

# *Evolution in Iterated Prisoner's Dilemma*

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- evolutionary dynamics on Iterated Prisoner's Dilemma
- selection out of a vast set of the repeated game strategies
- Axelrod (1997) round-robin tournaments: Tit-For-Tat winner
- **ecology** of submitted rules critical for success of this direct reciprocity norm
- Brandt and Sigmund (2006): ALLD, TFT, ALLC with Replicator Dynamics: RSP cycles

- Enriched ecologies of heuristics: GTFT, Pavlov(WSLS) +alternative evolutionary dynamic
- Pavlov= stimulus-response strategy, “Win Stay Lose Shift”
- GTFT= Generous reciprocator: cooperate with certain probability even after opponent defection

- Prisoner's Dilemma stage game
- Discuss iterated strategies
- Construct a 5x5 IPD game
- Discuss various sub-ecologies under Perturbed BR dynamics
  - 1 2x2 (*2-cycle*)
  - 2 3x3 (*limit cycle, chaos*)
  - 3 4x4 (*co-existence of attractors: RSP and chaotic*)
  - 4 5x5 (*"breaking of an invariant circle" route to chaos*)

# PD Stage Game

$$\begin{bmatrix} r/c & C & D \\ C & b-c, b-c & -c, b \\ D & b, -c & 0, 0 \end{bmatrix}$$

- $b$  benefits of cooperation
- $c$  costs associated with cooperative behavior
- $b > c > 0$

# Iterated PD strategies

- random pairing to play an (infinitely) repeated PD game
- focus on *memory-one* strategies
- for each time  $t$  state of play b/w two such randomly drawn opponents  $\Omega = \{CC, CD, DC, DD\}$
- Iterated strategy:
  - 1 start with a first random move  $C$  or  $D$
  - 2 play  $C$  with probability  $(r, s, t, p)$  conditional on realized state at time  $t - 1$  being  $CC, CD, DC, DD$ , respectively

# Deterministic Players

- unconditional cooperators:

$$AII C - (1, 1, 1, 1)$$

- unconditional defectors:

$$AIID - (0, 0, 0, 0)$$

- conditional cooperators "Tit-for-Tatters":

$$TFT - (1, 0, 1, 0)$$

- generous conditional cooperators "Generous-Tit-for-Tat":

$$GTFT - (1, m, 1, n)$$

- stimulus-response (Pavlov) "WinStayLoseShift":

$$WSLS - (1, 0, 0, 1)$$

# Stochastic Players

- stochastic players:  $\varepsilon$ -perturbations of the deterministic ones
- $\varepsilon$ -mistakes/errors in implementation of the deterministic strategies
- $S_1 = (r, s, t, p)$  vs.  $S_2 = (x, y, z, w)$  Markov chain on state space  $\Omega = \{CC, CD, DC, DD\}$  with transition matrix  $T$ .
- $\varepsilon > 0$ , ergodic chain, unique invariant distribution  $\Omega_\tau$
- $\Omega_\tau = \{\tau_{CC}, \tau_{CD}, \tau_{DC}, \tau_{DD}\}$ - fraction of time system spends in each of the four states in  $\Omega$
- $\pi_i(S_i, S_j), (\forall) i, j \in \{AIID, TFT, GTFT, WSLS, AIIC\}$



# IPD payoff matrix M

$r/c$	<i>AIID</i>	<i>TFT</i>	<i>GTFT</i>	<i>WSLS</i>	<i>AiIC</i>
<i>AIID</i>	$\varepsilon (b - c)$	$m_{12}$	$m_{13}$	$\frac{1}{2}b - c\varepsilon$	$b - b\varepsilon - c\varepsilon$
<i>TFT</i>	$m_{21}$	$\frac{1}{2}b - \frac{1}{2}c$	$m_{23}$	$\frac{1}{2}b - \frac{1}{2}c$	$m_{25}$
<i>GTFT</i>	$m_{31}$	$m_{32}$	$\frac{n}{n+\varepsilon} (b - c)$	$m_{34}$	$m_{35}$
<i>WSLS</i>	$b\varepsilon - \frac{1}{2}c$	$\frac{1}{2}b - \frac{1}{2}c$	$m_{43}$	$m_{44}$	$b - \frac{1}{2}c - b\varepsilon$
<i>AiIC</i>	$b\varepsilon - c + c\varepsilon$	$m_{52}$	$m_{53}$	$m_{54}$	$(1 - \varepsilon) (b - c)$

