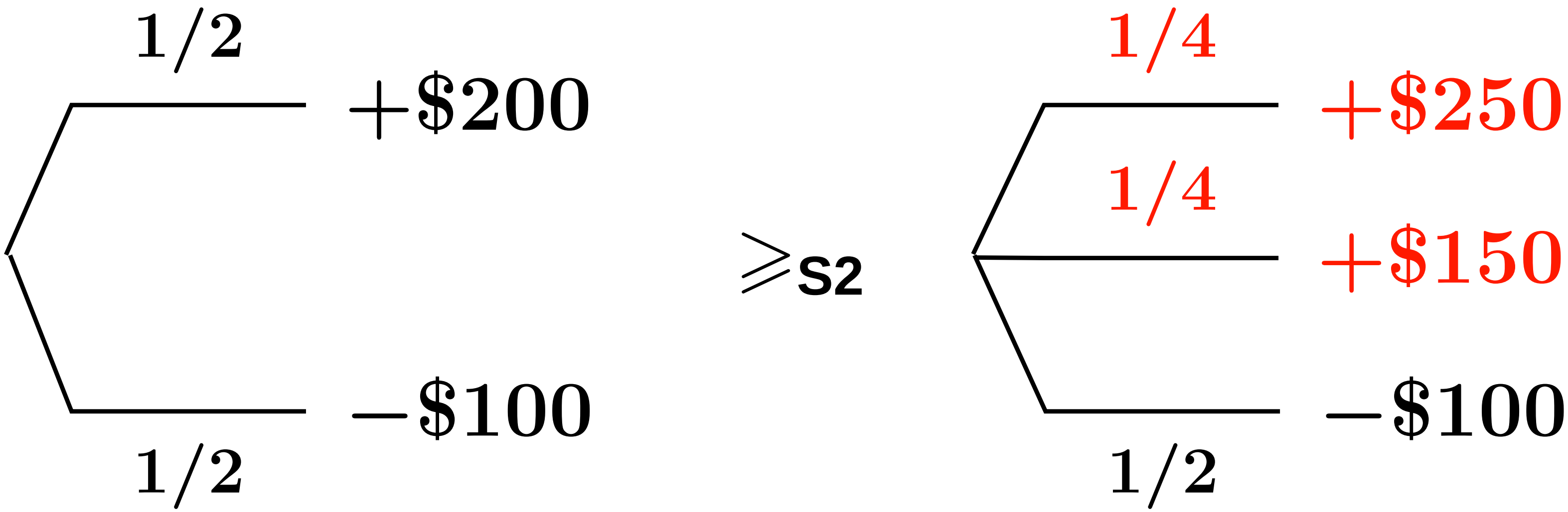

- from g to h: a **MEAN-PRESERVING SPREAD**
- $\geqslant_{\text{S2}} \;=\; \geqslant_{\text{S1}}$ + mean-preserving spreads

Acceptance and Rejection

Let g be a gamble.

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"risk-averse decision-makers are **LESS AVERSE** to g than to h" =

Comparing Risks: Take 2

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Comparing Risks by Rejection

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	REJECTED =
$g \geqslant_{\mathbf{A}} h$	**REJECTED** by u at w

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$g \geqslant_{\mathrm{WU}} h$	**REJECTED** by u at **ALL** w

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Comparing “Comparing Risks”

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$$g \geqslant_{\mathbf{S}} h$$

$$\Downarrow$$

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$$\swarrow \qquad \searrow$$

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Riskiness Orders: Results

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- **WEALTH-UNIFORM DOMINANCE**:

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Riskiness Orders: Results

- WEALTH-UNIFORM DOMINANCE:
  - is a ***complete*** order
- UTILITY-UNIFORM DOMINANCE:

Riskiness Orders: Results

- **WEALTH-UNIFORM DOMINANCE**:
  - is a ***complete*** order :
    for every g, h either $g \geqslant_{\mathrm{WU}} h$ or $h \geqslant_{\mathrm{WU}} g$
- **UTILITY-UNIFORM DOMINANCE**:

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R^{AS} and R^{FH}

$\mathbf{R^{AS}}$ and $\mathbf{R^{FH}}$

Aumann–Serrano index of riskiness R^{AS} :

$$\mathbf{E}\left[1 - \exp\left(-\frac{1}{\mathsf{R}^{\mathsf{AS}}(g)}\,g\right)\right] = 0$$

