Modelling Production with Undesirable Outputs

Carmen Fernández[1], Gary Koop[2], and Mark F.J. Steel[2]

[1] School of Mathematics and Statistics, University of St. Andrews, St. Andrews, KY16 9SS, U.K.
[2] Department of Economics, University of Edinburgh, Edinburgh EH8 9JY, U.K.

Abstract: Many production processes yield both good outputs and undesirable ones (*e.g.* pollutants). In this paper, we develop a generalization of a stochastic frontier model which is appropriate for such technologies. We discuss efficiency analysis and, in particular, define technical and environmental efficiency in the context of our model. Methods for carrying out Bayesian inference are described and applied to a longitudinal (or panel) data set of Dutch dairy farms.

Keywords: Bayesian methods; Dairy farms; Efficiency; Environment; Longitudinal data; Markov chain Monte Carlo; Stochastic frontiers.

1 Introduction

Stochastic frontier models are commonly used in the empirical study of production technology and the efficiency of economic agents, such as firms, individuals or countries. The seminal papers in the field are Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977), while a survey is provided in Bauer (1990). The ideas underlying this class of models can be applied to production models, but also to cost frontiers (by suitably redefining the quantities involved). The discussion here will focus on production frontiers, which aim to capture the maximum amount of output that can be obtained from a given level of inputs. Thus, they describe the best-practice technology for turning inputs into output. In practice, actual output of an individual production unit may fall below the maximum possible. The latter deviation from the frontier is a measure of inefficiency and is the focus of interest in many applications. The introduction of measurement or specification error is required by the fact that we do not know where the frontier is situated and have to estimate it from the available data. This makes the frontier stochastic, hence the term "stochastic frontier model". The standard stochastic frontier model addresses the situation where only one output is produced, with a set of inputs. Inference with such standard stochastic frontier models can be done using classical or Bayesian approaches. In previous work, we have introduced and argued in favour of a Bayesian approach (see *e.g.* van den Broeck, Koop, Osiewalski and Steel, 1994). Some theoretical foundations for Bayesian analysis in stochastic frontier models are presented in Fernández, Osiewalski and Steel (1997) and an introductory survey of Bayesian methods in such models can be found in Koop and Steel (2000). Classical methods are discussed in *e.g.* Bauer (1990) or Horrace and Schmidt (1996). The present paper will take a Bayesian view.

An important extension of this framework is to allow for more than one type of output to be produced simultaneously. A Bayesian model for these multiple-output production processes is proposed in Fernández, Koop and Steel (2000a). The present paper further extends the above model in order to deal with situations where an individual unit produces undesirable outputs (such as pollution) as an inevitable by-product of the production of desirable outputs. In the application used in this paper, for example, Dutch dairy farms produce not only good outputs, such as milk, but also undesirable outputs, such as excessive nitrogen due to the application of manure and chemical fertilizers. It is thus important to understand the nature of the best-practice technology available to farmers for turning inputs into good and bad outputs. Furthermore, it is important to see how individual farmers measure up to this technology. In other words, evaluation of farm efficiency, both in producing as many good outputs and as few undesirable outputs as possible, is of interest. Here, we describe how extensions of stochastic frontier models can be used to shed light on these issues. We begin by explaining the generalization of the standard single-output stochastic frontier model to allow for several good outputs, following Fernández *et al.* (2000a). Next, we consider the more challenging case where some of these outputs can be undesirable. In the fourth Section, we shall briefly outline the prior used in the Bayesian model and the inference procedure used. Finally, we present some of the results for our empirical application involving Dutch dairy farms.

2 A Stochastic Frontier Model with Multiple Good Outputs

In Fernández *et al.* (2000a), we developed extensions of stochastic frontier models to allow for efficiency analysis in the presence of multiple outputs. Note that previous work with multiple outputs has often involved either having data on prices (*e.g.* in order to estimate a demand system) or on costs (*e.g.* in order to estimate a cost function). However, particularly in the case when some of the outputs are not sold in markets (*e.g.* pollution), such price or cost information is not available. Hence, it is important to develop methods which involve only output and input data.

The theoretical starting point in most analyses of multiple-output technology is a transformation function:

$$f(y, x) = 0,$$

where y is a vector of p good outputs and x is a vector of inputs. If the transformation function is separable then we can write it as:

$$\theta(y) = h_g(x).$$

In the present paper, we assume a constant elasticity of transformation form for $\theta(y)$, but the basic ideas extend to any form.

