Many-agent interaction in an economy with diffusion and purchasing[1]

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Introduction

The diffusion equations appear in some models of information and innovation diffusion, price movements and expenditure diffusion, diffusion processes in ecological systems (Sahal, 1985; Petrosyan, Zaharov, 1997; Petit, Sanna-Randaccio and Tolwinski, 2000; Rogers, 2003; Cappoza, 2003, etc.).

The game-theoretical model of diffusion conflict process of manufacture and consumption was constructed in (Troeva and Malafeyev, 2000). An advanced economy model of the kind was considered in (Troeva, 2002) where the distributors activities were supposed to be present alongside with enterprises and trading network. Existence of the ε-equilibrium point was proved for both models.

In the paper proposed here the numerical solving for the differential game corresponding to the model with the distributors activities is considered.

The model

The economic region in which n enterprises (agents) are operating for finite period of time is considered. These enterprises produce the goods of the same type. The enterprises supply the goods to trading network independently from each other. The trading network sells and distributes the goods in the economic region. It is assumed that every enterprise has got some distributors which get the goods directly from the enterprise. The distributors supply the goods to trading network as well. The overhead charges in trading network are assumed to be equal to zero.

As for the operation of the trading network the following is assumed:

– The goods are passed on only from the network point with more goods to the point with less goods. Besides, the more the difference in goods amount, the more the rate of goods transition.

– The goods transition is executed by two means: 1) between neighbouring points of the network, 2) between enterprise and its distributors.

– Selling of the goods is proportional to their amount;

– The goods are not distributed outside of the economic region.

The aim of each enterprise is to maximize the total income from the sale of its products for finite period of time.

Differential game

The above mentioned problem is formalized by n-person differential game $\Gamma(c_0, T)$ with a prescribed duration $T < \infty$. Let $I = \{i\} = \{1, \dots, n\}$ be a set of the enterprises (agents).
Let us denote by $u_i(t)$ the goods production intensity of the enterprise $i \in I$ at the moment t. Let us assume that the goods production intensity satisfies the following conditions:

$$0 \le u_i(t) \le G_i(t), \quad i \in I, \tag{1}$$

at any moment $t \in [0, T]$. Here $G_i(t) > 0$ is a given square integrable function which describes the maximal goods production intensity of the enterprise i at the moment t. Let us assume that the costs of manufacture on goods unit of the enterprise i are constant and equal to $M_i > 0$, $i \in I$.
Let us denote by $c^i(x, y, t)$ the goods quantity of the agent i at the point $(x, y) \in \mathbf{R}^2$ at the moment t.
Let us consider the economic region as a bounded two-dimensional domain $\Omega \in \mathbf{R}^2$ with piecewise smooth boundary S, $\overline{\Omega} = \Omega \cup S$.

Differential game

The dynamics of the agent $i \in I$ in the game $\Gamma(c_0, T)$ is described by the initial boundary value problem for the following differential equation:

$$\frac{\partial c^i}{\partial t} = \frac{\partial}{\partial x}\left(D(x,y,t)\frac{\partial c^i}{\partial x}\right) + \frac{\partial}{\partial y}\left(D(x,y,t)\frac{\partial c^i}{\partial y}\right) - qc^i + u_i\psi_i(x,y) + $$
$$+ \sum_{i_l=1}^{d_i} \eta_{i_l}[c^i(x_i,y_i) - c^i(x,y)]\psi_{i_l}(x,y), \quad (x,y) \in \Omega,\ t > 0. \qquad (2)$$

Here $D(x,y,t) > 0$ is the diffusion coefficient characterizing the interaction between neighbouring points of the network; $q > 0$ is the coefficient characterizing consumer demand; u_i is a control parameter of the agent i; d_i is the number of the distributors of the enterprise i; $\eta_{i_l} \geq 0$ is the coefficient characterizing the interaction between the agent i and the distributor i_l, where $i_l = \overline{1, d_i}$. The function $\psi_i(x,y) = \delta(x - x_i, y - y_i)$ gives the location of the agent i inside the economic region. The function $\psi_{i_l}(x,y) = \delta(x - x_{i_l}, y - y_{i_l})$ gives the location of the distributor i_l.

