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Pricing of contingent convertibles under smile conform models

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We look at the problem of pricing contingent convertible bonds (CoCos) where the underlying risky asset dynamics are given by a smile conform model, more precisely, an exponential Lévy process incorporating jumps and heavy tails. A core mathematical quantity, needed in closed form in order to produce an exact analytical expression for the price of a CoCo, is the law of the infimum of the underlying equity price process at a fixed time. With the exception of Brownian motion with drift, no such closed analytical form is available within the class of Lévy processes that are suitable for financial modeling. Very recently, however,

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there has been some remarkable progress made with the theory of a large family of Lévy processes, known as β-processes. Indeed, for this class of Lévy processes, the law of the infimum at an independent and exponentially distributed random time can be written down in terms of the roots and poles of its characteristic exponent, all of which are easily found within regularly spaced intervals along one of the axes of the complex plane. Combining these results together with a recently suggested Monte Carlo technique, which capitalizes on the randomized law of the infimum, we show the efficient and effective numerical pricing of CoCos. We perform our analysis using a special class of β-processes, known as β-VG, which have similar characteristics to the classic variance-Gamma model. The theory is put to work by performing two case studies. After calibrating our model to market data, we price and analyze one of the Lloyds CoCos as well as the first Rabo CoCo.

1 INTRODUCTION

A contingent convertible bond (CoCo) is a bond issued by a financial institution where, upon the appearance of a trigger event, an automatic conversion into a predetermined number of shares takes place or where a partial write-down of the face value of the debt is imposed. In this paper, we will focus on CoCos where a conversion in new shares takes place, but the techniques can be easily applied to write-downs. The moment of conversion or write-down corresponds to a particular moment where the issuing bank gets into a state of possible nonviability. This is a situation where the future of the bank is questioned by the depositors, bondholders and regulators. In order to quantify such a life-threatening situation, the conversion or write-down is triggered by a particular predefined event. This automatic conversion makes this asset class attractive from a regulatory point of view. The conversion of debt into equity takes place automatically. No shareholder meeting is required and the bank does not need to issue new shares in difficult market conditions. A general and quantitative introduction to contingent capital and the structuring, pricing and dynamics of CoCos can be found in De Spiegeleer and Schoutens (2011a).

In a financial context, CoCos were first launched just before year-end 2009 by Lloyds Banking Group. Later, in 2010, Rabobank successfully issued their so-called senior contingent notes. In February 2011, we witnessed the surprise US$2 billion CoCo issue by Credit Suisse. This placement was very well-received by the market and it was many times oversubscribed. Yield-hungry investors showed a very large appetite for these instruments. In the spring of 2011, Bank of Cyprus issued their convertible enhanced capital securities, which combined convertible bond features (for an introduction to convertible bonds, see De Spiegeleer and Schoutens (2011b)) with a CoCo and are therefore often called convertible contingent convertibles (CoCoCos).

TABLE 1 List of issued CoCos.

Issuer	Issue date	Issue size	Accounting trigger	Trigger level (%)	Maturity	Callable
Lloyds	Nov 2009	13 700	CET1	5.00	10–22	
Rabo	Mar 2010	1250	Equity capital ratio	7.00	10	
Rabo	Jan 2011	2000	Equity capital ratio	8.00	Perpetual	After 5.5
Credit Suisse	Feb 2011	2000	CET1	7.00	30	After 5.5
Credit Suisse	Mar 2012	750	CET1	7.00	10	After 5
Bank of Cyprus	May 2011	890	CET1	5.00	Perpetual	After 5
ANZ	Sep 2011	1250	CET1	5.13	Perpetual	After 6
UBS	Feb 2012	2000	CET1	5.00	5	After 5
ZKB	Feb 2012	590	CET1	7.00	Perpetual	After 5.5
Macquarie	Mar 2012	250	CET1	5.125	45	After 5

"Issue size" measured in millions of USD; "Maturity" and "Callable" measured in years.

In 2012, the CoCo issuance continued with issues from ZKB, UBS and Credit Suisse. A summary of the different issues up until time of writing can be found in Table 1. CoCos have therefore not missed their entry into the market.

Two cornerstones of the CoCo-note construction are the specification of the trigger event and the conversion price. The latter is the price that a CoCo investor is implicitly paying at conversion. This price is likely to be much higher than the stock price at conversion time, and hence, in that case, the CoCo holder suffers a (significant) loss. Note that the conversion price also determines the number of new shares issued at conversion and therefore determines the amount of dilution of existing shareholders.

The CoCos issued so far have been set up with an accounting trigger. In this case, a comparison between the regulatory capital of the bank and the risk-weighted value of its assets can trigger a conversion of contingent debt into equity. The CoCos issued by Credit Suisse and Lloyds will convert into shares if their respective core Tier 1 (CT1) ratios fall below 7% and 5%, respectively. The Credit Suisse contingent convertibles come with a supplementary regulatory trigger, which is also called a nonviability trigger. Through this trigger, the Swiss regulator has the power to impose a conversion into shares if it deems the Credit Suisse Group to be nonviable without public financial support. The regulator could in theory trigger a conversion into shares even when the accounting ratio was above the trigger level. In the UK, the regulator itself is steering away from using the accounting-based triggers. Andrew Haldane, Executive Director of Financial Stability for the Bank of England, wants regulators to start looking at market triggers. The share price or average share price, for example, would define the trigger event very transparently (see Haldane 2011).

Many were expecting CoCos to play a role in the extra surcharge imposed by Basel to the globally systemically important banks (GSIBs). Although the capital surcharges imposed by the new Basel III guidelines for the GSIBs eventually did not allow for contingent capital, ie, CoCos, we estimate that the CoCo market could reach €150 billion over the next five years, which is in the region of the sizes of the liquid names in the convertible bond space. Basel III has not only imposed more capital but also focused on the quality; the CT1 ratio has been set at 7%. This includes a 4.5% minimum to which a 2.5% capital conservation buffer has been added. Large GSIBs will also have to meet a supplementary minimum capital surcharge of up to 2.5%. This extra layer has to be met in CT1 capital. This is an extra surcharge that brings the minimum CT1 level from 7% to 9.5% for the large international banks that carry a "too-big-to-fail" label.

The minimum overall capital ratio is set at 10.5% for all Basel III-compliant banks and up to 13% for the GSIBs. That is 3.5% extra capital on top of the CT1 capital. This bucket has to consist of 1.5% additional Tier 1 and 2% Tier 2 debt. It is in these two categories of regulatory capital that CoCos will play an important role. CoCos do not have CT1 status but remain part of the regulatory capital.

