How Hard is Competition for Rank?

Paul W. Goldberg

1Department of Computer Science
University of Liverpool, U. K.

University of Warwick
April 2010
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**Efficient algorithms**

If the time taken by an algorithm is proportional to $n$ (the input size), or $n^2$ or $n^3$ etc then the algorithm is “efficient” or “fast”; if it is something like $2^n$ then it is not efficient.
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Goldberg

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Easy and hard problems

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To prove problem $P$ is hard, take a problem $H$ that is already believed to be hard, and “efficiently encode” instances of $H$ in terms of $P$ so that the answer to $P$ tells you the answer to $H$ ...
Easy and hard problems

**NP**-complete problems: hard problems that encode *CIRCUIT SAT* (given a boolean circuit with one output, find an input vector that causes the output to be TRUE)

We believe NASH is hard, but it is due to Nash's theorem that we "can't" encode CIRCUIT SAT in terms of NASH!

We settle for PPAD-completeness...

END OF (THE) LINE (Papadimitriou 1991)

Given a graph $G$ of indegree/outdegree at most 1, and a vertex of degree 1, find another vertex of degree 1. The catch is, $G$'s edges are represented by boolean circuits that take any pair of endpoints in $\{0, 1\}^n$ and output whether an edge is present between them.

Goldberg How Hard is Competition for Rank?
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Overview

Nash equilibria are “hard” to find
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- Try looking for other solution concepts, e.g. correlated equilibria, approximate Nash equilibria
- Or, look for algorithms that are efficient and apply to limited kinds of game
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This talk

- Some intuition on the hardness of unrestricted NE
- A class of games that appears to be “realistic” for which we so far have some positive results
The “Dragons’ Den” Game

Two entrepreneurs, Alice and Bob, want to raise £100,000 from a venture capitalist. Each of them may decide to spend £2,000 on image consulting. Alice has a better business idea, and the only way Bob will receive the investment is if he buys the image consulting and Alice does not.

Question: which of them will buy the image consulting?
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Question: which of them will buy the image consulting?

look for mixed (randomised) strategies; the problem becomes: compute the probabilities
### Dragons’ den: payoff matrix

<table>
<thead>
<tr>
<th>Alice</th>
<th>Bob</th>
</tr>
</thead>
<tbody>
<tr>
<td>spend 5000</td>
<td>0</td>
</tr>
<tr>
<td>don’t spend 5000</td>
<td>45</td>
</tr>
<tr>
<td>spend 5000</td>
<td>0</td>
</tr>
<tr>
<td>don’t spend 5000</td>
<td>50</td>
</tr>
</tbody>
</table>

Numbers are multiples of £5,000; assume it is worth £50,000 to win the investment.
“Incentive direction” of the players

Alice

Bob

don’t spend
spend

spend
don’t spend

Goldberg

How Hard is Competition for Rank?
“Incentive direction” of the players

Alice

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How Hard is Competition for Rank?
Nash equilibrium

Brouwer’s fixpoint theorem: continuous functions from a compact domain to itself, have fixpoints. A non-constructive proof.

L.E.J. Brouwer (1881-1966)
Nash equilibrium

Brouwer’s fixpoint theorem: continuous functions from a compact domain to itself, have fixpoints. A non-constructive proof. Nash’s theorem: using Brouwer’s FPT, there always exists a solution, provided that players may randomize (any number of players, any number of actions).

John Forbes Nash
Nash equilibrium

Brouwer’s fixpoint theorem: continuous functions from a compact domain to itself, have fixpoints. A non-constructive proof.
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- standard notion of “outcome of the game”

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Brouwer’s fixpoint theorem: continuous functions from a compact domain to itself, have fixpoints. A non-constructive proof. Nash’s theorem: using Brouwer’s FPT, there always exists a solution, provided that players may randomize (any number of players, any number of actions).

- standard notion of “outcome of the game”
- each player is receiving optimal expected payoff in the context of the other players’ choices.
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- standard notion of “outcome of the game”
- each player is receiving optimal expected payoff in the context of the other players’ choices.

