Product differentiation in the presence of social interactions of consumers[a]

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Abstract

We present a dynamic game of location-price competition between two firms. Differently from other Hotteling's type models, we assume that consumers are positively influenced by the product choices of others and decide in groups of limited sizes where to consume from.

Our model suggests the existence of three types of oligopolies: one characterized by small distances between players, another characterized by intermediary distances between players, and the third one characterized by large distances between them. This result generalizes the standard result of location-price competition. It provides insights into product differentiation behaviors in cases where consumers enjoy consuming products in the company of others (Becker, 1991) and decide in groups where to consume from.

Motivation

Becker (1991):

"... *A popular seafood restaurant in Palo Alto, California, does not take reservations, and every day it has long queues for tables during prime hours. Almost directly across the street is another seafood restaurant with comparable food, slightly higher prices and similar service and other amenities. Yet this restaurant has many empty seats most of the time. Why doesn't the popular restaurant raise prices, which would reduce the queue for seats but expand profits?...*"

Beckers' explanation: social interaction of consumers

A slight increase in prices could not only eliminate the queue, but also cut an additional number of costumers who use to visit the restaurant just because it is permanently over-demanded. The resulting effect is that a slight increase in prices might reduce significantly (discontinuously) the restaurant's demand.

Contribution

We propose a model that supports and extends Beckers' explanation.

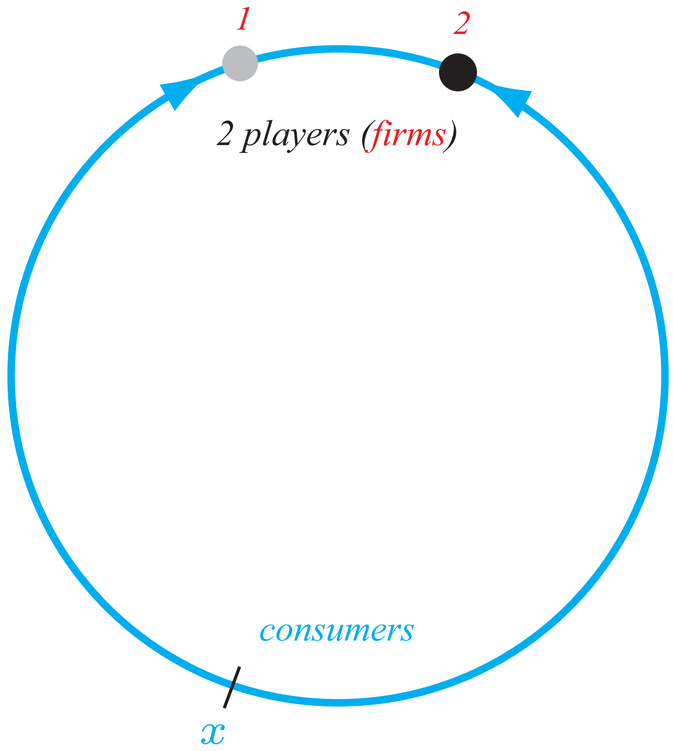
- We argue that the products proximity observed in Becker's restaurant case in fact ensures the demand polarization.

In light of this, we also answer the following question:

- Why would producers opt to be close to each other? (by coming close to each other some of them will be under demanded!)

The answer lies on a critical strength of social interactions among consumers. If the strength of social interactions is large enough and collations among consumers are sufficiently small, then the expected profit for all producers will be higher if they come close to each other than if they get distant from each other. The opposite result (where the maximal distance leads to maximal profits) is derived when the strength of social interactions is smaller than this critical value.