Differential game

Let the function $c^i(x,y,t)$ satisfies the following boundary condition:

$$D(x,y,t)\frac{\partial c^i}{\partial m} = 0, \qquad (3)$$
$$(x,y) \in S,\ t \in [0,T],$$

where m is a outward normal to the boundary S. The condition (3) corresponds to the missing of the goods flow through the boundary of the economic region. Let the function $c^i(x,y,t)$ satisfies the following initial condition:

$$c^i(x,y,0) = c_0^i(x,y), \qquad (4)$$
$$(x,y) \in \Omega,\ t = 0.$$

where $c_0^i(x,y)$ is some given function describing the initial distribution of the goods of the agent i in the region at the initial moment $t = 0$. A measurable function $u_i = u_i(t)$, satisfying the condition (1) for all $t \in [0,T]$ is called the admissible control of the agent $i \in I$. Let us denote by $\mathcal{U}_i$ the set of admissible controls (measurable functions) $u_i(t),\ t \in [0,T]$.

Differential game

Furthermore, let us assume that all coefficients of the equations (2)–(4) satisfy the conditions guaranteeing existence of the unique solution of the problem (2)–(4) in the space $W_2^{1,0}(\Omega \times (0,T))$ for any admissible control $u_i \in \mathcal{U}_i,\ i \in I$ and any initial condition $c_0^i \in W_2^1(\Omega),\ i \in I$. Here $W_2^{1,0}(\Omega \times (0,T))$ is the Sobolev space consisting of functions $c^i(x,y,t) \in L_2(\Omega \times (0,T))$, where $L_2(\Omega \times (0,T))$ is the space of functions which have got square integrable generalized derivatives of first order ([Ladyzhenskaya, 1973]).

Differential game

Let us assume that the Cournot oligopoly takes place in every point $(x, y) \in \overline{\Omega}$ at the every moment t where n agents producing the goods of the same type participate. Let price of the goods in every point $(x, y) \in \overline{\Omega}$ at the moment t is defined as follows:

$$P(x, y, t, c) = R \cdot (A - \sum_{i=1}^{n} c^i(x, y, t))/A, \qquad (5)$$

where $c = (c^1, c^2, \dots, c^n)$; $R =$ const, $A =$ const. Then the payoff of the agent i from the sale of its products at time T is defined by the following functional:

$$H_i(c, u_i) = \int_0^T \int_{\overline{\Omega}} qc^i(x, y, \tau)P(x, y, \tau, c)d\Omega d\tau - \int_0^T M_i u_i(\tau)d\tau. \qquad (6)$$

Let us denote by $F_i(c_0^i,\, t_0,\, t)$ the set of the points $c^i(x,y,t) \in W_2^1(\Omega)$ for which there exists an admissible control $u_i(t)$ such that the game goes from the state $c_0^i(x,y)$ to the state $c^i(x,y,t)$ for the time interval $[t_0,t]$. The set $F_i(c_0^i,\, t_0,\, t)$ is called attainability set of the agent i, $i = \overline{1,n}$ from initial state c_0^i on the time interval $[t_0,t]$.
Let us denote by $\widehat{F}_i(c_0^i,\, t_0,\, t),\ i \in I$ the set of trajectories $\widehat{c}^i(x,y,\cdot)$ of (2)–(4) which start at $c_0^i(x,y)$ at the moment t_0 and which are defined on the time interval $[t_0,t]$. The trajectories set $\widehat{F}_i(c_0^i,\, t_0,\, t)$ considering as a subset of the Banach space of square integrable functions from $[t_0,t]$ into $W_2^1(\Omega)$ is compact in uniform metric:

$$\widehat{\rho}_t(\widehat{c}(\cdot),\widehat{c}'(\cdot)) = \sqrt{\int\limits_{t_0}^{t} \left(\|\widehat{c}(\cdot,\tau) - \widehat{c}'(\cdot,\tau)\|_{2,\Omega}^{(1)} \right)^2 d\tau}.$$