The European Banking Authority (EBA) announced after the European summit of October 2011 that banks had until June 2012 to build a CT1 capital ratio of 9%, although they had to submit their plans on how they would reach this target before January 1, 2012. These stress tests revealed that around €100 billion extra capital would be needed to reinforce the balance sheets of European banks. The bulk of this effort needed to come from Greek, Spanish, Italian, Portuguese and French banks. The EBA clearly left banks the option of using CoCos in this capital raising exercise: "newly issued private contingent instruments will be accepted if in line with strict criteria defined in a common European term-sheet that the EBA is finalizing." (EBA 2011).

Conceptually, CoCos can help to create a system that will prevent the need for and reduce the cost of government bailouts of financial institutions. These securities could be used to fulfill higher post-credit-crunch crisis capital requirements without diluting common equity holders. However, care needs to be taken in the structuring of new CoCos. The size and the triggering mechanism are crucial and should be determined with great care. In this paper, we look at the issue of pricing a CoCo under a smile conform model using the equity derivative approach in De Spiegeleer and Schoutens (2012) and using credit default swap (CDS) quotes to calibrate the relevant equity data. For a discussion on the anatomy and construction of CoCos, see De Spiegeleer and Schoutens (2011b) and the references therein.

In this context, we also believe it is important to employ a variety of models to price and investigate the dynamics of this new asset class. For the moment, only a limited number of models are available. We mention a so-called rule-of-thumb credit-risk-based model as described in De Spiegeleer *et al* (2011); an equity derivatives approach model, as detailed in De Spiegeleer and Schoutens (2012), is also available. Finally, we mention a couple of firm value models such as Glasserman and Nouri (2012), Madan and Schoutens (2011) and Madan (2011). This paper follows the equity derivatives approach but goes beyond the traditional Black–Scholes modeling as employed in De Spiegeleer and Schoutens (2012). It employs a so-called smile-conform model from the class of Lévy processes, incorporating fatter tails and jumps in contrast with the Black–Scholes setting. It is well-documented that the Black–Scholes model has tails that are too light to accurately model financial data. Non-Brownian Lévy models in general and the VG and β-VG model in particular have naturally built-in fatter tail behavior via the underlying semi-heavy tail distribution, and hence more naturally take tail risk into account. This is reflected in the fact that such Lévy models can better capture the volatility smile (as is illustrated later in the paper). Since CoCos are instruments whose payoff is very sensitive to tail risk, Lévy models are, from a modeling perspective, suitable candidates to investigate the price dynamics of CoCos. In addition, the β-VG model, by its explicit Wiener–Hopf factorization, has the advantage that numerical calculations are more tractable and robust. Although this paper focuses on a particular example, the β-VG model, the technique can be applied to any Lévy process for which accurate knowledge of the Wiener–Hopf factors is available. Stable processes, subclasses of spectrally one-sided Lévy processes or the recently introduced large family of meromorphic Lévy processes, are examples for which the following methodology is applicable. We refer to Schoutens (2003) for a general introduction to Lévy processes in finance and to Seneta (2000) for a historical perspective. For the particular model in question, it is possible to calculate in semiclosed form the joint distribution of the process and its running minimum (maximum) processes. Using Wiener–Hopf theory as described in Kuznetsov *et al* (2011) allows us to very efficiently simulate the process and its running minimum at a series of time points. This fast Monte Carlo simulation makes the process extremely well-suited to numerically price CoCos under an equity derivative approach. Indeed, under the equity approach, the CoCo is decomposed into a series of equity derivatives with barrier features. In order to price such barrier options, we need access to the running minimum process to evaluate whether or not the barrier has been breached. We have already mentioned that the equity approaches and the abovementioned credit rule-of-thumb approach work under the framework of an implied stock price barrier. The breaching of the noncontinuously observable accounting trigger level (in the examples the CT1 ratio) is assumed to occur when the continuously observable stock price process falls under the implied stock price barrier.

As always, the more sophisticated the model, the harder it becomes to calibrate the model and get the numerics done simply and in a realistic amount of time. This paper therefore focuses precisely on the β-VG model employed in an equity derivatives context. This allows us to calibrate the more advanced model on market data and accomplish an accurate and fast pricing of CoCos.

The theory in this paper is illustrated on market data. The methods are illustrated on one of the Lloyds CoCos and the first Rabo issue and are readily adaptable to the other CoCos traded in the market. More precisely, we calibrated the β-VG model on relevant CDS market data of Lloyds and Rabo and priced by illustration with data as of October 14, 2011.

2 THE WIENER–HOPF MONTE CARLO TECHNIQUE

Let us assume that X is any Lévy process with law $\mathbb{P}$. Let us define $e(q)$ to be an exponentially distributed random variable with parameter q independent of X, $\bar{X}_t = \sup\{X_s \mid 0 \leqslant s \leqslant t\}$ and $\underline{X}_t = \inf\{X_s \mid 0 \leqslant s \leqslant t\}$. It is a fact that, for all Lévy processes,

$$X_{e(q)} \overset{\mathrm{d}}{=} S + I, \tag{2.1}$$

where the equality is in distribution, S and I are independent random variables equal in distribution to $\bar{X}_{e(q)}$ and $\underline{X}_{e(q)}$, respectively. This equality in distribution is known as the Wiener–Hopf decomposition.

We are interested in a straightforward method of performing Monte Carlo simulations for expectations of the type $\mathbb{E}[F(X_t, \bar{X}_t)]$ for suitable functions F. A straightforward Monte Carlo random walk approximation is an obvious way to proceed; however, it is well-understood that this can introduce significant numerical errors into the distribution of $\bar{X}_t$. The Wiener–Hopf Monte Carlo method offers an alternative solution to this problem by allowing for exact sampling from the law of $(X_g, \bar{X}_g)$ where g is a random time whose distribution can be concentrated arbitrarily closely around t depending on the tolerance chosen in the algorithm.

Suppose that $e_1(1), e_2(1), \ldots$ is a sequence of independent and identically distributed (iid) exponentially random variables with unit mean. The basis of the algorithm is the following simple observation, which follows directly from the strong law of large numbers. For all $t > 0$,

$$\sum_{i=1}^{n} \frac{t}{n} e_i(1) \to t \quad \text{as } n \uparrow \infty, \tag{2.2}$$

almost surely. Noting that the random variable on the left-hand side above can also be written as the sum of n independent random variables with an exponential distribution having rate n/t, it is equal in law to a Gamma random variable with parameters n and n/t, henceforth written as $g(n, n/t)$. For sufficiently large n, Kuznetsov *et al* (2011) argue that a suitable approximation to $\mathbb{P}(X_t \in \mathrm{d}x, \bar{X}_t \in \mathrm{d}y)$ is $\mathbb{P}(X_{g(n,n/t)} \in \mathrm{d}x, \bar{X}_{g(n,n/t)} \in \mathrm{d}y)$.