But, how to compute the probabilities? We would like an “efficient algorithm”. Next: how search for NE relates to search on large graphs

John Forbes Nash
“Incentive direction” of the players

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Bob

don’t spend

spend

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spend

Goldberg

How Hard is Competition for Rank?
“Incentive direction”, colour-coded

Alice

Bob

don’t spend

spend

spend

don’t spend

Goldberg

How Hard is Competition for Rank?
Now, pretend this triangle is high-dimension domain
...converges to Brouwer fixpoint
The corresponding graph
The corresponding graph
From graph search to NE computation

- Papadimitriou (1991): generic “END OF LINE” graph search problems seem to be hard
- They can encode/represent the difficulty of finding fixpoints of certain Brouwer functions.

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Daskalakis, G and Papadimitriou (2005-6) show that games can also represent/encode a class of Brouwer functions which themselves encode END OF LINE graph search. Basically, solving a game is equivalent to finding your way around a very large graph, one that allows efficient local exploration and consists of long paths.
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- 2-players (Chen, Deng and Teng ’06); 2-players, 0/1-valued payoffs (Abbott, Kane and Valiant ’05)
How to make a hard case of the problem

Chen and Deng ('06, '09): 2D-Brouwer
coming back to “Dragons’ Den”

(Current work with colleagues at Liverpool)
coming back to “Dragons’ Den”

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What if there are

- more than 2 competitors?
coming back to “Dragons’ Den”

(Current work with colleagues at Liverpool)
What if there are
- more than 2 competitors?
- many choices per competitor?
- more than one “prize” for winning?

Players compete for rank.
how hard is competition for rank?
Competition for rank

University League Table 2010

Published April 30th 2009

Create your own customised ranking, see the device below the main table.

To compare 2 or more universities, select the box next to the name and click Compare.
To create your own ranking see below

<table>
<thead>
<tr>
<th>Rank</th>
<th>Institution</th>
<th>Student Satisfaction</th>
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<th>Entry Standards</th>
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Auto Trader UK - Buy & Sell New & Used Cars, Car Loans, Car Insurance
The UK's #1 site to buy and sell new and used cars, bikes, vans, trucks and caravans with over 350000 vehicles online. Check Car news, reviews and obtain ...
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Recognised in the recent Research Assessment Exercises.

Following a Grade 5 rating in 2001, 75% of the Department's research activity was judged as 3* or 4* in 2008, putting it among the top 10 Computer Science departments in the country. All three research groups also won a best paper prize at a major conference in 2008.
Competition for rank

Six-figure scholars
Membership of £100K club is growing

Cut and thrust
Mandelson steadfast at memorial conference

Tainted by Climategate
Unfair suspicion falls on other UEA research

Critical dialogue
How to cultivate the Socratic spirit

Ways to rise in the league tables without breaking the bank
Britain's quality of life worse than former Communist countries

Britain's has fallen to 25th position on a list of best places in the world to live.

By Myra Butterworth, Personal Finance Correspondent
Published: 6:54PM GMT 06 Jan 2010

Our poor climate, soaring unemployment and congested roads mean we are now ranked behind countries such as the People's Republic of China.
Some background on ranking games

“Ranking games” (Brandt, Fisher, Harrenstein and Shoham)

each combination of strategies results in a ranking of the players; every player has a monotonically decreasing function from rank to utility.

Problem: unrestricted ranking games are still hard: a 3-player ranking game can easily encode an unrestricted 2-player 0/1 game. (as noted earlier, hard to solve)
Our idea: assume strategies are correlated with “competitiveness”
Each player has his own function from effort to performance.
The model

Player \( i \) has actions (pure strategies) \( a_1^i, \ldots, a_n^i \)
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Player \( i \) has actions (pure strategies) \( a_1^i, \ldots, a_n^i \). \( a_j^i \) has associated quantities \( c_j^i \) (the cost) and \( r_j^i \) (the “return”, or level of performance).

Players get ranked on the \( r_j^i \)-values they obtain.
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$c_j^i \leq c_{j+1}^i$ and $r_j^i \leq r_{j+1}^i$, i.e. lower-indexed strategies are less competitive.
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There are “prizes” awarded to players according to rank; the \( k \)-th prize has value \( u_k \).

If a player plays \( a_j \) and wins the \( k \)-th prize, his overall utility is \( u_k - c_j \).
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If a player plays $a_j$ and wins the $k$-th prize, his overall utility is $u_k - c_j$.