R^{AS} and R^{FH}

Aumann–Serrano index of riskiness R^{AS} :

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$$\mathbf{E}\left[\log\left(1 + \frac{1}{R^{FH}(g)}g\right)\right] = 0$$

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(the **CRITICAL WEALTH LEVEL**)

Riskiness Orders

$$g \geqslant_{\mathbf{S}} h$$

$$\Downarrow$$

$$g \geqslant_{\mathbf{A}} h$$

$$\swarrow \qquad \searrow$$

$$g \geqslant_{\mathbf{WU}} h \qquad\qquad g \geqslant_{\mathbf{UU}} h$$

Riskiness Orders

$$g \geqslant_{\mathbf{S}} h$$

$$\Downarrow$$

$$g \geqslant_{\mathbf{A}} h$$

$\swarrow\!\!\swarrow$ $\searrow\!\!\searrow$

$g \geqslant_{\mathbf{WU}} h$ * $g \geqslant_{\mathbf{UU}} h$ *

* = complete order

Riskiness Orders

$$g \geqslant_{\mathsf{S}} h$$

$$\Downarrow$$

$$g \geqslant_{\mathsf{A}} h$$

$$\swarrow \qquad \searrow$$

$$g \geqslant_{\mathsf{WU}} h \; * \qquad\qquad g \geqslant_{\mathsf{UU}} h \; *$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$\mathsf{R}^{\mathsf{AS}}(g) \leq \mathsf{R}^{\mathsf{AS}}(h) \qquad\qquad \mathsf{R}^{\mathsf{FH}}(g) \leq \mathsf{R}^{\mathsf{FH}}(h)$$

---

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Technical Details

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Technical Details

- **GAMBLE** g:
  - a real-valued random variable
  - $\mathbf{E}[g] > 0$
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  - finitely many values
- **UTILITY** u:
  - $u : \mathbb{R}_+ \longrightarrow \mathbb{R}$ (put $u(x) = -\infty$ for $x \leq 0$)
  - strictly increasing
  - concave

Technical Details

- **UTILITY** u (*continued*):

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    - *or:* DARA (condition 2 of Arrow, 1965)

Technical Details

- **UTILITY** u (*continued*):
  - rejection decreases with wealth:
    - g rejected at $w \Rightarrow$ g rejected at w', for $w' < w$
    - *or:* DARA (condition 2 of Arrow, 1965)
  - rejection increases with scale:

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  - rejection increases with scale:
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      λg rejected at λw, for $\lambda > 1$

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    - *or:* IRRA (condition 1 of Arrow, 1965)

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  - every gamble is sometimes rejected:

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  - every gamble is sometimes rejected:
    - for every g there is w where g is rejected

Technical Details

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  - rejection decreases with wealth:
    - g rejected at $w \quad \Rightarrow$
      g rejected at w', for $w' < w$
    - *or:* DARA (condition 2 of Arrow, 1965)
  - rejection increases with scale:
    - g rejected at $w \quad \Rightarrow$
      λg rejected at λw, for $\lambda > 1$
    - *or:* IRRA (condition 1 of Arrow, 1965)
  - every gamble is sometimes rejected:
    - for every g there is w where g is rejected
    - *or:* $u(0^+) = -\infty$

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$\Leftrightarrow$

there exist $p, q \in (0, 1]$ such that

$p * g$ **STOCHASTICALLY DOMINATES** $q * h$

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  - $2\,\mathbf{E}[u(w + g)] \geq \mathbf{E}[u(w + 2g)] + u(w)$
  - IF $\mathbf{E}[u(w + g)] \leq u(w)$
    THEN $\mathbf{E}[u(w + 2g)] \leq u(w)$
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g is less risky than h
whenever
g is uniformly rejected less than h
by risk-averse agents

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$$\Downarrow$$

$$g \geqslant_{\mathsf{A}} h$$

$$\swarrow\!\!\!\swarrow \qquad\qquad \searrow\!\!\!\searrow$$

$$g \geqslant_{\mathsf{WU}} h \ \ * \qquad\qquad g \geqslant_{\mathsf{UU}} h \ \ *$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$\mathsf{R}^{\mathsf{AS}}(g) \le \mathsf{R}^{\mathsf{AS}}(h) \qquad\qquad \mathsf{R}^{\mathsf{FH}}(g) \le \mathsf{R}^{\mathsf{FH}}(h)$$

---

* = complete order

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  (**ALL** risk-averse agents reject λg more than g)