This approximation gains practical value in the context of Monte Carlo simulation when we take advantage of the Wiener–Hopf factorization in its probabilistic form (2.1). The following theorem was proved in Kuznetsov *et al* (2011) using the stationary and independent increments of the underlying Lévy process.

THEOREM 2.1 *Fix $n \geqslant 1$. Then*

$$(X_{g(n,n/t)}, \bar{X}_{g(n,n/t)}) \stackrel{\mathrm{d}}{=} (V(n,n/t), J(n,n/t)),$$

where $V(n,n/t)$ and $J(n,n/t)$ are defined for $n \geqslant 1$ by

$$V(n,n/t) = \sum_{j=1}^{n} (S^{(j)} + I^{(j)}), \tag{2.3}$$

$$J(n,n/t) = \max\left\{\sum_{j=1}^{k-1} (S^{(j)} + I^{(j)}) + S^{(k)} \colon k = 1, \dots, n\right\}. \tag{2.4}$$

Here, $\{S^{(j)} \colon j \geqslant 1\}$ is an iid sequence of random variables with common distribution equal to that of $\bar{X}_{e_1(n/t)}$ and $\{I^{(j)} \colon j \geqslant 1\}$ is another iid sequence of random variables with common distribution equal to that of $\underline{X}_{e_1(n/t)}$.

Given (2.2) it is clear that the pair $(V(n,n/t), J(n,n/t))$ converges in distribution to $(X_t, \bar{X}_t)$. This suggests that we only need to be able to simulate iid copies of the distributions of $S^{(1)}$ and $I^{(1)}$ and then, by the simple functional transformation given in (2.3) and (2.4), we may produce a realization of the random variables $(X_{g(n,n/t)}, \bar{X}_{g(n,n/t)})$. Given a suitably nice function F and using standard Monte Carlo methods we estimate, for large k,

$$\mathbb{E}[F(X_t, \bar{X}_t)] \simeq \frac{1}{k} \sum_{m=1}^{k} F(V^{(m)}(n,n/t), J^{(m)}(n,n/t)), \tag{2.5}$$

where $(V^{(m)}(n,n/t), J^{(m)}(n,n/t))$ are iid copies of $(V(n,n/t), J(n,n/t))$. Indeed, the strong law of large numbers implies that the right-hand side of (2.5) converges almost surely as $k \uparrow \infty$ to $\mathbb{E}(F(X_{g(n,n/t)}, \bar{X}_{g(n,n/t)}))$, which in turn converges as $n \uparrow \infty$ to $\mathbb{E}(F(X_t, \bar{X}_t))$.

3 THE β-VG MODEL

The classical model for the evolution of the value of a risky underlying, eg, $S = \{S_t\}_{t \geqslant 0}$, that we shall use in this paper is that of an exponential Lévy process (for an introduction to Lévy processes in finance, see Schoutens (2003)). That is to say, for all $t \geqslant 0$,

$$S_t = S_0 \mathrm{e}^{X_t},$$

where $X = \{X_t\}_{t \geqslant 0}$ is a Lévy process. Recall that the law of every Lévy process is characterized through a triplet (μ, σ, ν), where $\mu \in \mathbb{R}$, $\sigma \geqslant 0$ and ν is a measure concentrated on $\mathbb{R} \setminus \{0\}$ such that $\int_{\mathbb{R}} (1 \wedge x^2) \nu(\mathrm{d}x) < \infty$. More precisely, for all $z \in \mathbb{C}$, we have

$$\mathbb{E}[\mathrm{e}^{\mathrm{i}z X_t}] = \mathrm{e}^{t \psi(\mathrm{i}z)},$$

where

$$\psi(z) = \mu z + \tfrac{1}{2}\sigma^2 z^2 + \int_{\mathbb{R}\setminus\{0\}} (\mathrm{e}^{zx} - 1 - zx \mathbf{1}_{\{|x|<1\}}) \nu(\mathrm{d}x). \tag{3.1}$$

The Lévy processes in which we are interested are called β-VG processes and are a special example of so-called β-processes introduced by Kuznetsov (2010). Rather than giving a general overview of the latter class, we shall give a brief description of the former. The defining characteristic of the β-VG class is the choice of Lévy measure, which is given by

$$\nu(\mathrm{d}x) = \left(c \frac{\mathrm{e}^{-\alpha_1 x}}{(1 - \mathrm{e}^{-x})} \mathbf{1}_{\{x>0\}} + c \frac{\mathrm{e}^{\alpha_2 x}}{(1 - \mathrm{e}^{x})} \mathbf{1}_{\{x<0\}} \right) \mathrm{d}x, \tag{3.2}$$

where $\alpha_1, \alpha_2, c > 0$. The above Lévy measure has characteristics which are extremely close to the Lévy measure of VG processes, which is given by

$$\nu(\mathrm{d}x) = \left(C \frac{\mathrm{e}^{-Mx}}{x} \mathbf{1}_{\{x>0\}} + C \frac{\mathrm{e}^{Gx}}{x} \mathbf{1}_{\{x<0\}} \right) \mathrm{d}x,$$

where C, G and M are all strictly positive real numbers. By choosing $c = C$, $\alpha_1 = M$ and $\alpha_2 = G$ we can easily verify that the Lévy density of the β-VG process exhibits the same exponential decay for large values of $|x|$ as well as the same polynomial growth for $|x|$ close to the origin. A comparison of the goodness of fit can be found in Schoutens and Damme (2010). Note that β-VG processes can also be found under the name of Lamperti–Stable processes (see Caballero *et al* 2010).

With the choice of the Lévy density in (3.2), it turns out that we can always arrange the characteristic exponent (3.1) so that it can be written in the form

$$\psi(z) = \mu z + \tfrac{1}{2}\sigma^2 z^2 + c[\Psi(\alpha_1) - \Psi(\alpha_1 - z)] + c[\Psi(\alpha_2) - \Psi(\alpha_2 + z)], \tag{3.3}$$

where $\Psi(x) = \Gamma'(x)/\Gamma(x)$, $\Gamma(x)$ is the usual Gamma function and $\mu \in \mathbb{R}$, which is different in value to the μ in the representation (3.1).

