

# Quantitative results on a multi-allelic Moran type model with mutation

Workshop on Models and Inference in Population Genetics

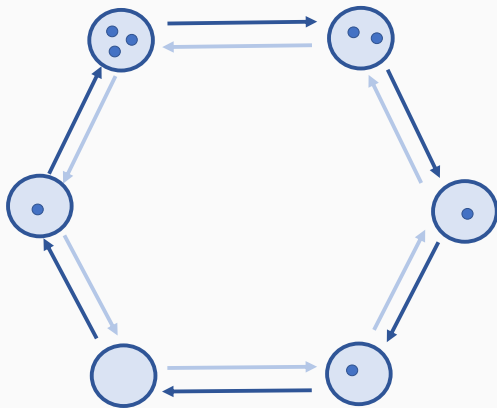
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Josué M. Corujo

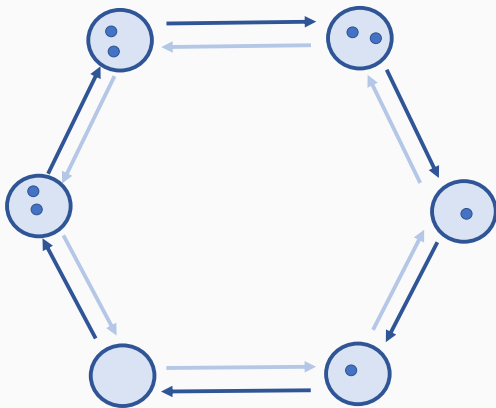
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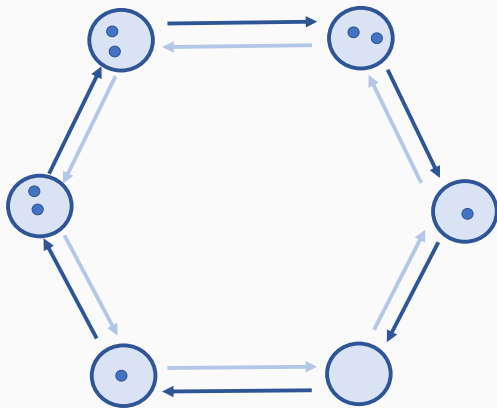
# Neutral multi allelic Moran type model on the cycle graph



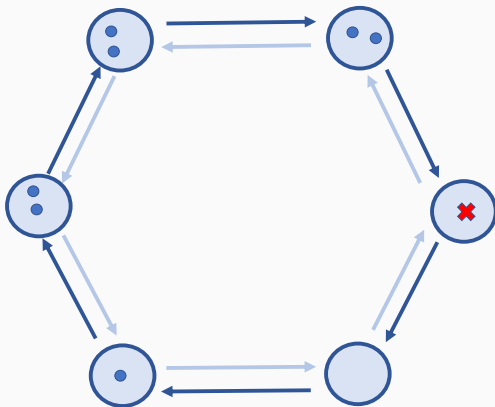
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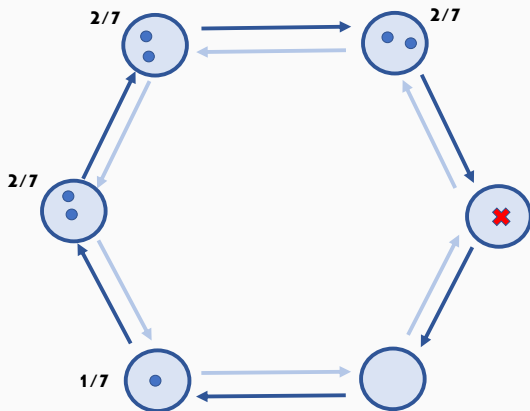
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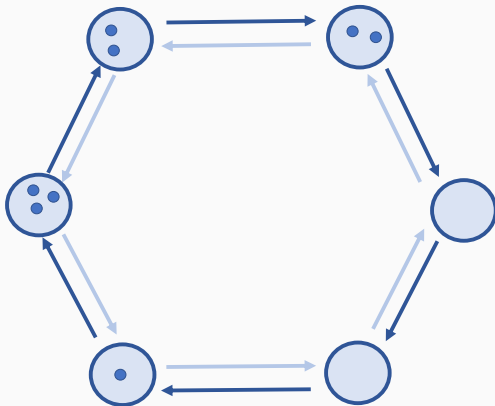
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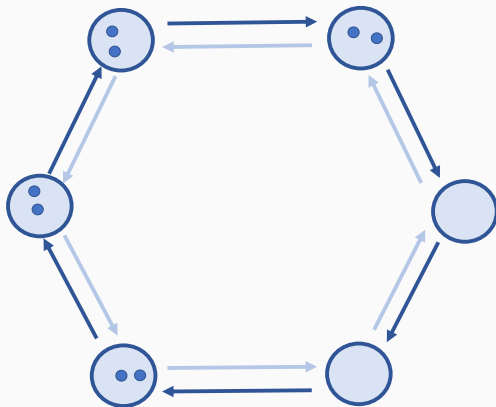
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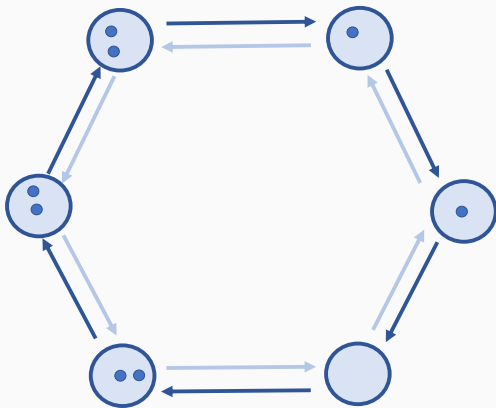


