

Abstract

The **Multiregression Dynamic Model (MDM)** is a multivariate graphical model for a multidimensional time series that allows the estimation of time-varying effective connectivity (Costa *et al.*, 2013). An MDM is a state space model where connection weights reflect the instantaneous interactions between brain regions. Because the marginal likelihood has a closed form, model selection across a large number of potential connectivity networks is easy to perform. With application of the Integer Programming Algorithm, we can quickly find optimal models that satisfy acyclic graph constraints. Also, due to a factorisation of the marginal likelihood, the search over all possible directed (acyclic or cyclic) graphical structures is even faster. These methods are illustrated using recent resting-state and steady-state task fMRI data.

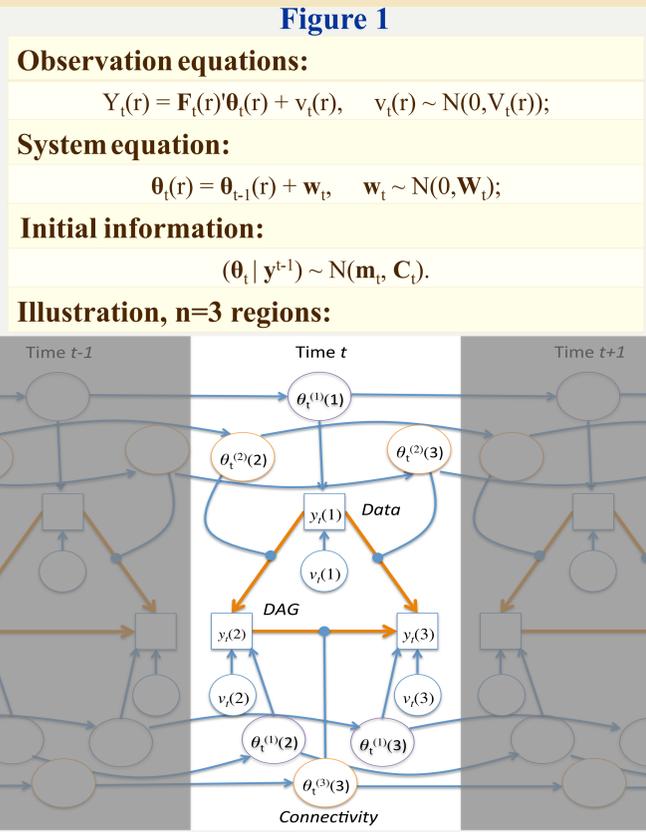
The Multiregression Dynamic Model

The linear MDM is defined by equations shown in Figure 1 (top). $\mathbf{Y}_t' = (Y_t(1), \dots, Y_t(n))$ is the time series data at time t for n regions, and $\boldsymbol{\theta}_t(r)$ is the

vector of time-varying intercept and connectivity parameters; $\mathbf{F}_t(r)$ are the “regressors” at time t , a vector comprised of the value 1 (for the intercept) and values of data for the parent nodes of region r at the current time t .

Note that state space models do not assume nonstationarity, though when state covariance $\mathbf{W}_t(r) = 0$ for all t and r , the usual static regression model is obtained. When $\mathbf{W}_t(r)$ is unknown, it can be defined via a scalar discount factor (DF), such that a DF=1 produces a static model. The conditional forecast distribution of $Y_t(r)$ given the past up to time $t-1$ at its parents is a Student t distribution, and thus the joint log predictive likelihood (LPL) can be calculated as the sum of this distribution over nodes.

Figure 2 (bottom) illustrates a 3-node DAG MDM, considering region 1 as the parent of region 2 and region 3; and region 2 as the parent of region 3. The orange ovals represent the (time-varying) effective connectivity strength between two regions, the purple ovals the intercepts, and the blue circles the observation noise.