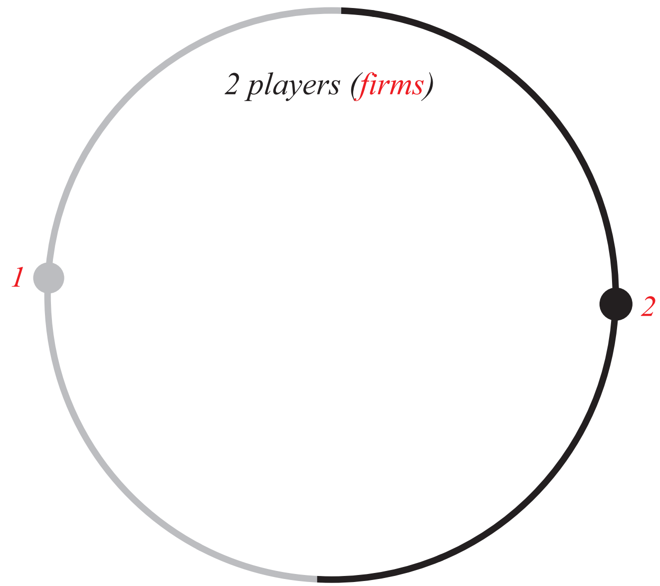
Model of spatial product differentiation



$$U_x(i) = u - P^{(i)} - T_x^{(i)}$$

$u > 0$ $\quad$ $P^{(i)}$, *price of* i $\quad$ $T_x^{(i)}$, *transport cost*

D'Aspremont, Gabszewicz, Thisse (1979)

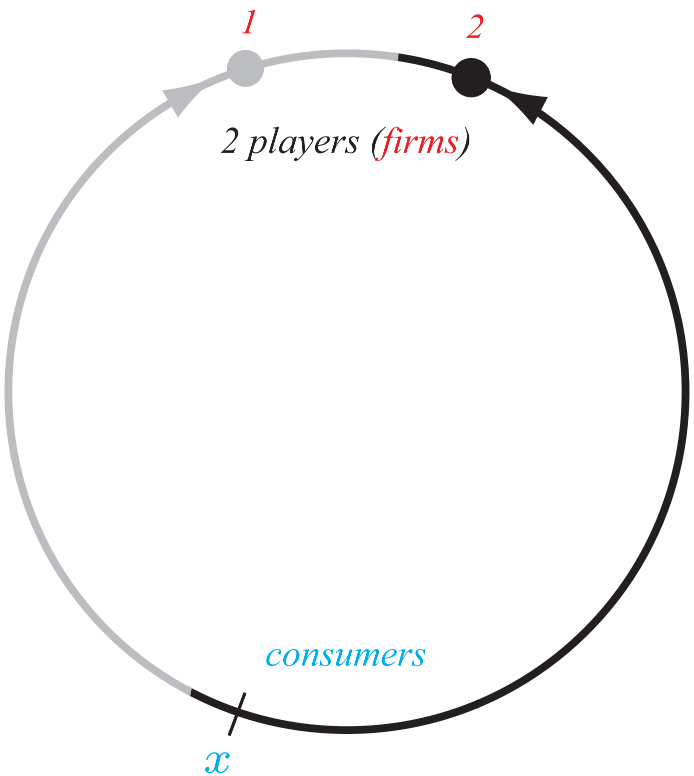


$$\mathbf{d}_* = \mathbf{d}_{max}, \qquad P_*^{(1)} = P_*^{(2)} = T(\mathbf{d}_{max}), \qquad \bar{N}_*^{(1)} = \bar{N}_*^{(2)} = 1/2$$

$\bar{N}_*^{(1)}$, $\bar{N}_*^{(2)}$ *fractions of consumers that choose firms 1 and 2*

Result. In Nash equilibrium, the distance between players is maximal, and the players share the market symmetrically.