Using the binomial expansion it is not difficult to show that the density in (3.2) can otherwise be written in the form

$$\sum_{n \geqslant 1} a_n \rho_n \mathrm{e}^{-\rho_n x} \mathbf{1}_{\{x>0\}} + \sum_{n \geqslant 1} \hat{a}_n \hat{\rho}_n \mathrm{e}^{\hat{\rho}_n x} \mathbf{1}_{\{x<0\}}, \tag{3.4}$$

where, for $n \geqslant 1$, the coefficients a_n, $\hat{a}_n$ are all positive and $\rho_n = \alpha_1 + (n-1)$ and $\hat{\rho}_n = \alpha_2 + (n-1)$. We can also easily show that all the poles of the exponent ψ are simple and positioned precisely at $\{\rho_n\}_{n\geqslant 1}$ and $\{\hat{\rho}_n\}_{n\geqslant 1}$. For any $q \geqslant 0$, the roots of the equation $\psi(z) - q$ are also all positioned on the real line, say $\{\zeta_n\}_{n\geqslant 1}$ and $\{\hat{\zeta}_n\}_{n\geqslant 1}$ as sequences ordered by magnitude on the positive and negative half lines respectively. Moreover, these roots and poles respect the remarkable interlacing property

$$\cdots < -\hat{\rho}_2 < -\hat{\zeta}_2 < -\hat{\rho}_1 < -\hat{\zeta}_1 < 0 < \zeta_1 < \rho_1 < \zeta_2 < \rho_2 < \cdots .$$

It turns out that these roots and poles play a crucial role in describing the distribution of the running supremum and running infimum of X.

For the case of β-VG processes, we can develop the decomposition (2.1) further by specifying the distribution of the variables $\bar{X}_{e(q)}$ and $\underline{X}_{e(q)}$ explicitly (see Kuznetsov *et al* 2012). Indeed, it can be shown (cf. Ferreiro-Castilla and Utzet 2010, Proposition 2.1) that

$$\bar{X}_{e(q)} \stackrel{\mathrm{d}}{=} \sum_{n\geqslant 1} \Theta_n, \tag{3.5}$$

such that, for each $n \geqslant 1$, random variable Θ_n has distribution given by

$$\frac{\zeta_n}{\rho_n}\delta_0(\mathrm{d}x) + \left(1 - \frac{\zeta_n}{\rho_n}\right)\zeta_n \mathrm{e}^{-\zeta_n x}\,\mathrm{d}x, \quad x \geqslant 0, \tag{3.6}$$

where $\delta_0(\mathrm{d}x)$ is the Dirac delta measure that places a unit atom at 0. Note that, by the interlacing property, $\zeta_n < \rho_n$, and we may interpret the above distribution the result of tossing a coin with probability ζ_n/ρ_n of landing on "heads". If the latter happens, then Θ_n takes value zero. Otherwise, the value of Θ_n is sampled from an independent exponential distribution with rate ζ_n.

In the case of β-VG processes, we can use (3.5) with $q = n/t$ to sample from $S^{(1)}$. The reader will note that an obvious analogous approach holds for sampling from $I^{(1)}$. It is clearly impossible to draw samples from an infinite number of the distributions given in (3.6) and then add them together. However, as a suitable approximation, it is possible to draw samples from a finite but large number of such distributions and then add them together. Strictly speaking, this may introduce a very small artificial atom at zero in the approximate distribution for $\bar{X}_{e(q)}$ in the event that the real distribution has no atom at zero. However, we also note that there is convergence in distribution and in the L^2 norm of the approximation (see Ferreiro-Castilla and Schoutens 2012).

For a comprehensive study of the numerical analysis of the model as well as for the implementation issues of the algorithm, we refer the reader to Schoutens and Damme (2010), Ferreiro-Castilla *et al* (2012) and Ferreiro-Castilla and Schoutens (2012).

4 CASE STUDY I : THE PRICING OF A LLOYDS CONTINGENT CONVERTIBLE

This section focuses on the calibration and the pricing of the CoCo with ISIN XS0459089255. This CoCo has a maturity of 8.48 years.

4.1 Pricing formula

As indicated above, we model the risk-neutral stock price process as an exponential β-VG process:

$$S_t = S_0 e^{(r-q)t} \frac{\exp(X_t)}{E[\exp(X_t)]}, \quad S_0 > 0,$$

where r is the risk-free rate and q is the dividend yield.

In the Lloyds CoCo, conversion takes place when the CT1 ratio drops below a certain minimum level (5%). However, CT1 is an accounting ratio and is not continuously observable. We therefore develop a proxy model, in which we replace the event that CT1 falls below a certain level, with an equivalent event where the stock price drops below a barrier S^*.

De Spiegeleer and Schoutens (2012) explain how a CoCo bond can be decomposed in a series of barrier-type derivatives. The decomposition arises from the following reasoning. When no trigger event takes place during the lifetime of the CoCo, at maturity, the notional N is returned to the investor, who has received at the coupon dates t_i, $i = 1, \dots, k$, a series of amounts c_i. We can easily calculate the present value of the cashflows by simple discounting. However, if the barrier is breached, we have a conversion trigger and this has two effects. First, all future coupons are canceled, and second, the investor does not get his notional back at maturity but instead receives a number of stocks at trigger point. The exact amount is described in the terms of the CoCo bond and equals $C_r = N/C_p$, with C_p hence equal to conversion price (in the Lloyds example, C_p equals 59 pence and corresponds to the price of the Lloyds stock when these CoCo bonds were issued). The effect of all this is modeled via a short position in a series of digital barrier options corresponding to each coupon and knocking in when the barrier level S^* is breached, together with a down-and-in barrier forward, which also knocks in at the same barrier level. Hence, when the barrier is hit, the digital barriers eliminate any post-conversion scheduled coupon payments and the down-and-in barrier forward replaces the notional payoff with an equity position.

