Weak convergence for a sequence of coalescent processes

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For each $n \in \mathbb{N}$,

- $\{\mathbf{H}^{(n)}(k)\}_{k\in\mathbb{N}} \subset \mathbb{N}^d$: the block counting process of the Kingman coalescent, with parent independent mutations, evolving backwards in time (d types, mutation rate θ, mutation probabilities $(Q_j)_{j=1}^d$)
- $\mathbf{X}^{(n)}(k) = \frac{1}{n}\mathbf{H}^{(n)}(k \wedge \tau^{(n)}) \quad k \in \mathbb{N}$
- $\{\mathbf{M}^{(n)}(k)\}_{k\in\mathbb{N}} = \{(M_{ij}^{(n)}(k))_{i,j=1}^d\}_{k\in\mathbb{N}}$ with
  $\mathbf{M}_{ij}^{(n)}(k)$: number of mutations from type i to type j occurred in $\mathbf{H}^{(n)}$ up to time k

Theorem (weak convergence): if $\mathbf{X}^{(n)}(0) \sim \mu_0^{(n)} \Rightarrow \mu_0$,

$$\{(\mathbf{X}^{(n)}(\lfloor nt\rfloor), \mathbf{M}^{(n)}(\lfloor nt\rfloor))\} \Rightarrow_{n\to\infty} \{(\mathbf{X}(t), \mathbf{M}(t))\}_{t\geq 0}$$

- $\mathbf{X}$ is a deterministic process

And given $\mathbf{X}$, for each $i, j = 1, \ldots, d$,

- $\mathbf{M}_{ij}$ is an independent Possion processes with varying intensity
  $\lambda_{ij}(t) = \dfrac{\theta Q_j X_i(t)}{\|\mathbf{X}(t)\|_1^2}$.