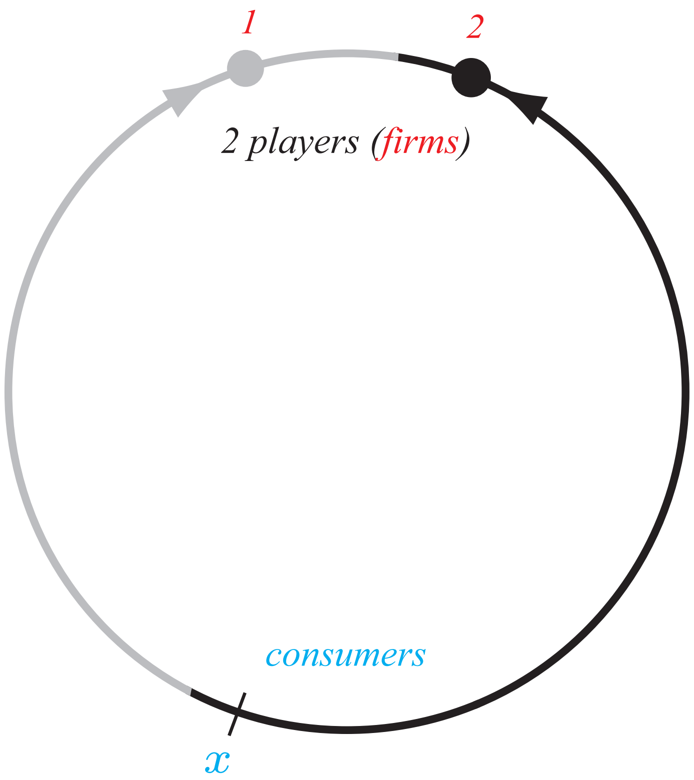
Introducing positive externalities



$$U_x(i) = u - P^{(i)} - T_x^{(i)} + \boxed{J\bar{N}^{(i)}}$$

$J > 0$, $\bar{N}^{(i)}$ *fraction of consumers that choose firm* $i = 1, 2$

Consumers coalitions



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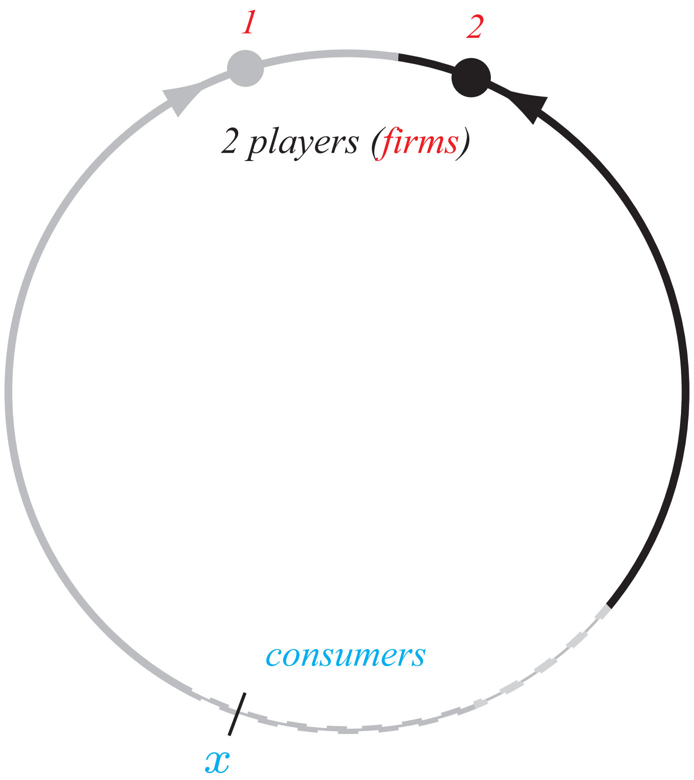
Consumers coalitions



$$U_x(i) = u - P^{(i)} - T_x^{(i)} + \boxed{J\bar{N}^{(i)}}$$

$J > 0$, $\quad \bar{N}^{(i)}$ *fraction of consumers that choose firm* $i = 1, 2$