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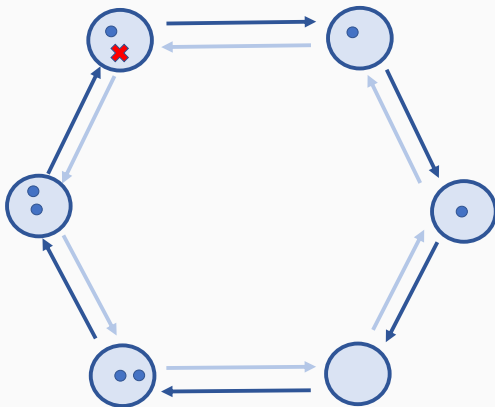




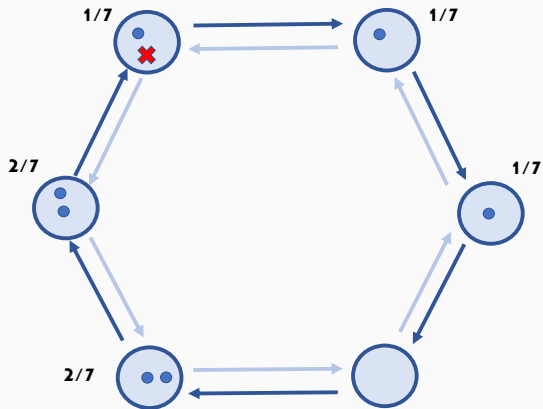
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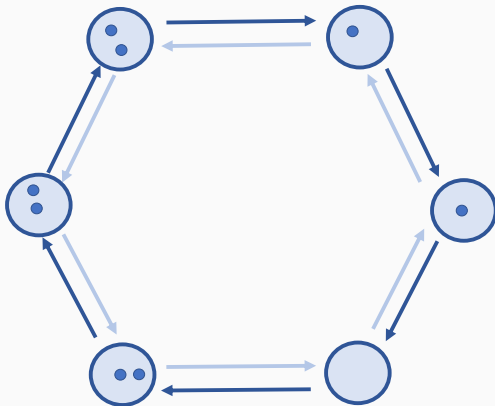
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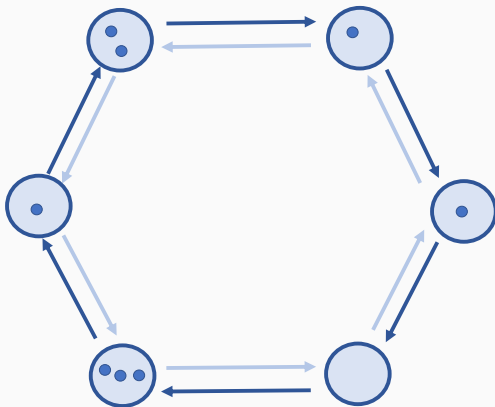
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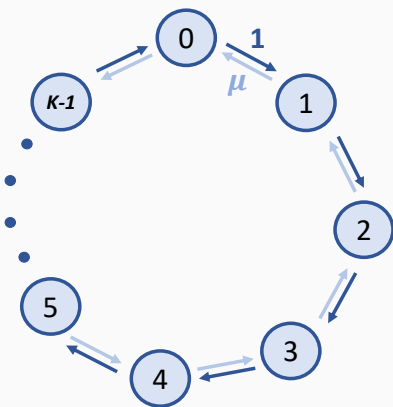
## Neutral multi allelic Moran type model on the cycle graph



## Mutations: asymmetric random walk on the cycle graph

Let us denote by  $(X_t)_{t \geq 0}$  the asymmetric random walk on the cycle graph, i.e. with infinitesimal generator

$$\mathcal{H}f(x) = f(x+1) - f(x) + \mu[f(x-1) - f(x)], \quad \text{for all } x \in \mathbb{Z}/K\mathbb{Z}.$$



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## Moran type process of $N$ individuals on the $K$ cycle graph

- $N$  individuals “moving” independently on  $\mathbb{Z}/K\mathbb{Z}$  as copies of  $(X_t)_{t \geq 0}$ .
- When one of them dies (at rate  $p$ ), another, chosen uniformly among the other  $N - 1$  individuals, reproduces.
- $\eta_t(k)$  is the number of individuals in site  $k$  at time  $t$ , for every  $k \in \mathbb{Z}/K\mathbb{Z}$ .

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### Problem

Study the **speed of convergence** of  $\eta_t^{(N)}(k)/N$  when  $t$  and  $N$  tend to infinity, for every  $k \in \mathbb{Z}/K\mathbb{Z}$ .



# Speed of convergence

The (random) empirical distribution  $m(\eta_t^{(N)})$  converges in distribution to  $\nu_{st}$  when  $t$  and  $N$  goes to infinity. We are interested in studying the speed of the following convergences:

$$\begin{array}{ccc} m(\eta_t^{(N)}) & \xrightarrow[t \rightarrow \infty]{} & \nu_N \\ N \downarrow & & \downarrow N \\ \mathcal{L}(X_t) & \xrightarrow[t \rightarrow \infty]{} & \nu_{st} \end{array}$$

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