Differential game

At the every moment $t \in [0, T]$ of the game $\Gamma(c_0, T)$ the agents know the state, the dynamics and the duration T of the game.
Let $\widehat{c}^i(x, y, \cdot) \in \widehat{F}_i(c_0^i, 0, T)$ is the trajectory of (2)–(4) arising from a control u_i and $\Pi_\delta^i(\widehat{c}^i)$ is the trajectory arising from the same control u_i delayed by δT.
The following lemma describing relation between these trajectories is valid.
Lemma 1. For each $\delta \in (0, 1]$ there exists a mapping $\Pi_\delta^i : \widehat{F}_i(c_0^i, 0, T) \to \widehat{F}_i(\cdot)$ such that if $\widehat{c}^i(x, y, \tau) = \widehat{c}'^i(x, y, \tau)$ for $\tau \in [0, t]$ then $\Pi_\delta^i(\widehat{c}^i)(\tau) = \Pi_\delta^i(\widehat{c}'^i)(\tau)$ for $\tau \in [0, t + \delta T]$. Moreover,

$$\varepsilon^i(\delta) = \sup_{\widehat{c}^i \in \widehat{F}_i(\cdot)} \|\widehat{c}^i - \Pi_\delta^i(\widehat{c}^i)\| \xrightarrow[\delta \to 0]{} 0.$$

Differential game

Let us fix the permutation $p = (i_1, \ldots, i_k, \ldots, i_n)$ and consider n-person multistep game $\Gamma_p^\delta(c_0, T)$ on the every step which the agents $i_1, \ldots, i_n$ choose in sequence controls $u^{i_1}, \ldots, u^{i_k}, \ldots, u^{i_n}$.
Definition 1. *The strategy*

$$^\delta\varphi_{i_k}^p : \widehat{F}_{i_k}^*(\cdot) = \prod_{j \neq i_k} \widehat{F}_j(\cdot) \to \widehat{F}_{i_k}(\cdot),$$

of the agent i_k in the game $\Gamma_p^\delta(c_0, T)$ is a mapping such that if $\widehat{c}^j(x, y, \tau) = \widehat{c}'^j(x, y, \tau)$ for $j < i_k$, $\tau \in [0, l\delta T]$ and if $\widehat{c}^j(x, y, \tau) = \widehat{c}'^j(x, y, \tau)$ for $j > i_k$, $\tau \in [0, (l-1)\delta T]$, then $^\delta\varphi_{i_k}^p(\widehat{c}^{*i_k}(x, y, \tau)) = {}^\delta\varphi_{i_k}^p(\widehat{c}^{*'i_k}(x, y, \tau))$, $\tau \in [0, l\delta T]$. Here $\delta = 1/2^N,\ l = 1, 2, \ldots, 2^N$.

Differential game

Let us denote by ${}^{\delta}\Phi^{p}_{i_k}$ the set of the strategies of the agent i_k in the game $\Gamma^{\delta}_{p}(c_0, T)$.
In the game $\Gamma^{\delta}_{p}(c_0, T)$ the agents $i_1, ..., i_n$ choose in sequence the strategies ${}^{\delta}\varphi^{p}_{i_1}, \ldots, {}^{\delta}\varphi^{p}_{i_n}$. The trajectory $\chi({}^{\delta}\varphi^{p})$ is uniquely defined for every n-tuple ${}^{\delta}\varphi^{p} = ({}^{\delta}\varphi^{p}_{i_1}, \ldots, {}^{\delta}\varphi^{p}_{i_n})$ stepwise on successive intervals $[0, \delta], \ldots, [T - \delta T, T]$. The payoff function of the agent $i \in I$ in the game $\Gamma^{\delta}_{p}(c_0, T)$ is defined as follows:

$$H^{\delta}_{i}(c_0, {}^{\delta}\varphi^{p}) = H_i(\chi^{\delta}({}^{\delta}\varphi^{p})), \tag{7}$$

here $H_i(\cdot)$ is the functional (6).

Differential game

So, n-person differential game with prescribed duration T is defined in a normal form :

$$\Gamma_p^\delta(c_0, T) = \langle I,\, \{{}^\delta\Phi_i^p\}_1^n,\, \{H_i^\delta\}_1^n\rangle.$$

In the game $\Gamma_p^\delta(c_0, T)$ there exists equilibrium point in virtue of the results of the work [Malafeyev, 1993].
The previous lemma implies the following lemma.
Lemma 2. If $i_k > i_1$, ${}^\delta\varphi_{i_k}^p \in {}^\delta\Phi_{i_k}^p$, then $\Pi_\delta^{i_k} \cdot {}^\delta\varphi_{i_k}^p \in {}^\delta\Phi_{i_k}^{p_{i_k}}$, where $p_{i_k} = (i_k, \widetilde{p})$, $\widetilde{p}$ is a permutation of the set $I \setminus i_k$; moreover, for $\widehat{c}^{*i_k} \in \widehat{F}_{i_k}^*(\cdot)$

$$\|{}^\delta\varphi_{i_k}^p(\widehat{c}^{*i_k}) - (\Pi_\delta^{i_k} \cdot {}^\delta\varphi_{i_k}^p)(\widehat{c}^{*i_k})\| \le \varepsilon(\delta) \underset{\delta\to 0}{\longrightarrow} 0.$$