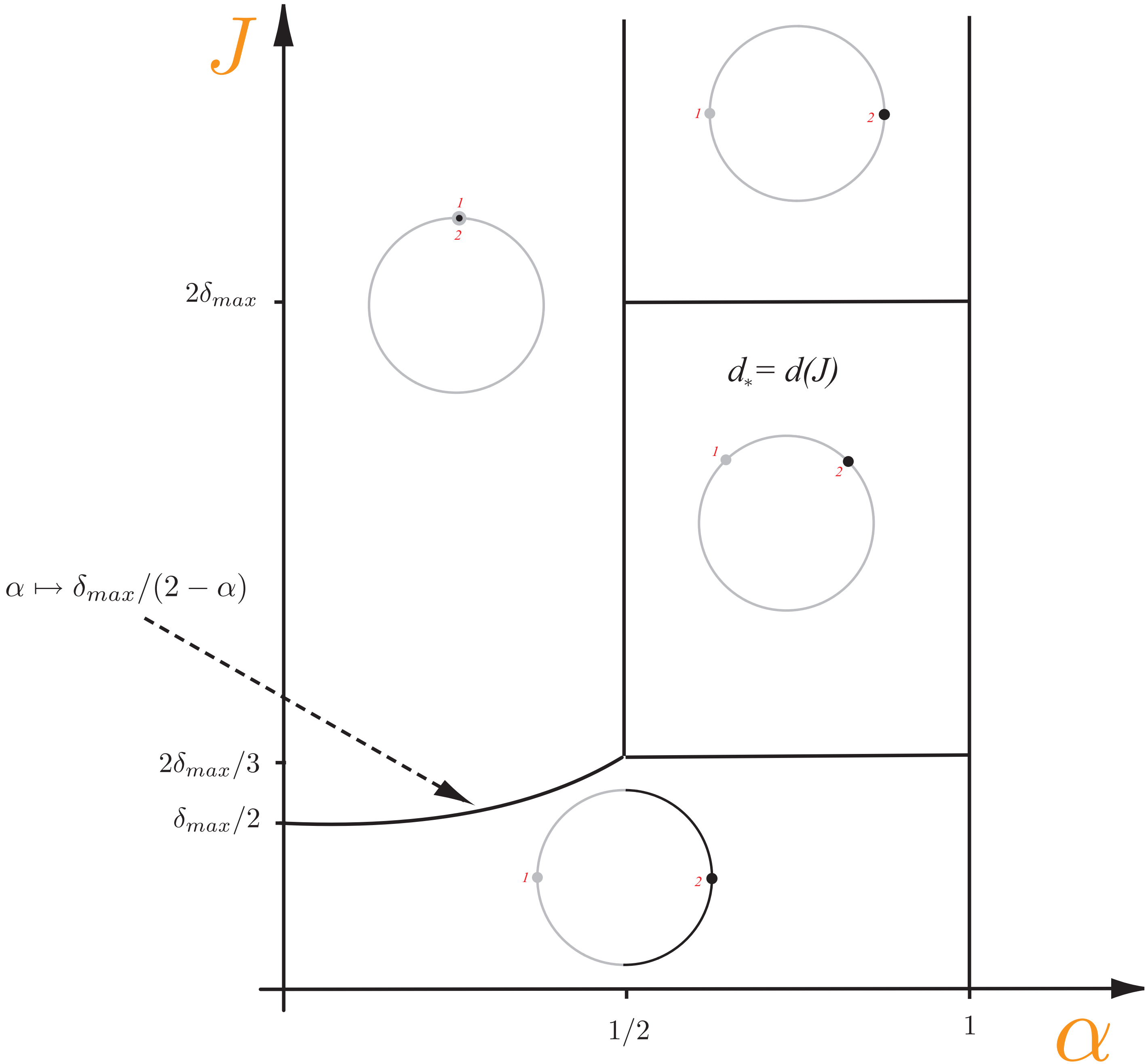
Product differentiation in Nash equilibria

Assumptions:

1. Firms play non decreasing sequences of prices $P_t^{(1)}, P_t^{(2)}$ over time $t = 1, 2, \ldots$
2. Consumers change their decisions according to best coalition responses. Deviating coalitions are not larger than α (due to prohibitive coordination costs among players).

New results related to the distances between players and market shares in Nash equilibrium.

Distances and market shares in Nash equilibrium will now depend on the model paramenters J (the strength of positive externalities in consumers decisions), α (the maximal measure of consumers coalitions) and δ_{max} (the maximal transportantion cost incurred by a consumer).