Under the Black–Scholes setting with a given flat volatility σ, closed-form formulas exist for the mentioned barrier derivatives and the price of the CoCo, P, is given by

$$P = A + B + C,$$

$$A = N \exp(-rT) + \sum_{i=1}^{k} c_i \exp(-rt_i),$$

$$\begin{aligned} B &= C_r \times [S_0 \exp(-qT)(S^*/S_0)^{2\lambda} \Phi(y_1), \\ &\qquad - K \exp(-rT)(S^*/S_0)^{2\lambda-2} \Phi(y_1 - \sigma\sqrt{T}) \\ &\qquad - K \exp(-rT) \Phi(-x_1 + \sigma\sqrt{T}) + S_0 \exp(-qT) \Phi(-x_1)], \\ C &= -\sum_{i=1}^{k} c_i \exp(-rt_i)[\Phi(-x_{1i} + \sigma\sqrt{t_i}) + (S^*/S_0)^{2\lambda-2} \Phi(y_{1i} - \sigma\sqrt{t_i})], \end{aligned}$$

with

$$\left.\begin{aligned} K &= C_p, \\ C_r &= \frac{N}{C_p}, \\ x_1 &= \frac{\log(S_0/S^*)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}, \\ y_1 &= \frac{\log(S^*/S_0)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}, \\ x_{1i} &= \frac{\log(S_0/S^*)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i}, \\ y_{1i} &= \frac{\log(S^*/S_0)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i}, \\ \lambda &= \frac{r - q + \sigma^2/2}{\sigma^2}. \end{aligned}\right\} \tag{4.1}$$

A is the simple discounting of coupons and notional under no conversion, B reflects the price of the down-and-in barrier forward (which corresponds to the price of a down-and-in call minus the price of a down-and-in put, both with strike equal to the conversion price C_p) and C equals the price of the short position in k digital barriers, one corresponding to each scheduled coupon payment. Function $\Phi(x)$ denotes the cumulative distribution function of a standard normal variable. For our Lévy process, A remains the same, while the definitions of B and C are

$$\begin{aligned} B &= C_r \times \exp(-rT) \times \mathbb{E}[(S_T - K)\mathbf{1}_{\{\inf_{0 \leqslant t \leqslant T} S_t < S^*\}}] \\ &= C_r \times \exp(-rT) \\ &\qquad \times [\mathbb{E}[(S_T - K)^+ \mathbf{1}_{\{\inf_{0 \leqslant t \leqslant T} S_t < S^*\}} - \mathbb{E}[(K - S_T)^- \mathbf{1}_{\{\inf_{0 \leqslant t \leqslant T} S_t < S^*\}}], \\ C &= -\sum_{i=1}^{k} c_i \exp(-rt_i) \mathbb{E}[\mathbf{1}_{\{\inf_{0 \leqslant t \leqslant t_i} S_t < S^*\}}]. \end{aligned}$$

Note that the representation of B is a simplification since it does not incorporate the time of conversion of the CoCo. However, the impact of this simplification is minor. When conversion is taking place, the actual company is in distress, and hence it is very natural for the company not to pay any more dividends. The impact has been studied in detail in De Spiegeleer *et al* (2013) and it is shown to be negligible.

Under more advanced models like the β-VG model, no closed-form formulas for barrier-type options are available and we have to rely on other numerical techniques. Existing methods are based around plain vanilla Monte Carlo, solving partial-differential-integral equations and Fourier-based methods (see, for example, Schoutens and Cariboni 2009). In this paper we instead explore numerical methods based around the Wiener–Hopf Monte Carlo method described in Section 2.

4.2 Calibration on CDS data

Stochastic models need to be calibrated on market data in order to obtain the parameters of the model that best reflect the current market situation. We should of course also pay attention to the real use of the model and hence use data that reflects and incorporates essential market information relevant to the pricing task. In our situation, we are dealing with a product that is sensitive to tail risk and in particular to extreme down moves of the stock price level. In addition, CoCo bonds are typically long-dated, with maturities exceeding at least five years in most cases. Since we are dealing with an equity model, ideally we should hence include instruments (say call or put options) with similar maturities and sensitivity to extreme down moves in the calibration of the model. However, such vanilla products, such as far out-of-the-money (OTM) puts with maturities exceeding, eg, five years, are not that available or not really liquid for individual companies (there is some liquidity in these instruments for some of the major indexes). Therefore, we need to look for alternative instruments to calibrate our model and CDSs can be of use here. Indeed, they are suitable instruments for pricing extreme down-side risk in the long term. Moreover, they are available for many individual names, especially for companies active on the bond markets. CDSs are actual credit risk derivatives, typically handled with credit derivative models. Since we are employing an equity model, we need to map the information on CDSs into useful stock price information. This is done by equating the upfront price of the insurance of default of the company, which can be directly calculated from the CDS spread, into a premium to be paid for a deep OTM digital put. The payoff of such a digital put and the triggering of a default are highly correlated events, and hence CDSs contain essential information for our models. To be precise, we follow the route also proposed in JP Morgan's CoCo pricing method by linking CDS quotes with deep OTM digital put options. The CDS spread under an assumption of 40% recovery is translated into a zero-recovery upfront premium. Then, this upfront premium is assumed to correspond with the premium of a deep OTM (European) digital put with the same maturity. The difference from JP Morgan's case is twofold. First, we take into account the CDS term structure and not only use the CDS quotes closest to the CoCo bond maturity,

TABLE 2 Implied volatilities under Black–Scholes of the CDS premium for Lloyds.

	T						
	1	**2**	**3**	**4**	**5**	**7**	**10**
CDS (bps)	253.20	286.40	299.30	315.80	325.90	330.20	337.50
r (%)	1.73	1.38	1.53	1.73	1.94	2.35	2.81
Digital put price (%)	4.89	9.88	14.51	19.30	23.66	30.51	38.79
σ (%)	114.05	95.96	87.31	82.92	80.19	77.08	77.16

and second, we match the CDSs not with the 95% OTM digital put option, but set the strike of the digital put on the basis of a mapping between instruments, for which we have much more information. More precisely, we compare EuroStoxx's fifty weighted averaged CDS quotes, translate these into zero-recovery upfront premiums and then compare them with the digital put price range. We observe a match at the strike 94% OTM. We hence use that level for the Lloyds (and Rabo) calibration as well, ie, we translate CDS spreads into implied volatilities at the 6% strike.