Observation

We can concisely represent games with many players/strategies, in contrast with unrestricted ranking games.
We can pre-process a $d$-player game so as to assume that $u_1 = 1$, $u_d = 0$; all costs $c^i_j$ lie in range $[0, 1]$; costs and returns are strictly monotonic in $j$, else we would have dominated actions; each player’s weakest action has cost 0.

**Theorem**

Suppose there is just one prize ($u_1 > 1$; $u_j = 0$ for $j > 1$). Suppose ties are impossible (if all $r^i_j$-values are distinct, or equivalently there is a tie-breaking rule).

Then there is just one player who gets positive payoff (all others get zero); namely the player who has the strongest action.
Some results

Proof.

- If $a^1_n$ is the strongest action in the game, note that player 1 can ensure a payoff of $u_1 - c^1_n$. 
Some results

Proof.

- If \( a^n_1 \) is the strongest action in the game, note that player 1 can ensure a payoff of \( u_1 - c^n_1 \).
- In a NE, for each player \( i \) let \( a^i_W \) be the weakest action that \( i \) plays with positive probability. All but one of these actions are guaranteed to lose (payoff: \( -c^i_W \)).
Some results

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- In a NE, for each player $i$ let $a^i_W$ be the weakest action that $i$ plays with positive probability. All but one of these actions are guaranteed to lose (payoff: $-c^i_W$).
- So, all but one player get a non-positive payoff (since a player’s payoff is his expected payoff for any action he uses with positive probability. $i$ can get payoff 0 by playing $a^i_1$, so presumably his overall payoff is 0.

Goldberg

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Some results

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- So, all but one player get a non-positive payoff (since a player’s payoff is his expected payoff for any action he uses with positive probability. $i$ can get payoff 0 by playing $a^i_1$, so presumably his overall payoff is 0.
- Finally, we found precisely one player who can get positive payoff.

What if the strongest action has cost 1? What about $> 1$ prizes?
Some results

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Suppose there is just one prize ($u_1 > 1$; $u_j = 0$ for $j > 1$). Suppose ties are impossible (if all $r_{ij}$-values are distinct, or equivalently there is a tie-breaking rule).

Then if you know the support of the solution, you can compute it easily; also, the solution is all in rational numbers.
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Then if you know the support of the solution, you can compute it easily; also, the solution is all in rational numbers.

(So, that’s like 2-player normal-form games! Is that interesting?)
Some results

- How about poly-time algorithms? We have some for special cases...
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- 2-player games are easy; no, they are not zero-sum; it’s quite a cute algorithm
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- 2-player games are easy; no, they are not zero-sum; it’s quite a cute algorithm
- $d$-players, $n$ actions, where $d$ is constant: Approximate NE can be found in poly-in-$n$ time by brute-force approach.
How about poly-time algorithms? We have some for special cases...

2-player games are easy; no, they are not zero-sum; it’s quite a cute algorithm

$d$-players, $n$ actions, where $d$ is constant: Approximate NE can be found in poly-in-$n$ time by brute-force approach.

FPTAS for $d$ players, 1 prize (in the paper, done for just 2 players) Dynamic programming approach
Suppose the $k$-th prize has value $a - bk$ where $a$ and $b$ are positive constants. We can solve as follows.

Each player gains $b$ for every other player he beats. So, express his payoff as the negation of the cost of his action, plus the sum of payoffs from a bunch of zero-sum 2-player games. His payment of that cost can be considered as a 2-player game against "nature" (a dummy player) who collects the cost but does not influence the player.

So, we have reduced the game to a zero-sum polymatrix game, which is known to be solvable in poly-time (Daskalakis and Papadimitriou '09).
Linear-prize ranking games

Suppose the $k$-th prize has value $a - bk$ where $a$ and $b$ are positive constants. We can solve as follows. Each player gains $b$ for every other player he beats. So, express his payoff as the negation of the cost of his action, plus the sum of payoffs from a bunch of zero-sum 2-player games.
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unrestricted games (designed by a notional adversary to be difficult to solve) indeed “cannot” be solved by efficient algorithms.

For these games, continue by looking for decentralised algorithms (a solution is implausible if it needs to be found centrally and then handed out to the players).

Another direction: weaken the objective – “approximate equilibria” replace “no incentive for a player to change” with “only a small incentive to change” — an interesting and challenging problem, both for centralised and decentralised algorithms!
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