SEQUENTIAL MONTE CARLO FOR FREDHOLM EQUATIONS OF THE FIRST KIND

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Fredholm Integral Equations

$$a(y) = \int g(y \mid x) b(x) \, dx \qquad (1)$$

describe problems in science where the output data a are distorted observations from the object of interest b.

Solutions of (1) are difficult to find, but good approximations can be found iteratively

$$b_n(x) = \int K(x, x') b_{n-1}(x') \int \frac{g(y|x') a(y)}{\int b_{n-1}(x'') g(y|x'') \, dx''} \, dy \, dx'. \qquad (2)$$

Positron Emission Tomography

Positron Emission Tomography (PET) is a medical diagnosis technique that is used to analyse internal biological processes to detect medical conditions such as schizophrenia, cancer, Alzheimer's and coronary artery disease [2].

► PET scanners give data a on the position and on the quantity of a radioactive tracer injected intravenously.

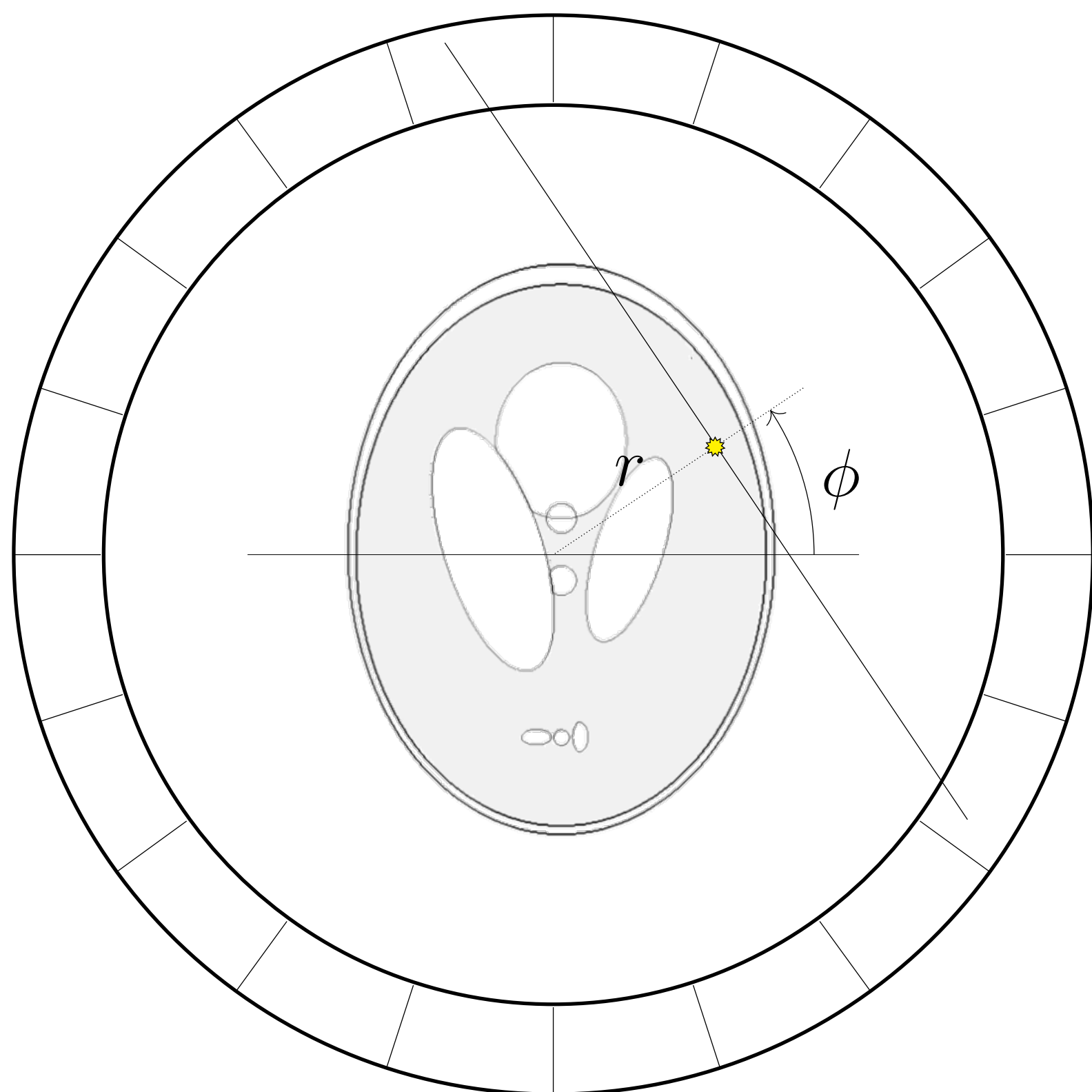► The physics of the scanner is described by g.