To establish some terminology, note that $\theta(y) = constant$ maps out the output combinations that are equivalent. Hence, it is referred to as the production equivalence surface, which is $(p-1)$-dimensional. By analogy with the single output case, $h_g(x)$ defines the maximum output (as measured by $\theta(y)$) that can be produced with inputs x and is referred to as the production frontier.

Since the empirical application used in the present paper involves (unbalanced) longitudinal or panel data, we assume that we have a set of NT observations corresponding to outputs of N different firms, where firm i is observed for time periods $t = 1, \ldots, T_i$. The output of firm i $(i = 1, \ldots, N)$ at time t $(t = 1, \ldots, T_i)$ is p-dimensional and is given by the vector $y_{(i,t)} = (y_{(i,t,1)}, \ldots, y_{(i,t,p)})' \in \Re^p_+$. We use the following transformation of the p-dimensional output vector:

$$\theta_{(i,t)} = \left(\sum_{j=1}^{p} \alpha_j^q \, y_{(i,t,j)}^q \right)^{1/q}, \tag{1}$$

with $\alpha_j \in (0,1)$ for all $j = 1, \ldots, p$ and such that $\sum_{j=1}^{p} \alpha_j = 1$ and with $q > 1$. For fixed values of $\alpha = (\alpha_1, \ldots, \alpha_p)'$, q and $\theta_{(i,t)}$, (1) defines a $(p-1)$-dimensional surface in $\Re^p_+$ corresponding to all the p-dimensional vectors of outputs $y_{(i,t)}$ that are technologically equivalent. In other words, (1) plots the production equivalence surface.

Given the transformation from the multivariate output vector $y_{(i,t)}$ to the univariate quantity $\theta_{(i,t)}$ (the parameters of which we estimate from the data), the basic problem of finding firm-specific efficiencies is essentially the same as in the single-output case. If we interpret the value $\theta_{(i,t)}$ as a kind of "aggregate output", and group these transformed outputs in an NT-dimensional vector

$$\log\theta = (\log\theta_{(1,1)}, .., \log\theta_{(1,T_1)}, \ldots, \log\theta_{(N,T_N)})', \tag{2}$$

we model $\log\theta$ through the following stochastic frontier model:

$$\log\theta = V\beta - Dz + \varepsilon_g. \tag{3}$$

In the latter equation, $V = (v(x_{(1,1)}), \ldots, v(x_{(N,T_N)}))'$ denotes an $NT \times k$ matrix of exogenous regressors, where $v(x_{(i,t)})$ is a k-dimensional function of the inputs $x_{(i,t)}$ corresponding to firm i at time t. The particular choice

of $v(\cdot)$ defines the specification of the production frontier: *e.g.* $v(x_{(i,t)})$ is the vector of an intercept and all logged inputs for a Cobb-Douglas technology, whereas a translog frontier also involves squares and cross products of these logs. The corresponding vector of regression coefficients is denoted by $\beta \in \mathcal{B} \subseteq \Re^k$. Often, theoretical considerations will lead to regularity conditions on β, which will restrict the parameter space $\mathcal{B}$ to a subset of $\Re^k$, still k-dimensional and possibly depending on x. For instance, we typically want to ensure that the marginal products of inputs are positive.
Technical inefficiency is captured by the fact that firms may lie below the frontier, thus leading to a vector of inefficiencies $\gamma \equiv Dz \in \Re_+^{NT}$, where D is an exogenous $NT \times M$ $(M \leq NT)$ matrix and $z \in \mathcal{Z}$ with $\mathcal{Z} = \{z = (z_1, \ldots, z_M)' \in \Re^M : Dz \in \Re_+^{NT}\}$. Through different choices of D, we can accommodate various amounts of structure on the vector γ of inefficiencies. For instance, taking $D = I_{NT}$, the NT-dimensional identity matrix, leads to an inefficiency term which is specific to each different firm and time period. For a balanced panel (*i.e.* $T_i = T, i = 1, \ldots, N$), $D = I_N \otimes \iota_T$, where ι_T is a T-dimensional vector of ones and $\otimes$ denotes the Kronecker product, implies inefficiency terms which are specific to each firm, but constant over time (*i.e.* "individual effects"). In our application we make the latter choice for D (but with the obvious generalization to an unbalanced panel). Since we are working in terms of $\log \theta$, the log of the aggregate output, the *technical efficiency* corresponding to firm i (at any period) will be defined as $\tau_{1i} = \exp(-z_i)$ where z_i is the appropriate element of z. For more discussion and alternative definitions of efficiency measures, see Fernández, Koop and Steel (2000b). The term ε_g in (3) is meant to capture all other influences, such as measurement or specification error, and is accordingly not restricted in its sign.
Stochastics will be introduced into the sampling model through distributions on z (which could, equivalently, be considered part of the prior) and ε_g. Here, we use a choice of D that makes $\mathcal{Z} = \Re_+^N$ and we assume independence across observations and between z and ε. In order to fully specify a likelihood function for the p-dimensional outputs when $p > 1$, we also introduce a distribution on the weighted output shares, defined as