J
$2\delta_{max}$
$2\delta_{max}/3$
$\delta_{max}/2$
$\alpha \mapsto \delta_{max}/(2-\alpha)$
$d_* = d(J)$
1
2
$1/2$
1
α

Dynamical game of (social) product differentiation

In stage $t = 0$

Players choose $l^{(1)}$ and $l^{(2)}$ (locations of products)

In stages $t = 1, 2, 3 \ldots,$

Players choose simultaneously $P_t^{(1)}$, $P_t^{(2)} \geq 0$

Pay-offs:

$$\pi^{(i)} = \mathbf{E}\Big\{ \lim\inf_{T\to\infty} \frac{1}{T} \Big[\sum_{t=1}^{T} \bar{N}_t^{(i)} P_t^{(i)} \Big] \Big\}, \quad i \in \{1, 2\} \tag{1}$$

where $\bar{N}_t^{(i)}$ denotes de Lesbegue measure of $N_t^{(i)}$ along the circle.

UTILITY OF CONSUMER $x \in N_t^{(i)}$ AT TIME t

$$U(P_t^{(i)}, T_x^{(i)}, N_t^{(i)}) = u - P_t^{(i)} - T_x^{(i)} + J\bar{N}_t^{(i)}, \;\; i \in \{1, 2\} \qquad (2)$$

where

- $P_t^{(i)} \in [0, \infty)$, price of product $i \in \{1, 2\}$ at time t
- $T_x^{(i)} \in [0, \infty)$, transportation cost
- $\bar{N}_t^{(i)} \in [0, 1]$, measure of the set of consumers that choose $i \in \{1, 2\}$ at time t
- $J > 0$, strength of social interactions

We assume

$$T_x^{(i)} \sim [d(x, l^{(i)})]^2$$

where $d(x, l^{(i)})$ is the distance between consumer x and product i.

Dynamics of consumers coalitions

A time $t = 0$

$$\bar{N}_0^{(0)} = 1, \quad \bar{N}_0^{(1)} = 0, \qquad \bar{N}_0^{(2)} = 0$$

At each time $t = 1, 2, \ldots$, we choose at random a deviating coalition $C_t^{(i \to j)}$, satisfying

$$\bar{C}_t^{(i \to j)} < \alpha \quad (\alpha \text{ is a model parameter })$$

and set

$$N_t^{(j)} = N_{t-1}^{(j)} \cup C_t^{(i \to j)}, \qquad N_t^{(i)} = N_{t-1}^{(i)} - C_t^{(i \to j)}$$

$$N_t^{(k)} = N_{t-1}^{(k)} \text{ for } k \notin \{i, j\}$$

If there is no deviation coalition, we set $N_t^{(i)} = N_{t-1}^{(i)}$, $i = 0, 1, 2$

Deviating consumers coalitions

Let $D_t^{(i\to i)}$ be the set of all subsets $C \subset N_{t-1}^{(i)}$, that satisfy

$$U(P_t^{(i)}, T_x^{(i)}, N_{t-1}^{(i)}) < U(P_t^{(j)}, T_x^{(j)}, N_{t-1}^{(j)} \cup C) \quad \forall x \in C$$

For $D_t^{(i\to j)} \neq \emptyset$, we define $C_t^{(i\to j)} \in D_t^{(i\to j)}$, $i \neq j$:

- $C_t^{(i\to j)} = C(\tau)\Big|_{\tau=\tau_t}$
- $C(\tau) = \left\{ x \,:\, T_x^{(j)} - T_x^{(i)} \leq \tau \right\} \cap N_{t-1}^{(i)}$
- $\tau_t = \sup \left\{ \tau \,:\, \bar{C}(\tau) \leq \alpha \quad \text{and} \quad C(\tau) \in D_t^{(i\to i)} \right\}$

Preis Strategies

Players choose

$$P_t^{(i)} = X_{s_t}^{(i)} F^{(i)}(h_t) \tag{3}$$

Where

1. $s_l = \sum_{l=1}^{t} F^{(i)}(h_l), \quad h_t = \left\{ (\bar{N}_l, P_l) \right\}_{l=1}^{t-1}$
2. $F^{(i)}(h_t) \in \{0, 1\}$ is a function of the game history h_t where

$$\forall t > 1, \forall h_t : \; F^{(i)}(h_t) = 1 \quad \text{if} \quad P_{t-1}^{(i)} > 0$$

3. $X_s^{(i)}$, $s = 1, 2, \ldots$ is a non decresing sequence of positive numbers, which does not depend on the game history.