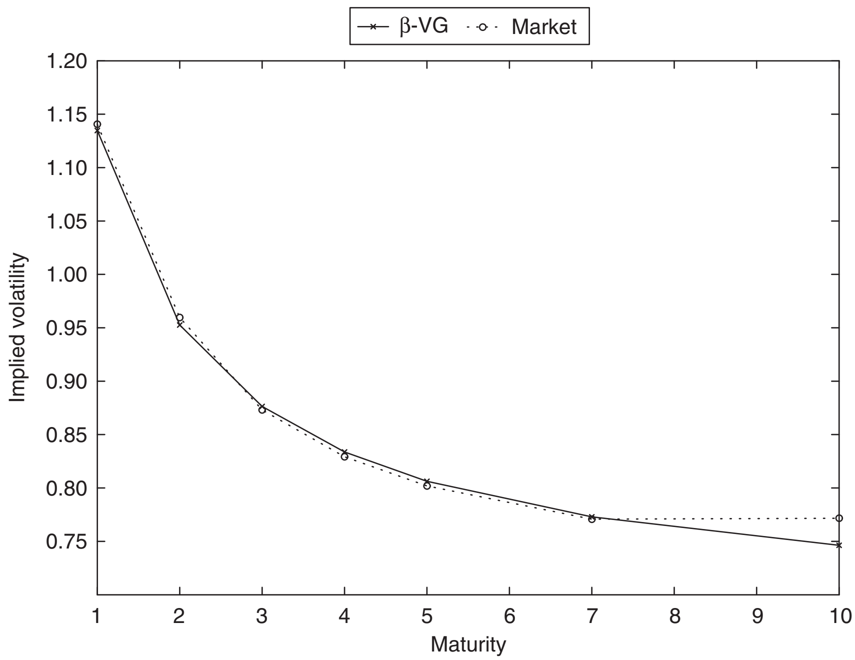
In Table 2, we survey the data used as of October 14, 2011. We give the CDS term structure, the risk-free yield curve (GBP: semi-annual swap spreads). We use a flat dividend yield of 1.50%. Lloyds stock was trading at 33.25 pence. In the CDS premium, we have assumed a 40% recovery. The table also gives us the corresponding prices of the digital put option and the corresponding (Black–Scholes) implied volatilities at the 6% strike. It is on the latter that we will calibrate our β-VG model.

The results of the calibration are shown in Figure 1 on the next page. The optimal parameter set is given by $c = 0.8446$, $\alpha_1 = 14.9964$ and $\alpha_2 = 1.3660$.

4.2.1 Pricing of the Lloyds contingent convertible

Next, we price the Lloyds CoCo under the β-VG model and compare with the traditional Black–Scholes model as developed in De Spiegeleer and Schoutens (2012). For the latter, we use a flat volatility of 77.11%, which equals the volatility determined out of the CDS quotes interpolated at the CoCo's maturity of 8.19 years. In Figure 2 on page 135, we plot the CoCo bond price (with a notional of £1000) for the calibrated β-VG model and the flat volatility Black–Scholes model for a range of conversion barriers. The implied barrier is the barrier level at which the market quote (in our case £1099.00) coincides with the model price. We have an implied barrier level of 3.31 pence (which is equivalent to an implied recovery of about 9.95%) in the β-VG setting and an implied barrier level of 3.03 pence under Black–Scholes (an implied recovery of about 9.11%). Under the equity derivative approach presented

FIGURE 1 β-VG calibration on Lloyds implied volatilities backed out of CDS term structure as of October 14, 2011.



here, the trigger depends on the price of the CoCo and vice versa. Figure 2 shows how this relation is modified when changing the underlying stochastic process from the Black–Scholes model to the β-VG setting.

5 CASE STUDY II: THE PRICING OF A RABO CONTINGENT CONVERTIBLE

This section focuses on the calibration and the pricing of Rabobank's senior contingent notes with ISIN XS0496281618. The pricing date is again October 14, 2011. On that day this Rabobank CoCo had a maturity of 8.44 years; the maturity at issue was equal to 10 years. The CoCo triggers if the equity capital/RWA ratio falls below 7%. If it is triggered, we have not a conversion into shares (because Rabobank does not have traded equity), but a write-down of 75% of the bond's notional. The CoCo pays a yearly coupon of 6.875%; at trigger investors receive a rebate of 25% of the outstanding notional.

FIGURE 2 Bond price of Lloyds CoCo XS0459089255, under Black–Scholes and β-VG dynamics, as of October 14, 2011.

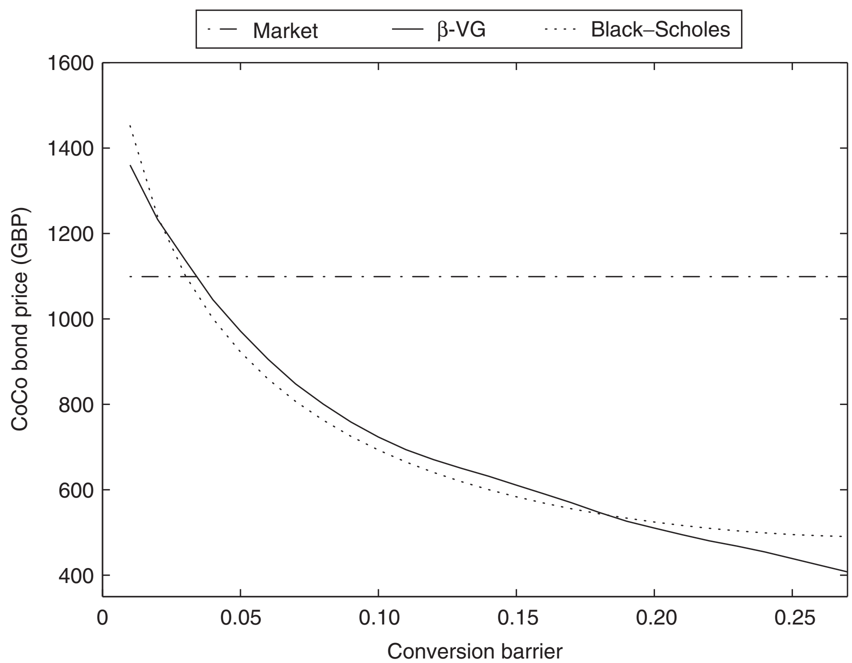


TABLE 3 Implied volatilities under Black–Scholes of the CDS premium for Rabo.

	T						
	1	**2**	**3**	**4**	**5**	**7**	**10**
CDS (bps)	50.70	73.70	97.60	109.20	116.30	122.40	127.10
r (%)	2.12	1.62	1.75	1.93	2.13	2.47	2.77
Digital put price (%)	1.00	2.63	4.97	7.13	9.17	12.66	17.19
σ (%)	92.35	77.26	71.22	67.03	64.17	60.47	57.89

The CoCo was trading on October 14, 2011 at a price of 88.84%. The Rabo CDS term structure was given as in Table 3. In that table we also give corresponding interest rate structure as well as the corresponding “equity” volatility obtained using the same procedure as in the Lloyds example.

The results of the calibration are shown in Figure 3 on page 137. The optimal parameter set is given by $c = 0.7013$, $\alpha_1 = 15.6750$ and $\alpha_2 = 1.7141$.