$$\eta_{(i,t,j)} = \frac{\alpha_j^q y_{(i,t,j)}^q}{\sum_{l=1}^p \alpha_l^q y_{(i,t,l)}^q}, \; j = 1, \ldots, p, \tag{4}$$

In particular, we group them into $\eta_{(i,t)} = (\eta_{(i,t,1)}, \ldots, \eta_{(i,t,p)})'$, and assume independent sampling from

$$p(\eta_{(i,t)}|s) = f_D^{p-1}(\eta_{(i,t)}|s), \tag{5}$$

where $s = (s_1, \ldots, s_p)' \in \Re_+^p$ and $f_D^{p-1}(\cdot|s)$ is the p.d.f. of a Dirichlet distribution with parameter s.

3 A Stochastic Frontier Model with Good and Bad Outputs

Important issues in environmental policy hinge on multiple output production technologies where some of the outputs are undesirable. For instance, we have data on farms which produce good outputs (*e.g.* dairy products) for the market and undesirable outputs (pollutants). We will refer to undesirable outputs as "bads". Efficiency analysis using stochastic frontier models can be used to shed light on practical policy questions, involving both the goods and the bads. For instance, if we find dairy farms to be environmentally efficient then pollution can only be reduced by reducing production at dairy farms. However, if many dairy farms are highly environmentally inefficient, then by adopting best-practice technology pollution can be reduced without harming production of milk.

The question now arises as to how to adapt the analysis of the previous section to allow for undesirable outputs and both technical and environmental inefficiency. Following Fernández *et al.* (2000a), we make one particular adaptation which we argue is reasonable. Others are clearly possible, and these are a topic of past and current research. For instance, Koop (1998) and Reinhard, Lovell and Thijssen (1999) assume that undesirable outputs can be treated as inputs. Fernández *et al.* (2000b) adapt the aggregator function in (1) to accommodate bad outputs. Here, we model the good outputs as in the previous section, but add a second frontier for the bad outputs. Environmental efficiency is then measured relative to this second frontier.

If we let b indicate a vector of m bad outputs, the most general description of best-practice technology is given by:

$$f(y, x, b) = 0.$$

We assume this transformation function can be broken down into:

$$\theta(y) = h_g(x),$$

and

$$\kappa(b) = h_b(y).$$

In other words, the general transformation function can be broken down into two equations involving a "goods production equivalence surface" $\theta(y)$, a "goods production frontier" $h_g(x)$, a "bads production equivalence surface" $\kappa(b)$, and a "bads production frontier" $h_b(y)$. The assumption that the amount of good outputs produced depends on the inputs, while production of bad outputs depends on the amount of good outputs is likely to be reasonable in many cases. If not, modifications of the present model can be implemented.

We begin with the model for the good outputs described in the previous section given by equations (1)-(5). We further let $b_{(i,t)} = (b_{(i,t,1)}, \ldots, b_{(i,t,m)})'$ be the vector of m bad outputs for firm i in period t. We define the environmental production equivalence surface through a similar constant elasticity of transformation form:

$$\kappa_{(i,t)} = \left(\sum_{j=1}^{m} \gamma_j^r \, b_{(i,t,j)}^r \right)^{1/r}, \tag{6}$$

with $\gamma_j \in (0,1)$ for all $j = 1, \ldots, m$ and such that $\sum_{j=1}^{m} \gamma_j = 1$ and with $0 < r < 1$.

Environmental inefficiency is measured using a stochastic frontier model with (6) as dependent variable. That is, we define $\log \kappa$ similarly to $\log \theta$ and set

$$\log \kappa = U\delta + Mv + \varepsilon_b \tag{7}$$

where $U = (u(y_{(1,1)}), \ldots, u(y_{(N,T_N)}))'$ is a function of the good outputs. U plays a similar role to V in equation (3) and, hence, the particular choice of $u(\cdot)$ defines the specification of the bads production frontier. Environmental inefficiencies are given by $Mv \in \Re_+^{NT}$. M plays an analogous role to D in the previous section and here we set $M = D$ which implies that the technical and environmental efficiency of each firm is constant over time. Thus, *environmental efficiency* of firm i will be defined as $\tau_{2i} = \exp(-v_i)$.

