Discussion of the paper
Semiparametric Bernstein-von Mises theorem for LAE models under a mixture prior

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Discussion

I enjoy reading the paper!

Arnold and Press (JoE, 1983) about Pareto family:

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I would suggest a couple of references here:

- Tressou (BA, 2008) on BNP for heavy tailed distribution (application to food risk assessment).
- Liang (CSDA, 1993) on convergence rates for e-B, Seo and Song (AMM, 2019) on BNP, Li, Lin and Dunson (arXiv, 2018) on posterior consistency of tail index.

Exposition

Motivation

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Motivation

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2. You mention Pareto distribution. It would be nice to have a list densities satisfying the assumptions. E.g. Pareto I-IV, Weibull, Gumbel, Burr XII, Generalized Burr-Gamma,....

Localization

Change of variable and localization

Set $n(\theta - \theta_0) = h \in (-\infty, \zeta)$ and assume $-nM\epsilon_n \leq h \leq nM\epsilon_n$
$\Rightarrow$ from

$$\int_{-\infty}^{X_1 - t/n} e^{l_n(\eta_0,\theta)} J(\theta)\pi_\theta(\theta) d\theta$$

in Eq. 7 should be:

$$\int_{-\infty}^{\zeta - t} e^{l_n(\eta_0,\theta_0 + h/n)} J(\theta_0 + h/n)\pi_\theta(\theta_0 + h/n)\mathbb{I}(-nM\epsilon_n \leq h \leq nM\epsilon_n) dh$$
$$= \int_{-nM\epsilon_n}^{\min\{nM\epsilon_n, \zeta - t\}} e^{l_n(\eta_0,\theta_0 + h/n)} J(\theta_0 + h/n)\pi_\theta(\theta_0 + h/n) dh$$

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Is ϵ_n chosen as a function of ζ?

Bernstein von Mises

Choice of the prior

The author consider the prior process P

$$dP(\varepsilon) = \frac{p\varepsilon^2 dQ^{(0)}(\varepsilon) + (1-p)dQ^{(1)}(\varepsilon)}{p\int_0^\delta \varepsilon^2 dQ^{(0)}(\varepsilon) + (1-p)}$$

with

- $Q^{(0)} \sim DP(M^{(0)}, G^{(0)})$ with $G^{(0)}$ on $[0,\delta)$ and $Q^{(1)} \sim DP(M^{(1)}, G^{(1)})$ with $G^{(1)}$ on $[\delta,\infty)$
- $p/(1-\delta_n) \sim \mathcal{B}e(\alpha,\beta)$, with $\alpha,\beta \in (0,1)$

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1. Why do you need the ε^2? Also, can it be ε^4?
2. Have you a prior for $M^{(j)}, j=0,1$? Or do you keep it fixed?
3. Can you deal with PY priors?

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For the parameter p in the prior process

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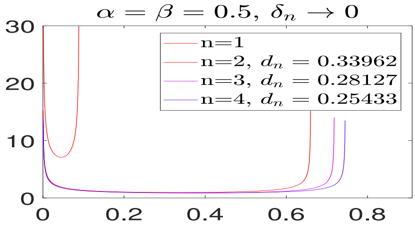

Figure: It is not clear to me why you need $\alpha, \beta \in (0,1)$?

Implementation

Gibbs sampling

The full conditional distribution of θ

$$f(\theta|...) \propto \prod_{i=1}^{n} (X_i - \theta)^{z-1} \exp\{\frac{z\theta}{\epsilon_{c_i}\xi_i}\} \mathbb{I}_{\{\theta \leq X_{(1)}\}} \pi(\theta) \qquad (1)$$

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What sampling method did you consider for θ?

Application

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Is the cumulative distribution of the cost G identified? Does identification assumptions of Theorem 1 in the Supplemnet of KS2011 apply to your model? I think you need a killer example here. What is the main advantage in applying your inference approach? Do you get different results?

Application and results

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In the context of auctions see Gugler, Weichselbaumer and Zulehner (EER, 2015) and references therein.