# Evolutionary dynamics of IPD strategies

- "population games" interpretation
- *IPD* strategy revision opportunities
- updating/switching according to fitness
- *IPD* rules ecology evolution:

$$x_{i,t+1} = \frac{e^{\beta(M\mathbf{x})_{i,t}}}{\sum_{i=1}^5 e^{\beta(M\mathbf{x})_{i,t}}}, \quad \sum_{i=1}^5 x_{i,t} = 1$$

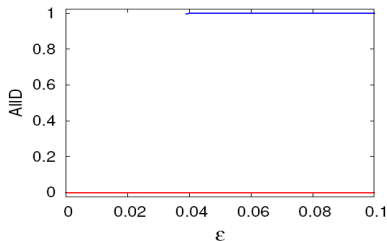
- $\beta \in [0, \infty)$  : random vs. best response

# 2x2 Ecologies

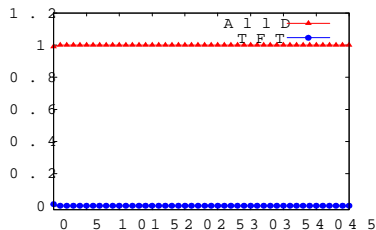
No.	2x2	Game	Bif.	Attractors	Path-Dependence
				$\beta$ large	
1	AIID-TFT	WDS,C	SN	multiple SS	yes, $\varepsilon = 0.05$
2	TFT-AIIC	HD	PD	2-cycle	no
3	AIID-AIIC	DS	none	unique SS	no
4	AIID-GTFT	C	SN	multiple SS	yes, $\varepsilon = 0.01$
5	AIID-WSLS	DS,C	SN	multiple SS	yes, $b = 4$
6	GTFT-AIIC	HD	PD	2-cycle	yes
7	GTFT-WSLS	C	SN	multiple SS	yes
8	TFT-GTFT	DS, C	PD	2-cycle	no
9	TFT-WSLS	WDS	none	unique SS	no
10	WSLS-AIIC	DS,HD	PD	2-cycle	no

- WDS(very small  $\varepsilon$ ), Coordination game

$$\left[ \begin{array}{cc} & \begin{array}{c} AIID \\ TFT \end{array} \\ \begin{array}{c} AIID \\ TFT \end{array} & \begin{array}{cc} \varepsilon(b-c) & -\varepsilon(c-2b+2b\varepsilon) \\ \varepsilon(b-2c+2c\varepsilon) & \frac{1}{2}b - \frac{1}{2}c \end{array} \end{array} \right]$$



(a)  $\beta = 200$ . Bifurcation diagram

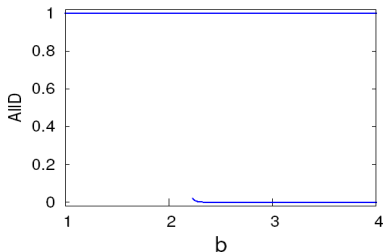


(b)  $\varepsilon = 0.05$ .

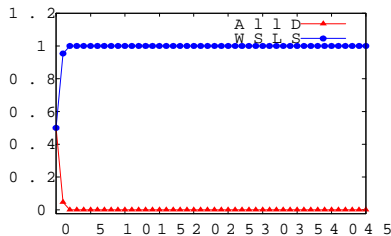
# AIID vs. Pavlov

- DS ( $b < b^*$ ), Coordination

$$\begin{bmatrix} AIID & \varepsilon(b-c) \\ WSLS & b\varepsilon - \frac{1}{2}c \end{bmatrix} \quad (b-c) \left(1 - 4\varepsilon^3 + 6\varepsilon^2 - 3\varepsilon\right)$$



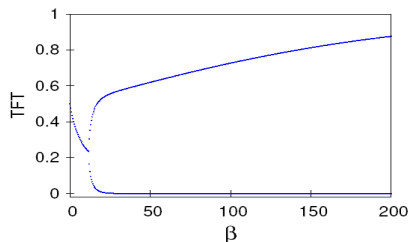
(a)  $\beta = 200, \varepsilon = 0.01$ .



