Estimating Small Probabilities

Exact Approaches for Continuous Time Problems

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Overview

Estimating the probability of rare events, and characterising how such events may arise in a given system, is of widespread interest in fields ranging from finance to molecular biology. However, simulating such events is non-trivial as (by definition) such events are rare and so naïve approaches are computationally infeasible.

Recently, considerable advances have been made in addressing such problems [Cérou and Guyader, 2007, Cérou et al., 2012, Bréhier et al., 2017]. However, although existing approaches are widely applied to continuous time systems (such as diffusion processes) they almost invariably depend upon first discretising those systems. Unbiased estimation is, consequently, possible only when the underlying system admits an exact, tractable discrete-time description.

A rare exact approach to estimating rare event probabilities could in principle be obtained by combining ideas from existing rare event simulation methods with those developed for the exact simulation of diffusion processes [Pollock et al., 2016]. This is the direction of the proposed project.

"Exact Algorithms" are focused on characterising, in a finite dimensional manner and without approximation error, entire sample paths of diffusions. It transpires that the characterisation possible for some classes of diffusion enables the simulation of upper and lower bounding processes which almost surely constrain certain classes of diffusion sample paths to any specified tolerance. Such constructions can be used in principle to design algorithms to simulate unbiasedly whether trajectories are contained within sets of interest (or not) as illustrated in Figure 1 – and so could possibly be incorporated within the framework of existing techniques as a starting point in this project. However, many interesting practical and theoretical questions remain open on the feasibility of this approach, and would lead naturally onto future doctoral research.

Selected References

[Bréhier et al., 2017] Bréhier, C., Gazeau, M., Goudenège, L., Lelièvre, T., and Rousset, M. (In Press, 2017). Unbiasedness of some generalized Adaptive Multilevel Splitting algorithms. *Annals of Applied Probability*.

[Cérou et al., 2012] Cérou, F., Del Moral, P., Furon, T., and Guyader, A. (2012). Sequential Monte Carlo for rare event estimation. *Statistics and Computing*, 22(3):795–808.

[Cérou and Guyader, 2007] Cérou, F. and Guyader, A. (2007). Adaptive multilevel splitting for rare event analysis. *Stochastic Analysis and Applications*, 25(2):417–443.

[Pollock et al., 2016] Pollock, M., Johansen, A., and Roberts, G. (2016). On the exact and ε-strong simulation of (jump) diffusions. *Bernoulli*, 22(2):794–856.

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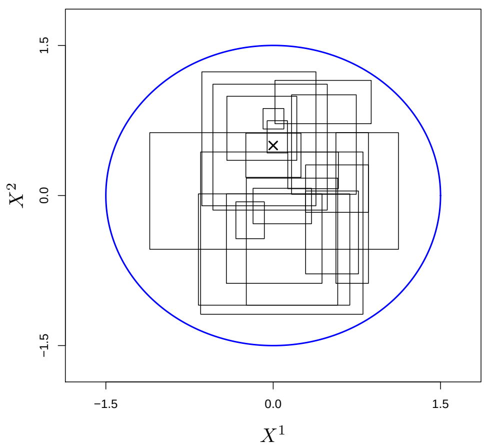
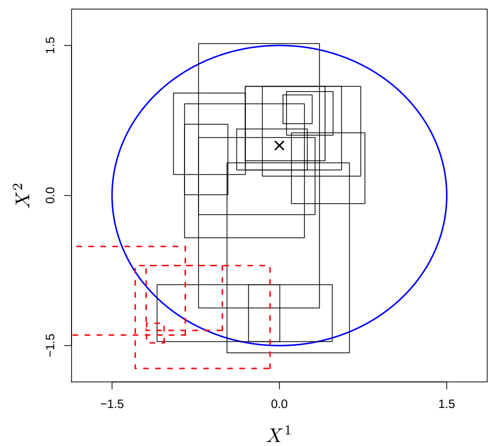


(a) Trajectory within set



(b) Trajectory not within set

Figure 1: Illustration of the determination of whether a 2-D diffusion sample path is contained within a circular set using a finite dimensional sample path skeleton. Inscribed rectangles denote regions where for some time interval sample paths are constrained. Black and infilled red rectangles denote intervals constrained entirely within or out-with the circle respectively. Dotted black and red rectangles denote intervals with undetermined or partial crossing respectively.