Equilibrium Selection and the Dynamic Evolution of Preferences

Tom Norman

Magdalen College, Oxford

GAM Workshop, Warwick
16th April 2010

Outline

Evolutionary Game Theory

- A population P is an *evolutionarily stable state* (ESS) if, for every "mutation" Q, there is an *invasion barrier* $\varepsilon(Q) > 0$ such that, for all $0 < \eta \leq \varepsilon(Q)$,

$$E(P, (1-\eta)P + \eta Q) > E(Q, (1-\eta)P + \eta Q).$$

  If the inequality is weak, P is a *neutrally stable state* (NSS).
- ESS $\Rightarrow$ Nash.
- Seen as appealing by virtue of their foundations in dynamic models,
- specifically the replicator dynamics, an example of the more general class of payoff-monotone dynamics.

Dynamic Analysis

- Letting $\sigma(x, Q) := E(\delta_x, Q) - E(Q, Q)$ be the success of strategy x if the population is Q,
- the replicator dynamics increase the frequency of strategies that are successful relative to the prevailing average fitness:

$$\frac{Q'(t)(x)}{Q(t)(x)} = \sigma(x, Q(t)),$$

or, more generally,

$$Q'(t)(A) = \int_A \sigma(x, Q(t))Q(t)(dx), \qquad \forall A \in \mathscr{B}.$$

Static–Dynamic Links

- Symmetric Nash equilibria are stationary under the replicator dynamics.
- Moreover, in the finite case with pairwise interactions, every ESS is asymptotically stable in the replicator dynamics, and every NSS is Lyapunov stable.
- In the infinite case, this is no longer true and we require stronger concepts.
- Bomze's "strong uninvadability," for example, is like evolutionary stability with respect to mutations that are "close" in the strong topology.

Preference Evolution

- "Indirect evolutionary approach": players play rationally for given preferences,
- but those preferences are free and subject to evolutionary selection according to their success in an underlying game of biological fitness.
- Specifically, a population of players is repeatedly matched to play a finite, symmetric 2-player fitness game.
- However, play is determined by a transformed payoff game.
- The payoffs $u \in U^2$ in this payoff game evolve according to the fitnesses induced by play in the payoff game.

Bias Example

(*a*)

	L	R
U	2 2	0 0
D	0 0	1 1

(*b*)

	L	R
U	$2 + x_c$ $2 + x_r$	0 x_r
D	x_c 0	1 1

Figure 1: (a) Coordination in fitnesses (b) Payoffs given biases

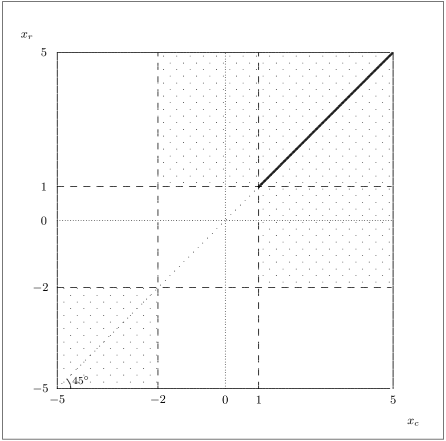
Results

- Divergence from fitness-maximizing preferences is then possible because of the resulting effect on opponents' play.
- Two key questions:
  1. What preferences would emerge if the whole range of possible preferences were allowed to compete, rather than some subset chosen for the example at hand?
  2. Can non-fitness-maximizing preferences emerge in the absence of preference observability?
- Dekel, Ely & Yilankaya (*REStud* 2007):
  - Efficient strict Nash (in fitnesses) $\Rightarrow$ stability (e.g. $\{U, L\}$).
  - Stability $\Rightarrow$ efficiency, given observability.
  - Absent observability, stability $\Rightarrow$ Nash in fitness game; strict Nash $\Rightarrow$ stability.

A Dynamic Model

- Let biases be shaped by the replicator dynamics.
- Given biases, Nash equilibrium in the payoff game determines play, and thus underlying fitnesses.
- If there is more than one equilibrium, each of them is assumed to be played with some given, strictly positive probability.
- Many biases are equivalent in terms of resulting fitnesses, so use setwise stability concepts (Norman, *GEB* 2008).

Bias Example



Equilibrium Selection

- More generally, we can think about any transformation from fitnesses to payoffs (not just biases),
- and we can think of numerous other rules for play in the presence of multiple equilibria;
- specifically, we can allow for any equilibrium-selection mechanism—e.g. global games.
- Gives a well-defined replicator dynamics (rather than a differential inclusion).

Results

- In common-interest fitness games, maximal efficient face $\rightsquigarrow$ asymptotically stable.
- For general fitness games, any face enforcing efficient strict Nash through dominant strategies is Lyapunov stable.
- and any face supporting a Pareto-dominated outcome is not Lyapunov stable for an appropriately chosen equilibrium-selection mechanism.
- For doubly symmetric fitness games (including some Hawk–Dove), “purified p^*-populations,” p^* an efficient MSE of the fitness game, satisfy a weaker form of stability.
- With unobservable preferences, maximal face supporting symmetric strict Nash outcome $\rightsquigarrow$ asymptotically stable.