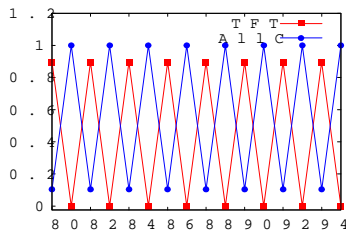
(b)  $b = 4. (x_0, y_0) = (50\%, 50\%)$

- HD

$$\left[ \begin{array}{cc} r/c & \text{TFT} \\ \text{TFT} & \frac{1}{2}(b-c) \\ \text{AIIC} & b-c-2b\varepsilon+c\varepsilon+2b\varepsilon^2 \end{array} \quad \begin{array}{c} \text{AIIC} \\ b-c-b\varepsilon+2c\varepsilon-2c\varepsilon^2 \\ (1-\varepsilon)(b-c) \end{array} \right]$$



(a)  $\varepsilon = 0.01$ . Bifurcation diagram



(b) large  $\beta = 220$ .

- IPD is a PD game itself

$$\begin{bmatrix} r/c & AIID & AIC \\ AIID & \varepsilon(b-c) & b - b\varepsilon - c\varepsilon \\ AIC & b\varepsilon - c + c\varepsilon & (1-\varepsilon)(b-c) \end{bmatrix}$$

- WSLS weakly dominates TFT

$$\left[ \begin{array}{ccc} r/c & \text{TFT} & \text{WSLS} \\ \text{TFT} & \frac{1}{2}b - \frac{1}{2}c & \frac{1}{2}b - \frac{1}{2}c \\ \text{WSLS} & \frac{1}{2}b - \frac{1}{2}c & (b - c) (1 - 4\varepsilon^3 + 6\varepsilon^2 - 3\varepsilon) \end{array} \right]$$



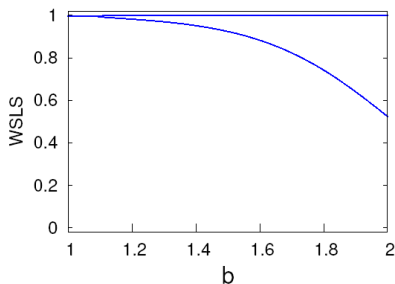
- Coordination

$$\left[ \begin{array}{cc} r/c & GTFT \\ GTFT & \frac{n}{n+\varepsilon} (b-c) \\ WSLS & m_{43} \end{array} \quad \begin{array}{c} WSLS \\ m_{34} \\ (b-c) (1 - 4\varepsilon^3 + 6\varepsilon^2 - 3\varepsilon) \end{array} \right]$$

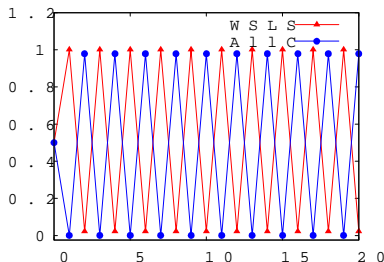
# Pavlov vs. AIIC

- DS ( $b < b^* \approx 2c$ ), HD

$$\left[ \begin{array}{cc} & \text{WSLS} \\ \text{WSLS} & (1 - \varepsilon)(1 - 2\varepsilon + 4\varepsilon^2)(b - c) & b - \frac{1}{2}c - b\varepsilon \\ \text{AIIC} & b - c - 2b\varepsilon + c\varepsilon + 2b\varepsilon^2 & (1 - \varepsilon)(b - c) \end{array} \right]$$



(a)  $\beta = 500$ . Bifurcation diagram

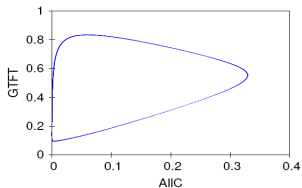


(b)  $\beta = 200$ .