Searching the MDM using IPA and DGM

First considering only acyclic models, we use an Integer Programming (IP) algorithm (gobnlp system; Bartlett and Cussens, 2013), to quickly find the DAG with the optimal LPL; we call this the MDM-IPA model. Without the acyclic constraint, we call our method a Directed Graph Model (MDM-DGM).

For instance consider a search problem for 3 variables. The Table 1 shows the local scores (LPL). The MDM-DGM finds the directed graph that maximizes the LPL for every node independently. Therefore, the best scoring model takes node 1 to have no parents and the nodes 2 and 3 with the each other node plus node 1 as its parents, see Figure 2(a). Note this is a cyclic graph.

The MDM-IPA also considers the acyclic constraints. Here, this constraint was violated in the cluster formed by nodes 2 and 3. The best solution leaves node 2 with the same set of parents as before, while node 3 becomes parentless; see Figure 2(b).

Table 1

Node	Parent	Score
1	No	-1469
	2	-1567
	3	-1646
	2 and 3	-1655
2	No	-1169
	1	-1140
	3	-1110
	1 and 3	-997
3	No	-1119
	1	-1193
	2	-1060
	1 and 2	-1056

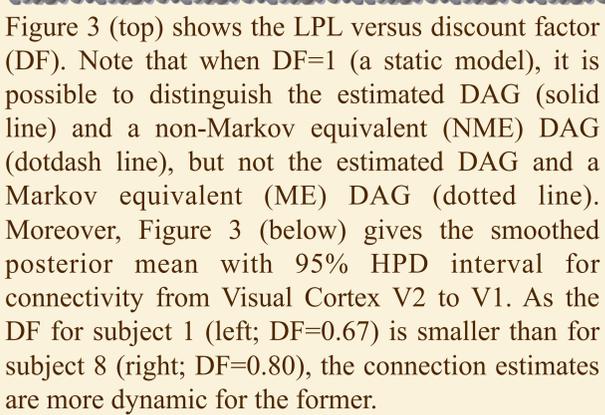
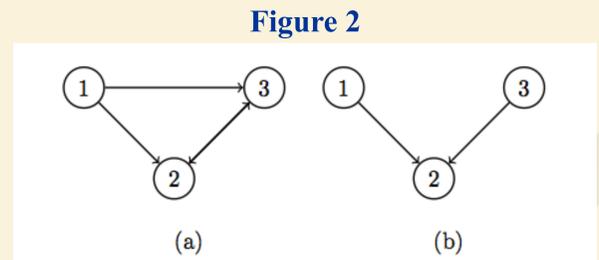
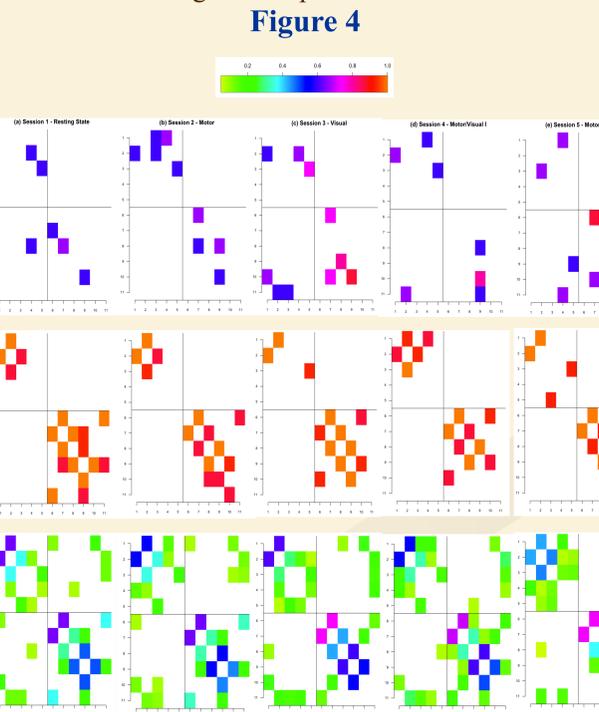


Figure 3

Figure 4 shows the prevalence of the edge $i \rightarrow j$, but only for those edges with significant Binomial tests after false discovery rate correction (FDR) at level $\alpha_{FDR} = 0.05$, where i indexes rows and j columns. The nodes are ordered according to the expected flow of information in the brain, and thus it is notable that we find significant edges between consecutive nodes. As expected, the significant MDM edges are a subset of the significant partial correlations.