To complete the sampling model, we shall introduce the following distributional assumptions. We link both frontiers by joint distributions on the inefficiency error terms and on the measurement error terms, while still retaining independence across observations. In particular, we assume a bivariate Normal distribution for $(\varepsilon_g, \varepsilon_b)$. That is, if we let $f_N^R(\varepsilon|a, A)$ denote the R-variate Normal p.d.f. with mean a and covariance matrix A, evaluated at ε, we take:

$$p(\varepsilon_g, \varepsilon_b|\Sigma) = f_N^{2NT} \left(\begin{array}{c} \varepsilon_g \\ \varepsilon_b \end{array} \Big| 0, \Sigma \otimes I_{NT} \right) \tag{8}$$

where Σ is a 2×2 P.D.S. matrix.

For the inefficiency error terms, we adopt a similar strategy, except that these have to be nonnegative. We assume independence between firms and for each $i = 1, \ldots, N$, we take a truncated Normal inefficiency distribution:

$$p(z_i, v_i|\mu, \Omega) = f_N^2((z_i, v_i)'|\mu, \Omega) f^{-1}(\mu, \Omega) I_{\Re_+^2}(z_i, v_i), \tag{9}$$

where $f(\mu, \Omega)$ is the integrating constant of the truncated Normal and $I_{\Re_+^2}(\cdot)$ is the indicator function for $\Re_+^2$.

Finally, we define a weighted vector of shares for the bads:

$$\zeta_{(i,t,j)} = \frac{\gamma_j^r b_{(i,t,j)}^r}{\sum_{l=1}^{m} \gamma_l^r b_{(i,t,l)}^r}, \; j = 1, \ldots, m, \tag{10}$$

stack them to form $\zeta_{(i,t)} = (\zeta_{(i,t,1)}, \ldots, \zeta_{(i,t,m)})'$, and assume independent sampling from

$$p(\zeta_{(i,t)}|h) = f_D^{m-1}(\zeta_{(i,t)}|h), \tag{11}$$

where $h = (h_1, \ldots, h_m)' \in \Re_+^m$

4 The Prior and Bayesian Inference

In the previous Sections, we have defined the sampling model, which depends on the parameters $(\beta, \delta, \Sigma, \mu, \Omega, \alpha, \gamma, q, r, s, h)$. We shall use the proper prior structure

$$p(\beta, \delta, \Sigma, \mu, \Omega, \alpha, \gamma, q, r, s, h) = p(\beta, \delta, \Sigma)p(\mu, \Omega)p(\alpha)p(\gamma)p(q)p(r)p(s)p(h)$$

The prior we assume on these parameters is chosen to be rather noninformative, except that we restrict β and δ to their respective regularity regions, and we impose that $q > 1$ and $0 < r < 1$, again for economic theory considerations.
In particular, we take an Inverted Wishart prior on Ω:

$$p(\Omega) = f_{IW}^2\left(\Omega|\Omega_0, \nu_0\right), \tag{12}$$

combined with

$$p(\mu|\Omega) = f_N^2(\mu|0, c\Omega), \tag{13}$$

Fernández, Koop and Steel (1999) report some simulation exercises to calibrate the prior of (μ, Ω) (*i.e.* choose values for the hyperparameters in (12) and (13)) so as to induce a reasonable prior on the efficiencies τ_{1i} and τ_{2i}.
For the other parameters we assume

$$p(\beta, \delta, \Sigma) \propto f_N\left(\begin{array}{c}\beta \\ \delta\end{array} |b_0, H_0^{-1}\right) f_{IW}^2(\Sigma|\Sigma_0, \lambda_0) I_{\mathcal{RR}}(\beta, \delta, \Sigma), \tag{14}$$

where $\mathcal{RR}$ indicates the regularity region,

$$p(\alpha) = f_D^{p-1}(\alpha|a_0), \tag{15}$$

$$p(\gamma) = f_D^{m-1}(\gamma|g_0), \tag{16}$$

$$p(q) \propto f_G(q|1, q_0) I_{(1,\infty)}(q), \tag{17}$$

where $f_G(.|a, b)$ denotes a Gamma density function with shape parameter a and mean a/b (if $a = 1$, we have an Exponential),