Differential game

The following lemma is valid [Malafeyev, 1993].
Lemma 3. Let the game $\Gamma_{H'} = \langle I, \{X'_i\}_1^n, \{H'_i\}_1^n \rangle$ is obtained from the game $\Gamma_H = \langle I, \{X_i\}_1^n, \{H_i\}_1^n \rangle$ by the epimorphic mapping $\alpha_i: \ X_i \to X'_i$, $i = 1, \ldots, n$, with

$$\|H(x) - H'(\alpha x)\| \leq \varepsilon, \qquad \alpha x = (\alpha_1(x_1), \ldots, \alpha_n(x_n)).$$

Then, if x is a equilibrium point of the game Γ_H then αx is the 2ε-equilibrium point of the game $\Gamma_{H'}$.

Differential game

Let us define the main game $\Gamma(c_0, T)$.
Definition 2. The pair $(\delta_i, \{{}^{\delta}\varphi_i^{p_i}\}_{\delta=1/2^N})$ is called *a strategy* of the agent i. Here $N \in Z$, δ_i is a range of dyadic partition of the time interval $[0, T]$ and ${}^{\delta}\varphi_i^{p_i}$ is the strategy of the agent i in the game $\Gamma_{p_i}^{\delta}(c_0, T)$ for the permutation $p_i = (i, \widetilde{p})$, $\widetilde{p}$ is the permutation of the set $I \setminus i$.

For n-tuple $\varphi = (\varphi_1, \ldots, \varphi_n)$ the game $\Gamma(c_0, T)$ is played as follows. The smallest $\delta_i = \delta$ is chosen and the trajectory $\chi(\cdot)$ is constructed for n-tuple ${}^{\delta}\varphi = ({}^{\delta}\varphi_1^{p_1}, \ldots, {}^{\delta}\varphi_n^{p_n})$. This trajectory is unique.
The game $\Gamma(c_0, T)$ is obtained from the game $\Gamma_p^{\delta}(c_0, T)$ by the epimorphic mapping which is defined in the lemma 2. Since in the game $\Gamma_p^{\delta}(c_0, T)$ there exist equilibrium points then existence of the ε-equilibrium point in the game $\Gamma(c_0, T)$ follows from the lemma 2 and the lemma 3.

Thus, the following theorem is valid [Troeva, 2002].
Theorem 1. There exist ε-equilibrium points in the n-person differential game $\Gamma(c_0, T)$ for all $\varepsilon > 0$.

Numerical Example

Let us consider the differential two-person game $\Gamma(c_0, T)$. The dynamics of the agent $i = 1, 2$ in the game $\Gamma(c_0, T)$ is described by the initial boundary value problem for the following differential equation on the domain $\overline{\Omega} = [0, l]$:

$$\begin{aligned} \frac{\partial c^i}{\partial t} &= D\frac{\partial^2 c^i}{\partial x^2} - qc^i + u_i\psi_i(x) + \\ &+ \eta_{i_l}[c^i(x_i) - c^i(x)]\psi_{i_l}(x), \quad x \in \Omega, \ t > 0. \end{aligned} \tag{8}$$

Let the function $c^i(x, t)$ satisfies the following boundary conditions:

$$D\frac{\partial c^i}{\partial x} = 0, \quad x = 0, \ t \in [0, T], \tag{9}$$

$$D\frac{\partial c^i}{\partial x} = 0, \quad x = l, \ t \in [0, T], \tag{10}$$

Let the function $c^i(x, t)$ satisfies the following initial condition:

$$c^i(x, 0) = c^i_0(x), \quad x \in \overline{\Omega}, \ t = 0. \tag{11}$$

Numerical Example

A measurable function $u_i = u_i(t)$, satisfying the condition $u_i = u_i(t) \in \overline{U_i} = [U_i^1, U_i^2],\ i = 1, 2$ for all $t \in [0, T]$ is called the admissible control of the agent $i \in I$, $U_i^1 =$ const, $U_i^2 =$ const.
The numerical method based on the dynamic programming method [Bellman (1960)] and the finite difference method [Samarsky (1989)] is proposed for the numerical solving of the auxuliary multistep game $\Gamma_p^\delta(c_0, T)$.

