

# Notes on Creating a Standardized Version of DVARS

Thomas Nichols<sup>1</sup>

1 Warwick Manufacturing Group and Department of Statistics,  
University of Warwick, Coventry, CV4 7AL, U.K.

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## Abstract

By constructing a sampling distribution for DVARS we can create a standardized version of DVARS that should be more similar across scanners and datasets.

## 1 Introduction

Power et al. (2012) proposed a measure to characterize the quality of fMRI data, an image-wide summary that produces a time series that can detect scans that are corrupted by artifacts. They called their measure DVARS, the per-image standard deviation of the temporal derivative of the data. Since at least 2006 Matthew Brett's Data Diagnostics webpage<sup>1</sup> has offered `tsdiffana.m`, a Matlab script that produces the same measure. For simplicity, I'll stick with the snappier name DVARS.

While DVARS does an excellent job of detecting bad scans—bad pairs of scans actually—it does not have any absolute units. The average value of DVARS (on good data) will depend on the temporal standard deviation and the temporal autocorrelation of the data. The purpose of this short note is to describe a formal description of DVARS, its nominal standard deviation, which leads to a

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<sup>1</sup><http://imaging.mrc-cbu.cam.ac.uk/imaging/DataDiagnostics>, viewed 28 October, 2012; it lists the “last edited” data as 31 July 2006. See also a new implementation in the `spmttools` toolbox, <http://sourceforge.net/projects/spmttools>

standardized version of DVARS which should be more comparable and interpretable between sites and scanners.

## 2 Methods

Let  $Y_{i,t}$  be fMRI time series data, for voxels  $i = 1, \dots, I$  and time points  $t = 1, \dots, T$ . A reasonable, approximate model for well-behaved (outlier-free data) is

$$Y_{i,t} = \mu_i + \epsilon_{i,t} \quad (1)$$

where  $\mu_i$  is the constant, T2\* image of the brain and  $\epsilon_{i,t}$  is the noise. Of course this model is wrong, as it neglects the experimental variation, but we expect such effects to be trivial relative to the artefactual variation of interest. The issue of drift and temporal autocorrelation will be addressed shortly.

DVARS is based on the spatial standard deviation of the temporal difference image<sup>2</sup>:

$$\text{DVARS}_t = \sqrt{\frac{1}{I} \sum_i (Y_{i,t} - Y_{i,t-1})^2}. \quad (2)$$

$$= \sqrt{\frac{1}{I} \sum_i (\epsilon_{i,t} - \epsilon_{i,t-1})^2}. \quad (3)$$

That is, the magic of DVARS is that the differencing cancels out  $\mu_i$ , the T2\* brain.

The problem of predicting a null, default behavior of DVARS, however, is that it depends on the variance of spatial noise, and the spatial noise structure is complicated and hard to model in general. In the time domain, however, we have a reasonable working model of the noise, the Auto Regressive order-1 (AR(1)) model. If we can assume that the spatial and temporal noise structure doesn't interact<sup>3</sup>, then standardizing the noise variance in the time domain will result in unit variance in the spatial domain.

First, we need to state the AR(1) model and see how it predicts the variance of the temporal

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<sup>2</sup> For simplicity this sample variance doesn't include the term where the mean of the difference is subtracted out, since  $E(Y_{i,t} - Y_{i,t-1})$  should be zero by our model.