Next, we price the Rabobank CoCo under the β-VG model and compare it with the traditional Black–Scholes model. The price given under the latter model is

$$
\begin{aligned}
\text{CoCo} &= A + B + C,\\
A &= N\exp(-rT) + \sum_{i=1}^{k} c_i \exp(-rt_i),\\
B &= -N\exp(-rT)(\Phi(-x_1 + \sigma\sqrt{T}) + (1-\text{D2T})^{2\lambda-2}\Phi(y_1 - \sigma\sqrt{T}))\\
&\qquad + RN((1-\text{D2T})^{a+b}\Phi(z) + (1-\text{D2T})^{a-b}\Phi(z - 2b\sigma\sqrt{T})),\\
C &= -\sum_{i=1}^{k} c_i \exp(-rt_i)[\Phi(-x_{1i} + \sigma\sqrt{t_i}) + (1-\text{D2T})^{2\lambda-2}\Phi(y_{1i} - \sigma\sqrt{t_i})],
\end{aligned}
$$

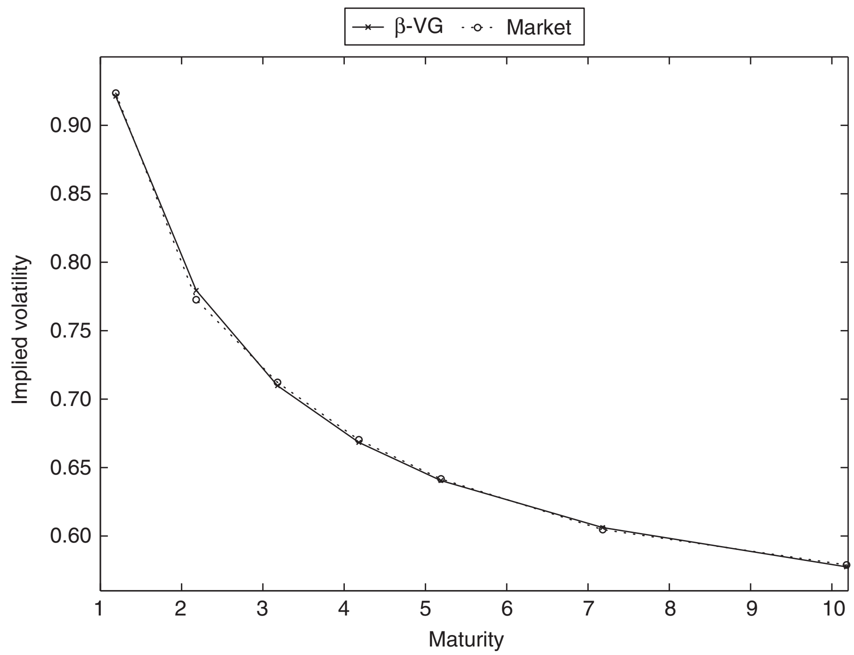
with

$$
\begin{aligned}
\text{D2T} &= (S_0 - S^*)/S_0 = 1 - S^*/S_0,\\
x_1 &= \frac{\log(1-\text{D2T})}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T},\\
y_1 &= \frac{\log(1-\text{D2T})}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T},\\
x_{1i} &= \frac{\log(1-\text{D2T})}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i},\\
y_{1i} &= \frac{\log(1-\text{D2T})}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i},\\
\lambda &= \frac{r - q + \sigma^2/2}{\sigma^2},\\
\mu &= r - q - \sigma^2/2, \quad a = \mu/\sigma^2, \quad b = \sqrt{\mu^2 + 2r\sigma^2}/\sigma^2,\\
z &= \frac{\log(1-\text{D2T})}{\sigma\sqrt{T}} + b\sigma\sqrt{T},
\end{aligned}
$$

where D2T is the distance to trigger. A corresponds to the discounting of notional and coupons, B contains the price of a short position in a digital down-and-in barrier option with a rebate of 25% of the notional at hitting time and, similarly, C reflects a stream of digital down-and-in barrier options on the coupons, which all knock when the CoCo gets triggered. For our Lévy process, A remains the same, while the definitions of B and C are

$$
\begin{aligned}
B &= -\Big(N\exp(-rT) + RN\exp\Big(-r\inf_{0\leqslant t\leqslant T} S_t < S^*\Big)\Big)\mathbb{E}[\mathbf{1}_{\{\inf_{0\leqslant t\leqslant T} S_t < S^*\}}],\\
C &= -\sum_{i=1}^{k} c_i \exp(-rt_i)\mathbb{E}[\mathbf{1}_{\{\inf_{0\leqslant t\leqslant t_i} S_t < S^*\}}].
\end{aligned}
$$

FIGURE 3 β-VG calibration on Rabobank's implied volatilities backed out of CDS term structure as of October 14, 2011.



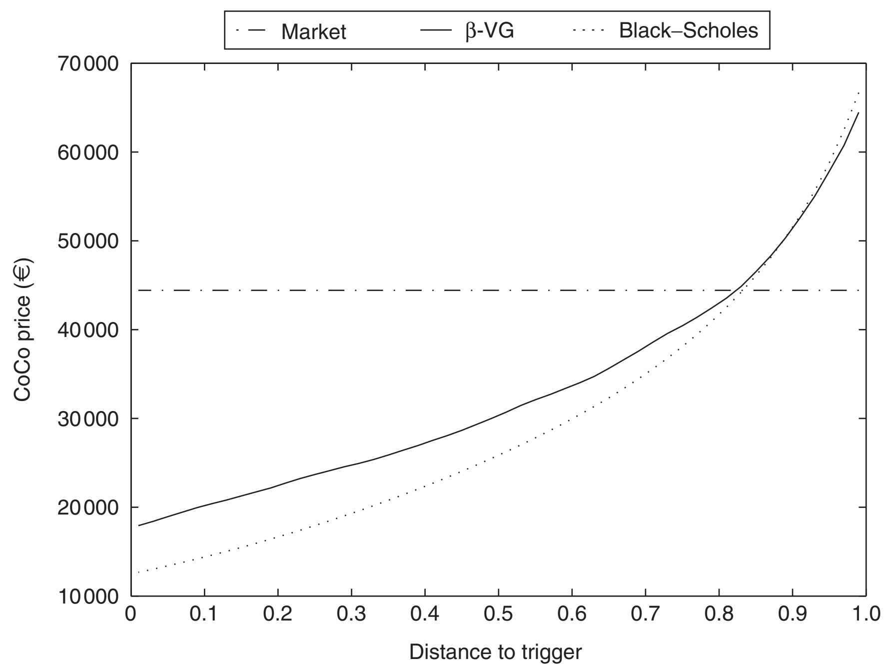
For the latter we use a flat volatility of 59.39%, which equals the volatility determined out of the CDS quotes interpolated at the CoCo's maturity of 8.44 years, and a credit spread of 124.37bps, again obtained by interpolation on the CDS curve at the maturity of the CoCo. In Figure 4 on the next page, we plot the CoCo bond price (with a notional of €50 000) for the calibrated β-VG model and the flat volatility Black–Scholes model. The X-axis is now the distance to trigger, defined as in the formula above.