$$p(r) \propto f_G(r|1, r_0) I_{(0,1)}(r), \tag{18}$$

$$p(s) = \prod_{j=1}^{p} f_G(s_j|1, k_j), \tag{19}$$

and, finally,

$$p(h) = \prod_{j=1}^{m} f_G(h_j|1, n_j). \tag{20}$$

We adopt noninformative choices for the hyperparameters in (14)-(20).
The resulting posterior from combining the sampling model with the prior just described does not lend itself to immediate analytical analysis. Instead, we shall use a Markov chain Monte Carlo (MCMC) algorithm on the space of the parameters augmented with the inefficiencies (z, v). The Markov chain will be constructed from Gibbs steps for $(z, v), (\beta, \delta), \Sigma$, where we can draw immediately from the conditionals, and Normal random walk Metropolis samplers for $\Omega, \mu, \alpha, \gamma, q, r, s, h$, since the conditionals for the latter do not have a well-known form. We fine-tune results from preliminary runs in order to select the variance for the increments in the random walk Metropolis samplers. The relevant conditional posterior distributions are described in detail in Fernández *et al.* (1999).

5 An Application to a Panel of Dutch Dairy Farms

We apply the model described in the previous Sections to a data set involving $N = 613$ Dutch dairy farms for the years 1991-94. It is an unbalanced panel with a total number of observations $NT = 1545$. For each farm, we have data on $p = 2$ good outputs, $m = 1$ bad output and 3 inputs:

- Good outputs: Milk (millions of kg) and Non-milk (millions of 1991 Guilders).
- Bad output: Nitrogen surplus (thousands of kg).
- Inputs: Family labor (thousands of hours), Capital (millions of 1991 Guilders) and Variable input (thousands of 1991 Guilders).

Variable input includes *inter alia* hired labor, concentrates, roughage and fertilizer. Non-milk output contains meat, livestock and roughage sold. The definition of capital includes land, buildings, equipment and livestock. Further detail on this data set is given in Reinhard *et al.* (1999).
Both the goods and bad production frontiers are here assumed to take Cobb-Douglas forms. A more detailed discussion of the empirical results can be found in Fernández *et al.* (1999).
Table 1 provides some characteristics of the posterior distribution. Note that the column labelled "Median" is the posterior median. The columns labelled "2.5%" and "97.5" are the 2.5% and 97.5% percentiles, respectively of the posterior distribution. "RTS" means returns to scale, which indicates the relative increase in aggregate output expressed as a fraction of a relative increase in all inputs (or good outputs for the bads frontier).

We also summarize results for the technical and environmental efficiencies of a typical or average farm, τ_{1f} and τ_{2f}. The latter results correspond to a predictive out-of-sample efficiency distribution, obtained by integrating out the distribution in (9) with the posterior of (μ, Ω). Our model allows for technical and environmental efficiencies to be correlated with one another and that correlation is evaluated at 0.25, indicating that there is a slight tendency for technically inefficient farms to also be environmentally inefficient.

Table 1: Posterior Results for Dutch Dairy Farm Data Set

	Median	2.5%	97.5%
β_1(Intercept)	-3.533	-3.694	-3.226
β_2(Labour)	0.120	0.090	0.150
β_3(Capital)	0.537	0.504	0.572
β_4(Variable)	0.487	0.463	0.509
RTS (Goods)	1.145	1.115	1.173
δ_1(Intercept)	2.578	2.262	2.890
δ_2(Milk)	0.889	0.858	0.921
δ_3(Non-milk)	0.081	0.065	0.098
RTS(Bads)	0.971	0.940	1.001
q	1.004	1.000	1.019
α_1	0.534	0.510	0.565
τ_{1f}	0.620	0.415	0.880
τ_{2f}	0.345	0.198	0.599

All results seem reasonably in accordance with economic intuition. Some of the more interesting results are:

- Firms tend to be more efficient technically than environmentally. In fact, the posterior median of the environmental efficiency for a typical farm is only 0.345, indicating that the typical farm produces roughly three times as much nitrogen surplus as would be consistent with best practice! Using the same data, Reinhard *et al.* (1999) and Fernández *et al.* (1999), who both use a single-frontier model, find similar results.
- The small positive correlation between both types of efficiencies indicates that farms which tend to be less efficient technically also tend to be less efficient environmentally. In contrast, the single frontier analysis of Fernández *et al.* (1999) finds a moderately negative correlation.
- However, there is a large spread of efficiencies across farms, which manifests itself in large differences between the 2.5 and 97.5th percentiles of both technical and environmental efficiencies.