Numerical Example

On the domain $\overline{\Omega} = [0, l]$ we construct the uniform net with steps h on x

$$\overline{\omega}_h = \quad \{x_k = kh,\ k = 0, \ldots, N_1;\ x_0 = 0,\ x_{N_1} = l\}\,. \qquad (12)$$

Here $x_i = \bar{x}_i$ is a location of the agent $i,\ i = 1, 2$, $x_{i_l} = \bar{x}_{i_l}$ is a location of the distributor $i_l,\ i_l = \overline{1, d_i}$.
On the every interval $[t_s, t_{s+1}], s = \overline{0, N_\sigma - 1}$ we construct the uniform net with step τ

$$\overline{\omega}_{\tau,s} = \{\bar{t}_j = j\tau,\ j = \overline{0, N_2};\ \bar{t}_0 = t_s,\ \bar{t}_{N_2} = t_{s+1}\}.$$

Here $t_s \in \sigma$, where σ is the time interval partition

$$\sigma = \{t_0 = 0 < t_1 < \ldots < t_{N_\sigma} = T\}.$$

On the admissible controls set $\overline{U_i} = [U_i^1, U_i^2],\ i = 1, 2$ we construct the following partition:

$$\Delta_i = \{u_{i,0} = U_i^1 < u_{i,1} < \ldots < u_{i,N_3} = U_i^2\},\ i = 1, 2.$$

Numerical Example

Let us denote by ${}^{i}y_k^{j,s}$ the function defined on the net $\overline{\omega}_{h\tau} = \overline{\omega}_h \times \overline{\omega}_{\tau,s}$. We construct for the problem (8)-(11) following purely implicit difference schems [Samarsky (1989)] on the net $\overline{\omega}_{h\tau} = \overline{\omega}_h \times \overline{\omega}_{\tau,s}$ for any pair of admissible controls $(u_{1,z_1}, u_{2,z_2}) \in \Delta_1 \times \Delta_2$, $z_i \in \overline{0, N_3}$:

$$\frac{{}^{i}y_0^{j+1,s} - {}^{i}y_0^{j,s}}{\tau} = 2D\,\frac{{}^{i}y_1^{j+1,s} - {}^{i}y_0^{j+1,s}}{h^2} - {}^{i}y_0^{j,s}q, \quad k = 0, \tag{13}$$

$$\begin{aligned} \frac{{}^{i}y_k^{j+1,s} - {}^{i}y_k^{j,s}}{\tau} &= D\,\frac{{}^{i}y_{k+1}^{j+1,s} - 2\,{}^{i}y_k^{j+1,s} + {}^{i}y_{k-1}^{j+1,s}}{h^2} - {}^{i}y_k^{j,s}q + \frac{1}{h}u_{i,z_i}^{s}\delta_{k,i} \\ &+ \frac{1}{h}\sum_{i_l=1}^{d_i}\eta_{i_l}({}^{i}y_i^{j,s} - {}^{i}y_k^{j,s})\delta_{k,i_l}, \ \ k = \overline{1, N_1 - 1}, \end{aligned} \tag{14}$$

$$\frac{{}^{i}y_{N_1}^{j+1,s} - {}^{i}y_{N_1}^{j,s}}{\tau} = -2D\,\frac{{}^{i}y_{N_1}^{j+1,s} - {}^{i}y_{N_1-1}^{j+1,s}}{h^2} - {}^{i}y_{N_1}^{j,s}q, \ \ k = N_1, \tag{15}$$

$$j = \overline{0, N_2 - 1},$$

Numerical Example

$$^{i}y_k^0 =^{i} y_k^{N_2,s},\ k = \overline{0, N_1},\ j = 0, \tag{16}$$

$$z_1 = \overline{0, N_3},$$

$$z_2 = \overline{0, N_3},$$

$$s = \overline{0, N_\sigma - 1},$$

$$^{i}y_k^{0,0} = c_0(x_k),\ k = \overline{0, N_1},\ s = 0,\ j = 0. \tag{17}$$

Here $\delta_{k,i}$ is the Kronecker symbol.
The constructed absolutely stable difference scheme (13)-(17) is solved by the sweep method [Samarsky (1989)].

