BAYESIAN INFERENCE IN GENETICS

MODEL

Wright-Fisher diffusion with selection and mutation

$$dX_t = \frac{1}{2}\left(sX_t(1-X_t) - \theta_2 X_t + \theta_1(1-X_t)\right)dt + \sqrt{X_t(1-X_t)}dW_t$$

DEFINITIONS

Bayesian estimator for selection

$$\tilde{s}_T = \arg\min_{\bar{s}_T} \int_{\mathcal{S}} \mathbb{E}^{(s)}\left[\ell\left(\sqrt{T}\left(\bar{s}_T - s\right)\right)\right] p(s)ds$$

Likelihood ratio function

$$Z_{T,s}(u) := \frac{d\mathbb{P}^{(s+\frac{u}{\sqrt{T}})}}{d\mathbb{P}^{(s)}}(X^T)$$

IBRAGIMOV-HAS'MINSKII CONDITIONS

1 $$\sup_{s\in\mathcal{K}} \mathbb{E}^{(s)}\left[\left|Z_{T,s}(u_2)^{\frac{1}{2}} - Z_{T,s}(u_1)^{\frac{1}{2}}\right|^2\right] \leq B(1+R^a)|u_2-u_1|^q$$

2 $$\sup_{s\in\mathcal{K}} \mathbb{E}^{(s)}\left[Z_{T,s}(u)^{\frac{1}{2}}\right] \leq e^{-g_T(|u|)}$$

3 Marginal convergence of $Z_{T,s}(u)$ uniform in the selection parameter to $Z_s(u) \in C_0(\mathbb{R})$

MAIN RESULT

The Bayesian estimator for selection is :

- **uniformly consistent over compact sets**
- **uniformly asymptotically normal over compacts sets**
- **displays pth moment convergence uniformly on compact sets for any p>0**
- **asymptotically efficient for suitable choice of loss function**

NEAT SIDE RESULT

Uniform in selection parameter ergodicity for WF

$$\lim_{T\to\infty}\sup_{s\in\mathcal{K}} \mathbb{P}^{(s)}\left[\left|\frac{1}{T}\int_0^T h(X_t)dt - \mathbb{E}^{(s)}\left[h\left(\xi\right)\right]\right| > \varepsilon\right] = 0$$