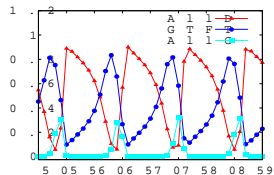
# 3x3 Ecologies

$3 \times 3$	Bif	Attractors	
		moderate $\beta$	$\beta \rightarrow \infty$
AIID-TFT-AIIC	NS	limit cycle	3-cycle
AIID-GTFT-WSLS	NS,LP	limit cycle	3 steady states
AIID-GTFT-AIIC	NS	limit cycle	1 steady state (AIID)
AIID-TFT-WSLS	NS	limit cycle	1 steady state (Pavlov)
AIID-TFT-GTFT	NS	limit cycle	1 steady state (GTFT)
AIID-WSLS-AIIC	PD	2-cycle, chaos	1 steady state (AIID)
TFT-WSLS-AIIC	PD	2-cycle	stable steady state
TFT-GTFT-WSLS	PD	2-cycle	2 steady states
GTFT-WSLS-AIIC	PD	2-cycle	2-cycle
TFT-GTFT-AIIC	PD	2-cycle	2-cycle

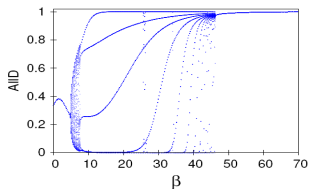
- RSP (moderate  $\beta$ ), DS



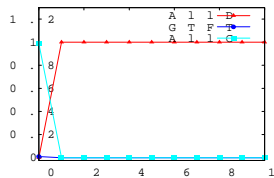
(a)  $b = 1.72$ .



(b)  $b = 1.72$ .

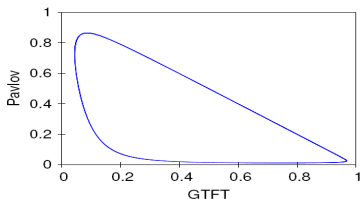


(c)  $\epsilon = 0.01$ .

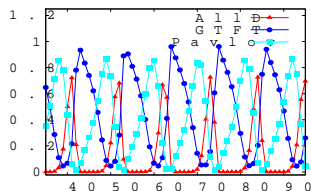


(d)  $\beta = 100$ .

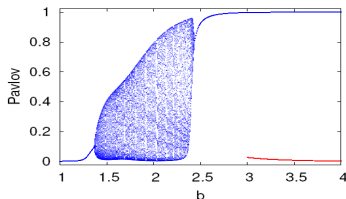
- RSP (low  $b$  and moderate  $\beta$ ), Coordination (high  $b, \beta$ )



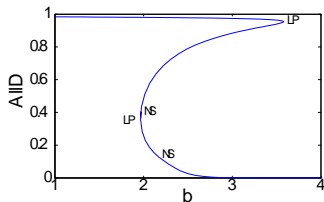
(a)  $b = 2.16$ .



(b)  $b = 2.16$ .

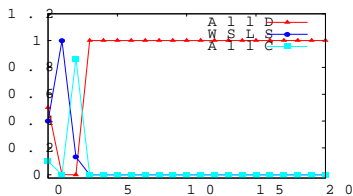


(c)  $\beta = 15$ .

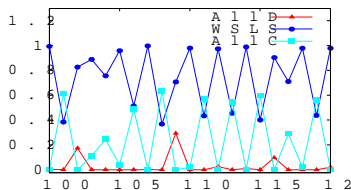


(d) Equilibria curve

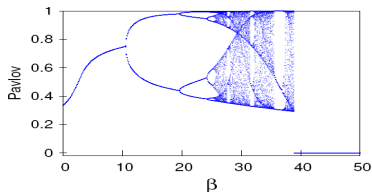
- chaotic co-existence (moderate  $\beta$ ), DS



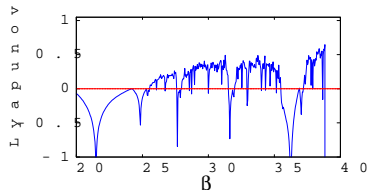
(a)  $b > 2c, \beta = 300$ .



(b)  $b > 2c, \beta = 30$ .



(e)  $b = 4.4$ .

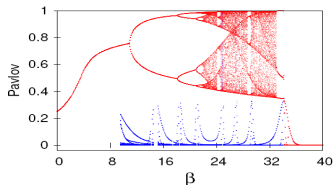


(f)  $b = 4.4$ .