Numerical Example

Let price of the goods in every point $(x, y) \in \overline{\Omega}$ at the moment t is defined as follows:

$$P(x, y, t, c) = R \cdot (A - c^1(x, t) - c^2(x, t))/A, \tag{18}$$

The payoff function of the agent i, $i = 1, 2$ in the game $\Gamma_p^\delta(c_0, T)$ is approximated as follows:

$$\begin{aligned} \underline{H}_i(u_1^0, \ldots, u_1^{N-1}, u_2^0, \ldots, u_2^{N-1}) &= \tau \cdot h \sum_{s=0}^{N_\sigma - 1} \sum_{j=0}^{N_2 - 1} \sum_{k=0}^{N_1 - 1} {}^i y_k^{j,s} q P \\ &- \tau \sum_{s=0}^{N_\sigma - 1} \sum_{j=0}^{N_2 - 1} M_i u_i^s, \end{aligned} \tag{19}$$

Numerical Example

Let $\underline{V}_i^\delta(\cdot)$ be value of the payoff function of the agent $i, \ i = 1, 2$ at equilibrium point

$$\underline{V}_i^\delta({}^1y^0, {}^2y^0, t_0) = \\ = \max_{u_1^0,\ldots,u_1^{N-1}} \max_{u_2^0,\ldots,u_2^{N-1}} \{H_i(u_1^0, \ldots, u_1^{N-1}, u_2^0, \ldots, u_2^{N-1})\}, \tag{20}$$

The following recurrence equations are valid:

$$\underline{V}_i^\delta({}^1y^{N_{\sigma-1}}, {}^2y^{N_{\sigma-1}}, t_{N_\sigma-1}) = \max_{u_{1,z}^{N_\sigma-1}\in\Delta_1} \max_{u_{2,z}^{N_\sigma-1}\in\Delta_2} \{H_i(u_{1,z}^{N_\sigma-1}, u_{2,z}^{N_\sigma-1})\}, \tag{21}$$

$$\underline{V}_i^\delta({}^1y^s, {}^2y^s, t_s) = \max_{u_{1,z}^s} \max_{u_{2,z}^s} \{H_i(u_{1,z}^s, u_{2,z}^s) \quad + \quad \underline{V}_i^\delta({}^1y_z^{s+1}, {}^2y_z^{s+1}, t_{s+1})\}, \\ s \quad = \quad \overline{N_\sigma - 2, 0} \tag{22}$$

Numerical Example

The numerical experiment were realized for the following input data: $D = 0.5$, $l = 20$, $h = 1$, $T = 52$, $\tau = 1$, $N_\sigma = 4$, $q = 0.02$, $\eta = 0.05$, $M_1 = 4$, $M_2 = 3$, $r = 10$, $A = 500$, $U_1^1 = 0$, $U_1^2 = 30$, $U_2^1 = 0$, $U_2^2 = 30$, $\bar{x}_1 = 4$, $\bar{x}_{1_1} = 12$, $\bar{x}_2 = 16$, $\bar{x}_{2_1} = 8$; the initial distribution of the goods $c_o(x) = 0$.

The results of numerical experiments are presented on the Figures 1-6:

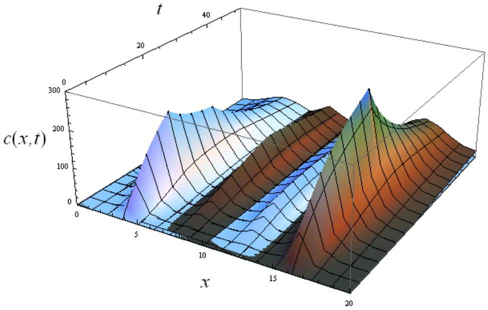

Figure 1: Distributions of the goods of agents 1 and 2 in trading network depending on time

Numerical Example

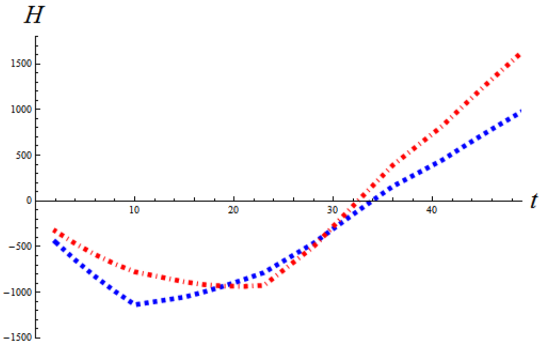

Figure 2: Values of the payoff functions of agents 1 (dot-dashed line) and 2 (dashed line) at equilibrium point depending on time

The payoff functions of the agents takes negative values at the initial moments. This is the result of costs for the goods production whereas selling of goods occurs with delay in the region.