Preis Strategies (Example)

Depending on $F^{(i)}$ and on the game histories $h_1, h_2, \ldots$, we may have:

$$\{P_1^{(i)}\}_{t=1}^{\infty} = 0, 0, 0, 0, X_1, X_2, X_3, \ldots \tag{4}$$

Example:

$$P_t^{(1)} = \begin{cases} 0 \quad \text{if} \quad \max\{P_{t-1}^{(2)}, P_{t-2}^{(2)}, \ldots, P_2^{(2)}, P_1^{(2)}\} < 10 \\ \\ 7 \qquad\quad \text{oderwise} \end{cases} \tag{5}$$

In (5) we have $X_t^{(i)} = 7, \;\; t = 1, 2, 3 \ldots$

Product differentiation

Define

$$\delta \stackrel{def}{=} \max_{x \in N} \left| T_x^{(1)} - T_x^{(2)} \right|, \qquad \mathbf{d} \stackrel{def}{=} d(l^{(1)}, l^{(2)})$$

It follows that

$$\delta = constant * \mathbf{d}(2\mathbf{d}_{max} - \mathbf{d})$$

δ is strictly increasing in $\mathbf{d}$.

It will be convenient to characterize the distance $\mathbf{d}_*$ in Nash equilibrium by the corresponding maximal difference in transport costs δ_*, where $\delta_* = \delta(\mathbf{d}_*)$.

NEW RESULT. There is a sub-game perfect Nash equilbrium given by: (it is unique if the set of price strategies is restricted to (3))

Nash equilibria (Market-share strategy)

if $J < J_c(\delta_{max}, \alpha)$, then

1. $\delta = \delta_{max}$.

   (The distance between firms is maximal).

2. $\lim_{t\to\infty} P_t^{(i)} = \delta_{max} - J,\ \ i = 1, 2.$

   (The firms play the same last prices in the long run).

3. $\lim_{t\to\infty} \bar{N}_t^{(i)} = 1/2,\ \ i = 1, 2.$

   (Market is shared symmetrically in the long rung).

4. $\pi_i^{(i)} = (\delta_{max} - J)/2,\ \ i = 1, 2$

   (Players receive the same pay-off).

Nash equilibria (Monopoly strategy)