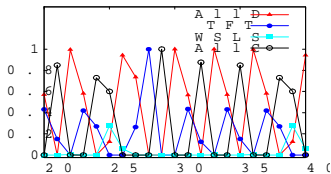
# Summary

4x4	Bif	Attractors	
		small $\beta$	large $\beta$
No AIID	PD	2-cycle	2-cycle/unique SS
No TFT	PD, NS	co-existence SS and chaos	unique SS (AIID)
No GTFT	PD, NS	co-existence RSP and chaos	unique SS (AIID)
No WSLS	NS	limit cycles, chaos	4-cycle
No AIIC	NS, PD	limit cycles, chaos	multiple SS

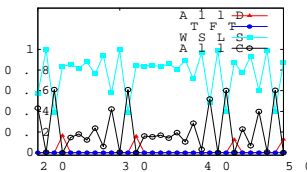
- Co-existence of attractors (RSP and chaos), moderate  $\beta$



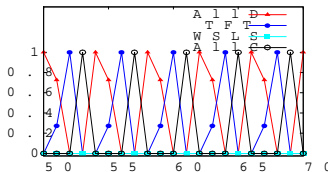
(a)  $b = 4$ .



(b) small  $\beta = 27$ .



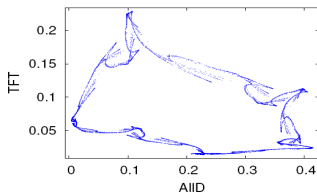
(c) small  $\beta = 27$ .



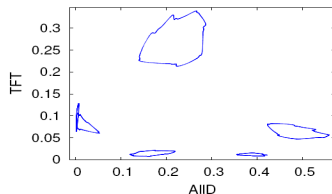
(d) large  $\beta = 100$ .



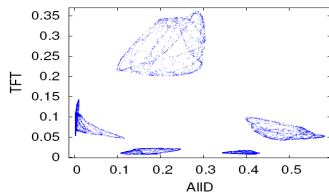
- Full 5x5: "breaking of an invariant circle" route to chaos



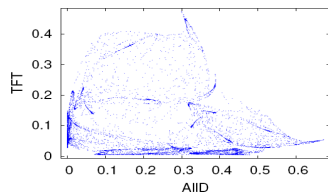
(a)  $\beta = 9.05$ . 1-piece



(b)  $\beta = 10.05$ . 6-piece

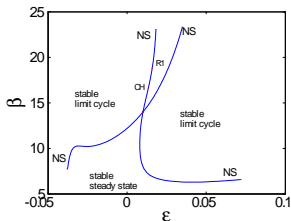
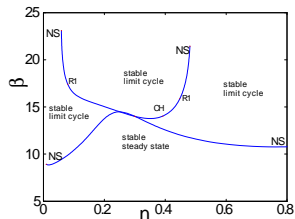
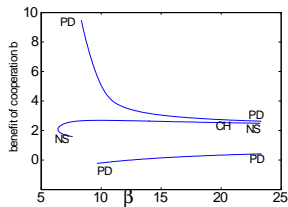
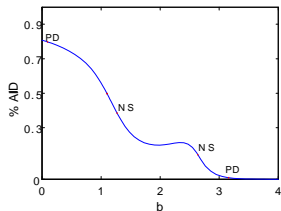


(c)  $\beta = 10.2$ . 6-piece



(d)  $\beta = 10.8$ . 1-piece

# Numerical Bifurcation Curves



# Concluding remarks

- Abundance of RSP patterns in the 3x3 ecologies of heuristics
- finite  $\beta$  leads to path-dependence and co-existence of cyclical/monomorphic and chaotic attractors
- AllC is detrimental to the discriminating types (TFT, GTFT and WSLS) in the 4x4 ecologies leading to an AllD monomorphism
- mixed evidence for Pavlov: wins the evolutionary competition in those 4x4 environments with hard defectors (AllD) but no AllC players...
- ...but, in the full 5x5 ecology almost goes extinct for  $\beta \rightarrow \infty$
- however, with boundedly rational players, fractions of Pavlov stay high even within the complete ecology of rules.

# Future directions

- beyond direct-reciprocity norms
- beyond memory-one strategies
- discounting future payoffs