Numerical Example

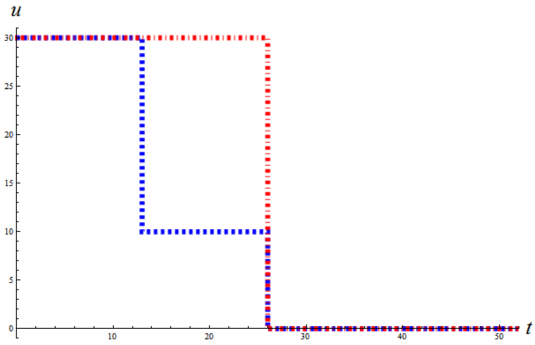

Figure 3: Agents' control functions at equilibrium point

The agent always is going to choose zero control function at the last step, it means the discontinuation of production. It follows on fixed duration of the game and necessity to maximize the payoff function. If the production will be stop on the last step then agent does not have expenditure and gets income from sales of product which has already distributed in the region.

Numerical Example

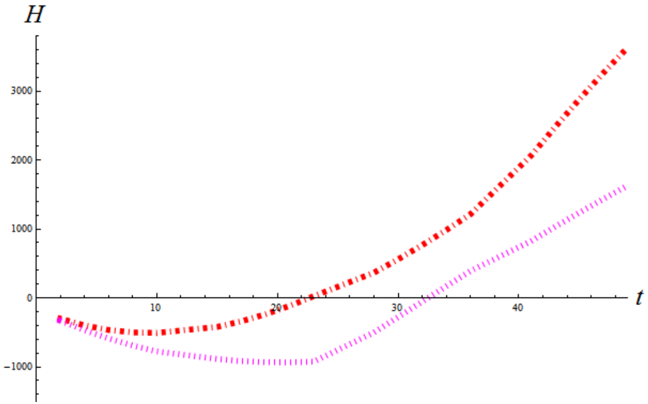

Figure 4: Values of the payoff function of the agent 1 for different values of consumer demand in the region: q=0.02 (dotted line) and q=0.04 (dot-dashed line)

Numerical Example

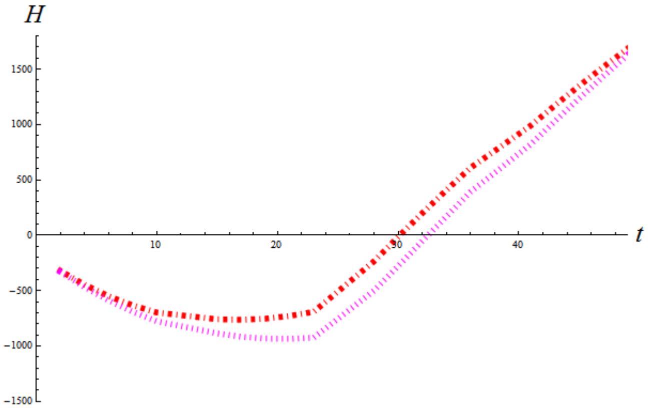

Figure 5: Values of the payoff function of the agent 1 for different values of diffusion coefficient: D=0.05 (dotted line) and D=0.5 (dot-dashed line)

Numerical Example

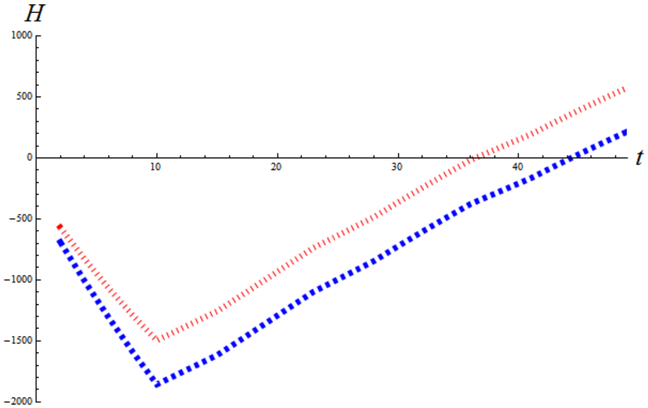

Figure 6: Value of the payoff function of the agent 1 for different values of the product cost: $M_1 = 5$ (dotted line) and $M_1 = 6$ (dashed line)

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