<sup>3</sup>Formally, "doesn't interact" means that the spatiotemporal correlation structure is separable into a product of spatial and temporal components. Again, for well-behaved data this is a reasonable assumption.

difference data. If each voxel's noise follows an AR(1) model then for voxel  $i$  we have

$$\text{Var}(Y_{i,t}) = \sigma_i^2 \quad (4)$$

$$\text{Corr}(Y_{i,t}, Y_{i,t-1}) = \rho_i. \quad (5)$$

These parameters are easily estimated as the usual standard deviation and  $\rho$  is even available from `fslmaths` with the `-ar1` option. To determine the variance of the temporal difference we need a basic result from probability: For two correlated random variables  $A$  &  $B$ , the variance of their difference is the sum of their variances minus twice their covariance:

$$\text{Var}(A - B) = \text{Var}(A) + \text{Var}(B) - 2\text{Cov}(A, B)$$

Thus if the time series follow an AR(1) model, the differenced times series have variance

$$\text{Var}(Y_{i,t} - Y_{i,t-1}) = 2\sigma_i^2 - 2\rho_i\sigma_i^2 \quad (6)$$

$$= 2(1 - \rho_i)\sigma_i^2. \quad (7)$$

This means we can predict the expected value of squared DVARS:

$$\text{E}(\text{DVARS}_t^2) = \frac{1}{I} \sum_i \text{E}((Y_{i,t} - Y_{i,t-1})^2) \quad (8)$$

$$= \frac{1}{I} \sum_i \text{Var}(Y_{i,t} - Y_{i,t-1}) \quad (9)$$

$$= \frac{1}{I} \sum_i 2(1 - \rho_i)\sigma_i^2. \quad (10)$$

This leads to the following revised definition of DVARS as

$$\text{DVARS}_t^* = \frac{\sqrt{\frac{1}{I} \sum_i (Y_{i,t} - Y_{i,t-1})^2}}{\sqrt{\frac{1}{I} \sum_i 2(1 - \rho_i)\sigma_i^2}}. \quad (11)$$

This is easily implemented because  $\sigma_i$  and  $\rho_i$  can be computed in `fslmaths`. Further, since we're worried about outliers, we can make use of robust estimators of  $\sigma_i$  and  $\rho_i$ . The simplest robust estimator of standard deviation is based on the Inter-Quartile Range (IQR), based on the following relationship for Normal variates:

$$\sigma = \text{IQR}/1.349$$

This is what I have implemented in my own DVARS script<sup>4</sup>. Unfortunately I haven't found a similar simple robust estimate for the AR(1) coefficient  $\rho_i$ , and thus have implemented the standard estimate.

Finally, with knowledge of the variance of the difference at each voxel  $i$ , we can also propose a new variant of DVARS based on voxel-wise standardized difference data:

$$\text{DVARS}_t^{**} = \sqrt{\frac{1}{I} \sum_i \left( \frac{Y_{i,t} - Y_{i,t-1}}{\sqrt{2(1 - \rho_i)\sigma_i^2}} \right)^2}. \quad (12)$$

However, this may not be as sensitive to problems because it will down-weight the voxels with high variance, i.e. those around the edge of the brain. On the other hand, since most of the edge-related variance is going to be due to motion and we already have the motion predictors,  $\text{DVARS}_t^{**}$  may be more useful for picking up problems that are *not* related to motion.

### 3 Discussion

The principal limitations of this work is that it depends on an estimates of standard deviation and AR-1 coefficient that are not themselves corrupted by bad data. Further work is needed to identify a robust estimator of  $\rho$ . Also, any sensible time series modelling effort begins by regressing out a linear from the data, and as fMRI is susceptible to drift, perhaps standard drift modelling should be done. As DVARS is driven by the most short-scale changes possible (from time  $t$  to  $t + 1$ ) it won't be affected by removal of drift, but it may result in more accurate modelling of the temporal correlation, and thus more accurate standardization.

Another limitation is that some users may *like* how DVARS reflects the underlying time series variance  $\sigma$ , and watch how the absolute value of DVARS changes with subsequent preprocessing steps. My response to this is that it is not just  $\sigma$  that changes but also the correlation, something that users may have less intuition on. And further, if the variance (and autocorrelation) are of interest, they ideally should be separately plotted and recorded, instead of indirectly inferred through DVARS.

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<sup>4</sup><http://go.warwick.ac.uk/tenichols/scripts/fsl/DVARS.sh>

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## **References**

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