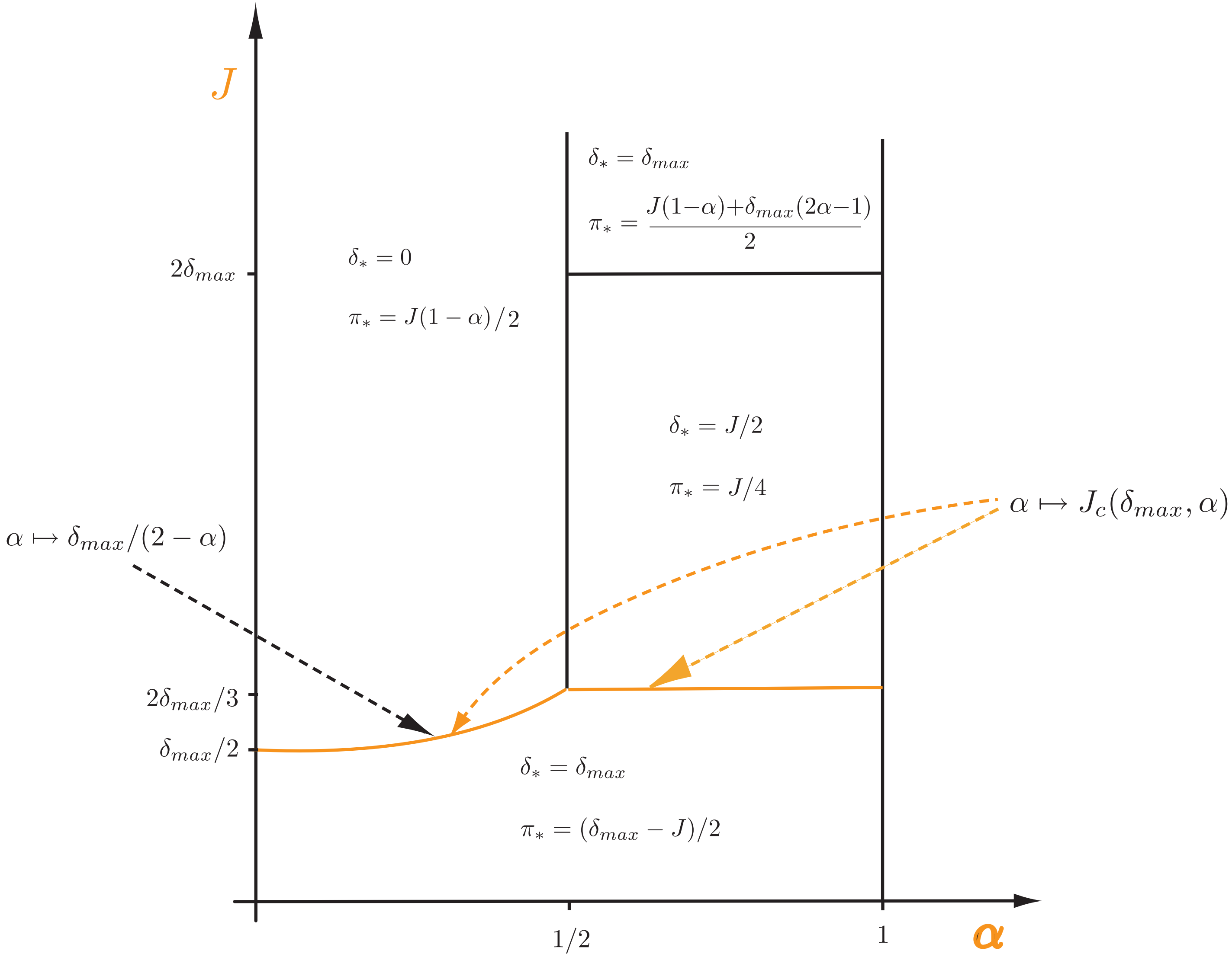
if $J > J_c(\delta_{max}, \alpha)$ and $\alpha \neq 1/2$, then

1. $\delta = \delta_*(\delta_{max}, \alpha, J)$.
   (The distance between firms depends on the model parameters).
2. $P_1^{(1)} = P_1^{(2)} = 0$, and for $t > 1$,

$$\begin{cases} P_t^{(i)} = 0 & \text{if} \quad \bar{N}_{t-1}^{(i)} < 1 \\ \\ P_t^{(i)} \uparrow P_*(\delta_{max}, \alpha, J) & \text{if} \quad \bar{N}_{t-1}^{(i)} = 1 \end{cases} \qquad (i = 1, 2) \tag{6}$$

3. $\lim_{t\to\infty} \bar{N}_{t-1}^{(i)} = 1$ and $\lim_{t\to\infty} \bar{N}_t^{(j)} = 0$, where $(i, j) = (1, 2)$ or $(i, j) = (2, 1)$. (One firm will become the monopolist).
4. Expected[a] pay-offs $\pi_i^{(i)} = P_*(\delta_{max}, \alpha, J)/2$, $i = 1, 2$

[a]Both players have equal probability to polarize the market.



J
$2\delta_{max}$
$2\delta_{max}/3$
$\delta_{max}/2$
$\delta_* = \delta_{max}$
$\pi_* = \frac{J(1-\alpha)+\delta_{max}(2\alpha-1)}{2}$
$\delta_* = 0$
$\pi_* = J(1-\alpha)/2$
$\delta_* = J/2$
$\pi_* = J/4$
$\alpha \mapsto J_c(\delta_{max}, \alpha)$
$\alpha \mapsto \delta_{max}/(2-\alpha)$
$\delta_* = \delta_{max}$
$\pi_* = (\delta_{max} - J)/2$
$1/2$
1
α

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