The analysis of fMRI Data

We analysed a rich fMRI study that have information for 5 steady-state acquisitions or sessions: Session 1 is a resting-state condition; session 2 is a motor condition in which individuals tapped something; session 3 is a visual condition in which individuals watched a movie; session 4 and session 5 are a combination between visual and motor condition, but the former is in a random way whilst in the latter, individuals tapping depending on random events in the movie. Data were acquired on 15 subjects, and each acquisition consists of 230 time points, sampled every 1.3 seconds, with $2 \times 2 \times 2 \text{ mm}^3$ voxels. We use 11 ROI's defined on 5 motor brain regions and 6 visual regions. The motor nodes used are Cerebellum, Putamen, Supplementary Motor Area (SMA), Precentral Gyrus and Postcentral Gyrus (nodes numbered from 1 to 5 respectively) whilst the visual nodes used are Visual Cortex V1, V2, V3, V4, V5 and task negative ($v1+v2$; nodes numbered from 6 to 11 respectively). The observed time series are computed as the average of BOLD fMRI data over the voxels of each of these defined brain areas.

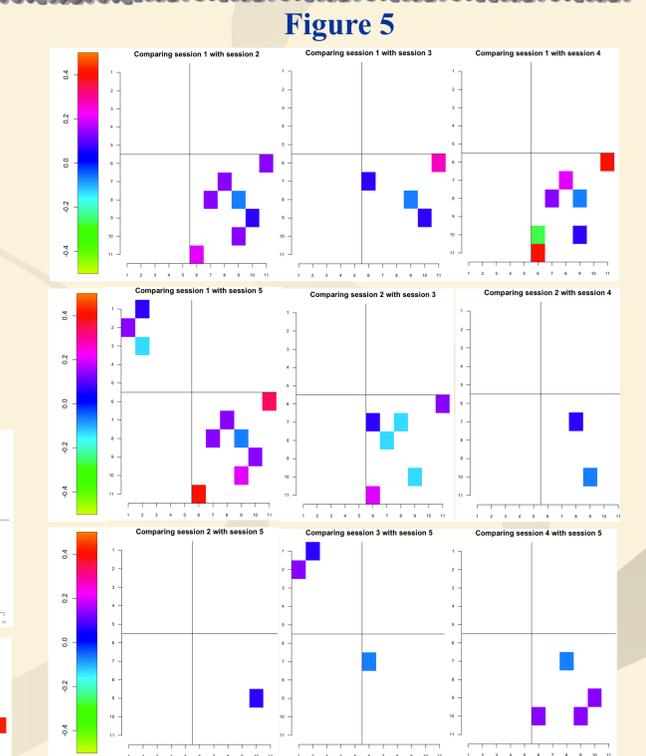


Figure 5 shows the significant difference of the proportion of subjects who have a particular edge for pairwise sessions using the MDM-DGM algorithm. Most of difference between session 1 and other sessions is positive and so connections exist in resting-state but not in other experimental condition. In general most of the difference between the sessions occurs in the connections between visual nodes (in the lower right square).

References:

Bartlett M., Cussens J. (2013). Advances in Bayesian network learning using integer programming. In *Proceedings of the 29th Conference on Uncertainty in Artificial Intelligence (UAI 2013)*. AUAI Press. To appear.

Costa L., Smith J.Q., Nichols T., Cussens J. (2013) Searching Multiregression Dynamic Models of Resting-State fMRI Networks using Integer Programming. *CRISM Res. Rep.* (Submitted to Bayesian Analysis Journal).

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