- Rather than conducting inference on the efficiency for a typical (unobserved) farm, we can also conduct inference on farm-specific efficiencies. Given that we have observed these farms, their efficiencies are less dispersed, and can lead to a ranking of firms, in the sense that *e.g.* the efficiencies of quartile firms (in the efficiency ranking) are quite well separated, and the posterior probability of these firms being reversed in the ranking is very low.

- Increasing returns to scale seem to exist for the production of good outputs, while slightly decreasing returns exists for bad output production.

- The elasticity of nitrogen production with respect to milk production (δ_2) is much larger than the elasticity with respect to non-milk production (δ_3). This finding indicates that it is the milk production side of dairy farming that is most associated with the production of nitrogen.

We hesitate to draw policy conclusions based solely on this one set of empirical results for one model specification. However, to illustrate the types of issues that our model can be used to address, we offer the following comments. The relatively large degree of environmental inefficiency indicates that pollution can be reduced in many farms at little cost in terms of foregone output. That is, if inefficient farms were to adopt best-practice technology and move towards their environmental production frontiers, production of pollutants could be reduced at no cost to milk or non-milk production. The positive correlation between the two types of efficiencies indicates that improving environmental efficiency could be associated with improvements in technical efficiency. Hence, policies aimed at improving efficiency (*e.g.* by educating farmers in best-practice technology) could have large payoffs. Furthermore, the pattern of returns to scale results indicate that larger farms have advantages. Hence, policies which promote rationalization of farms (*e.g.* encouraging larger farms to purchase smaller farms) could result both in more production of milk and non-milk outputs (due to increasing returns to scale in the good production frontier) and less pollution (due to decreasing returns in the environmental production frontier).

6 Conclusions

In this paper, we have shown how the standard stochastic frontier model with a single output can be extended to multiple outputs where some of the outputs are undesirable. The model we develop can be used to model production technologies which produce *e.g.* pollutants. The empirical application to Dutch dairy farms shows the practicality of this approach and highlights some important policy issues which our model can address.

7 References

Aigner, D.; Lovell, C.A.K. and Schmidt, P. (1977). Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics*, **6**, 21-37.

Bauer, P. (1990). Recent developments in the econometric estimation of frontiers. *Journal of Econometrics*, **46**, 39-56.

van den Broeck, J., Koop, G., Osiewalski J. and Steel, M.F.J. (1994). Stochastic frontier models: A Bayesian perspective. *Journal of Econometrics*, **61**, 273-303.

Fernández, C.; Koop, G. and Steel, M.F.J. (1999). Multiple output production with undesirable outputs: An application to nitrogen surplus in agriculture. manuscript.

Fernández, C.; Koop, G. and Steel, M.F.J. (2000a). A Bayesian analysis of multiple output production frontiers. *Journal of Econometrics*, forthcoming.

Fernández, C.; Koop, G. and Steel, M.F.J. (2000b). Alternative efficiency measures for multiple output production. manuscript.

Fernández, C.; Osiewalski, J. and Steel, M.F.J. (1997). On the use of panel data in stochastic frontier models with improper priors. *Journal of Econometrics*, **79**, 169-193.

Horrace, W. and Schmidt, P. (1996). Confidence statements for efficiency estimates from stochastic frontiers. *Journal of Productivity Analysis*, **7**, 257-282.

Koop, G. (1998). Carbon dioxide emissions and economic growth: A structural approach. *Journal of Applied Statistics*, **25**, 489-515.

Koop, G.; and Steel, M.F.J. (2000). Bayesian analysis of stochastic frontier models. Forthcoming in B. Baltagi, ed., *Companion in Theoretical Econometrics*, Basil Blackwell, Oxford.

Meeusen, W. and van den Broeck, J. (1977). Efficiency estimation from Cobb-Douglas production functions with composed errors. *International Economic Review*, **8**, 435-444.

Reinhard, S.; Lovell, C.A.K. and Thijssen, G. (1999). Econometric estimation of technical and environmental efficiency: An application to Dutch dairy farms, *American Journal of Agricultural Economics*, **81**, 129-153.

Acknowledgments: We acknowledge funding by the ESRC under grant number R34515 and we thank Stijn Reinhard and the Dutch Agricultural Economics Research Institute (LEI-DLO) for providing the data.