

# Discussion of “All About PEP” by Dimitris Fouskakis

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# Principles

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- ▶ What is the implied complexity prior induced by the Bayes factor? For example, in the case of PEP for the normal linear model, the expression for the BF can be used to derive this (using  $RSS_p = RSS_l$  and  $d_l = p - 1$ ) as a simple univariate integral, function of  $\delta$ . For comparison, in the “standard”  $g$ -prior setup, this penalty for an extra regressor is  $\sqrt{1 + g}$ .

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- ▶ What is the meaning of the truncation on the implied prior for  $g$ ? This can have practical consequences if it is truncated away from where most of the likelihood mass is. For example,  $g > n$  (for PEP in the normal linear model) is quite dramatic.

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  - ▶ Assigning zero prior probability to models for which  $p_j > n$ , which is the usual “informal” approach for  $g$ -priors
  - ▶ Mimicking formal approaches to use  $g$ -priors in situations where  $p > n$  such as Maruyama and George (2011) and Berger et al. (2016), based on different ways of generalizing the notion of inverse matrices?

Modelling with PEP

Sanity checks

large p

**Model Selection Consistency**

GLMs

Some further comments

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- ▶ Does consistency extend in a sensible way to the  $\mathcal{M}$ -open setting?

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- ▶ Li and Clyde (2018) use a truncated Compound Confluent Hypergeometric distribution on  $1/(1 + g)$ . How does this relate to the shifted generalized beta prime distribution proposed here (on  $g$ ) which implies a Beta truncated Pareto distribution for the shrinkage factor  $g/(1 + g)$ ? (covers the same special cases)

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## Some further comments

- ▶ Difference between PEP and PCEP seems to be conditioning on a common parameter, assigning a standard default prior to this. Why only condition on  $\sigma^2$  and not on  $(\beta_0, \sigma^2)$ ? Both are common and have convenient standard priors with nice properties.

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- ▶ Table 5 can be extended quite a bit. For example, the beta prime or gamma-gamma prior on  $g$  (possibly truncated) also covers the priors by
  - ▶ Maruyama and George (2011)
  - ▶ Bottolo and Richardson (2008) (right truncated)
  - ▶ horseshoe prior of Carvalho et al. (2010) (not consistent)
  - ▶ benchmark prior of Ley and Steel (2012)

and the original Zellner-Siow (1980) prior corresponds to an inverted gamma prior on  $g$ .