For the β-VG setting, which is the level corresponding to the market price of €44 420 (€50 000 $\times$ 88.84%), the implied distance to trigger equals 82.32%. For the Black–Scholes model the implied distance to trigger equals 83.22%.

6 CONCLUSIONS

In this paper, we showed how to price contingent convertibles under smile conform models with the β-family of Lévy processes introduced in Kuznetsov (2010). Our analysis focused in particular on the subclass of β-VG process introduced in Schoutens

FIGURE 4 Rabobank's senior contingent notes with ISIN XS0496281618 as of October 14, 2011.



and Damme (2010). We considered a derivative approach that reduces the pricing to a series of barrier options in which the trigger event is described by the underlying crossing the barrier. While a lot of literature can be found addressing the particular problem of pricing barrier options under Lévy models, the approach proposed here is novel, as it relies on recent results concerning the Wiener–Hopf approach (cf. Kuznetsov *et al* 2011).

Contingent convertibles are recent products, but their impact on the market makes them of considerable importance. For this reason, it is important to have a rich variety of models under which prices can be obtained. It is worth mentioning here that several studies are devoted to improving these models, from both the modeling point of view (Glasserman and Nouri 2012; Madan and Schoutens 2011; Madan 2011) and the numerical analysis point of view (Ferreiro-Castilla *et al* 2012). We compared our methodology against the Black–Scholes model employed in De Spiegeleer and Schoutens (2011b), showing that, on the one hand, the approach is feasible and, on the other, that our models better capture the intrinsic nature of these complex products.

REFERENCES

Bertoin, J. (1996). *Lévy Processes.* Cambridge Tracts in Mathematics, Vol. 121. Cambridge University Press.

Caballero, M. E., Pardo, J. C., and Pérez, J. L. (2010). On the Lamperti stable processes. *Probability and Mathematical Statistics* **30**(1), 1–28.

Carr, P. (1998). Randomization and the American Put. *Review of Financial Studies* **11**(3), 597–626.

De Spiegeleer, J., and Schoutens, W. (2011a). *Contingent Convertible (CoCo) Notes: Structuring and Pricing.* Euromoney Trading, London.

De Spiegeleer, J., and Schoutens, W. (2011b). *The Handbook of Convertible Bonds: Pricing, Strategies and Risk Management.* Wiley.

De Spiegeleer, J., and Schoutens, W. (2012). Pricing contingent convertibles: a derivatives approach. *The Journal of Derivatives* **20**(2), 27–36.

De Spiegeleer, J., Van Hulle, C., and Schoutens, W. (2011). Cracking the CoCo pricing conundrum. *Creditflux* (July), 20–21.

De Spiegeleer, J., Schoutens, W., and Van Hulle, C. (2013). *The Handbook of Hybrid Securities: Strategies, Pricing and Risk Management*, 2nd edn. Wiley.

European Banking Authority (2011). Questions and answers. News and communications, October 26. URL: www.eba.europa.eu/documents/10180/26923/Q-A-FINAL_2.pdf/5527374f-bf05-4963-8def-7c9d48fba5e7

Ferreiro-Castilla, A., and Schoutens, W. (2012). The β-Meixner model. *Journal of Computational and Applied Mathematics* **236**(9), 2466–2476.

Ferreiro-Castilla, A., and Utzet, F. (2010). Inversion of analytic characteristic functions and infinite convolutions of exponential and Laplace densities. *Journal of Theoretical Probability* **25**(1), 205–230.

Ferreiro-Castilla, A., Kyprianou, A. E., Scheichl, R., and Suryanarayana, G. (2012). *Multilevel Monte Carlo simulation for Lévy processes based on the Wiener–Hopf factorization.* Preprint. URL: http://arxiv.org/abs/1210.5868.

Glasserman, P., and Nouri, B. (2012). Contingent capital with a capital-ratio trigger. *Journal of Management Science* **58**(10), 1816–1833.

Haldane, A. G. (2011). Capital discipline. Speech given at the American Economic Association, Denver, CO (January).

Kuznetsov, A. (2010). Wiener–Hopf factorization and distribution of extrema for a family of Lévy processes. *Annals of Applied Probability* **20**(5), 1801–1830.

Kuznetsov, A., Kyprianou, A. E., Pardo, J. C., and van Schaik, K. (2011). A Wiener–Hopf Monte Carlo simulation technique for Lévy process. *Annals of Applied Probability* **21**(6), 2171–2190.

Kuznetsov, A., Kyprianou, A. E., and Pardo, J. C. (2012). Meromorphic Lévy processes and their fluctuation identities. *Annals of Applied Probability* **22**(3), 1101–1135.

Kyprianou, A. E. (2006). *Introductory Lectures on Fluctuations of Lévy Processes with Applications.* Springer.

Madan, D. B. (2011). On pricing contingent capital notes. In *Recent Advances in Financial Engineering 2011, Proceedings of the KIER-TMU International Workshop on Financial Engineering*, Kijima, M., Muromachi, Y., and Nakaoka, H. (eds). World Scientific.

Madan, D. B., and Schoutens, W. (2011). Conic coconuts: the pricing of contingent capital notes using conic finance. *Mathematics and Financial Economics* **4**(2), 87–100.

Maes, S., and Schoutens, W. (2012). Contingent capital: an in-depth discussion. *Economic Notes* **41**(1–2), 59–79.

Sato, K. (1999). *Lévy Processes and Infinitely Divisible Distributions*. Cambridge University Press.

Schoutens, W. (2003). *Lévy Processes in Finance*. Wiley.

Schoutens, W., and Cariboni, J. (2009). *Lévy Processes in Credit Risk*. Wiley.

Schoutens, W., and Damme, G. (2010). The β-variance gamma model. *Review of Derivatives Research* **14**(3), 263–282.

Seneta, E. (2000). The early years of the variance–gamma process. In *Advances in Mathematical Finance*, Fu, M. C., Jarrow, R. A., Yen, J. J., and Elliott, R. J. (eds). Birkhäuser, Boston, MA.