Fig. 1: Physics of PET. PET scanners are formed by detectors that count simultaneous gamma-ray emissions. The pair of detectors registering the emission gives the position (angle ϕ and distance from the centre r) and density of the tracer in the organ.

► Solving equation (1) gives a cross-section image b of the organ.
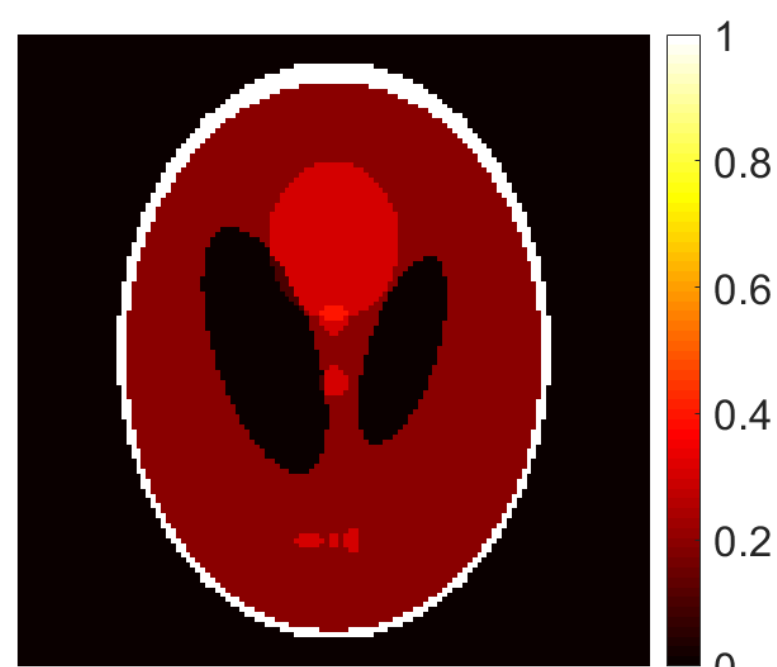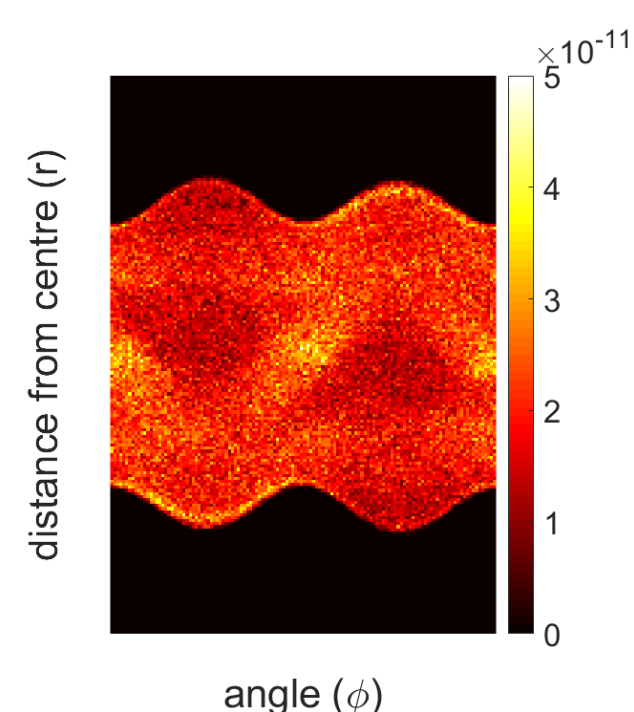

Fig. 2: Reference Image   Fig. 3: PET Data

The data used in this work (Figure 3) are obtained from the reference image in Figure 2, a simplified imitation of the brain's metabolic activity (e.g. [3]).

Sequential Monte Carlo

Sequential Monte Carlo (SMC) is a statistical approximation method often used to recover a signal from a set of noisy observations [1]. The signal is approximated by a population evolving in time. At each time step:

► individuals randomly mutate;

► mutations that produce fitter individuals are more likely to survive (selection);

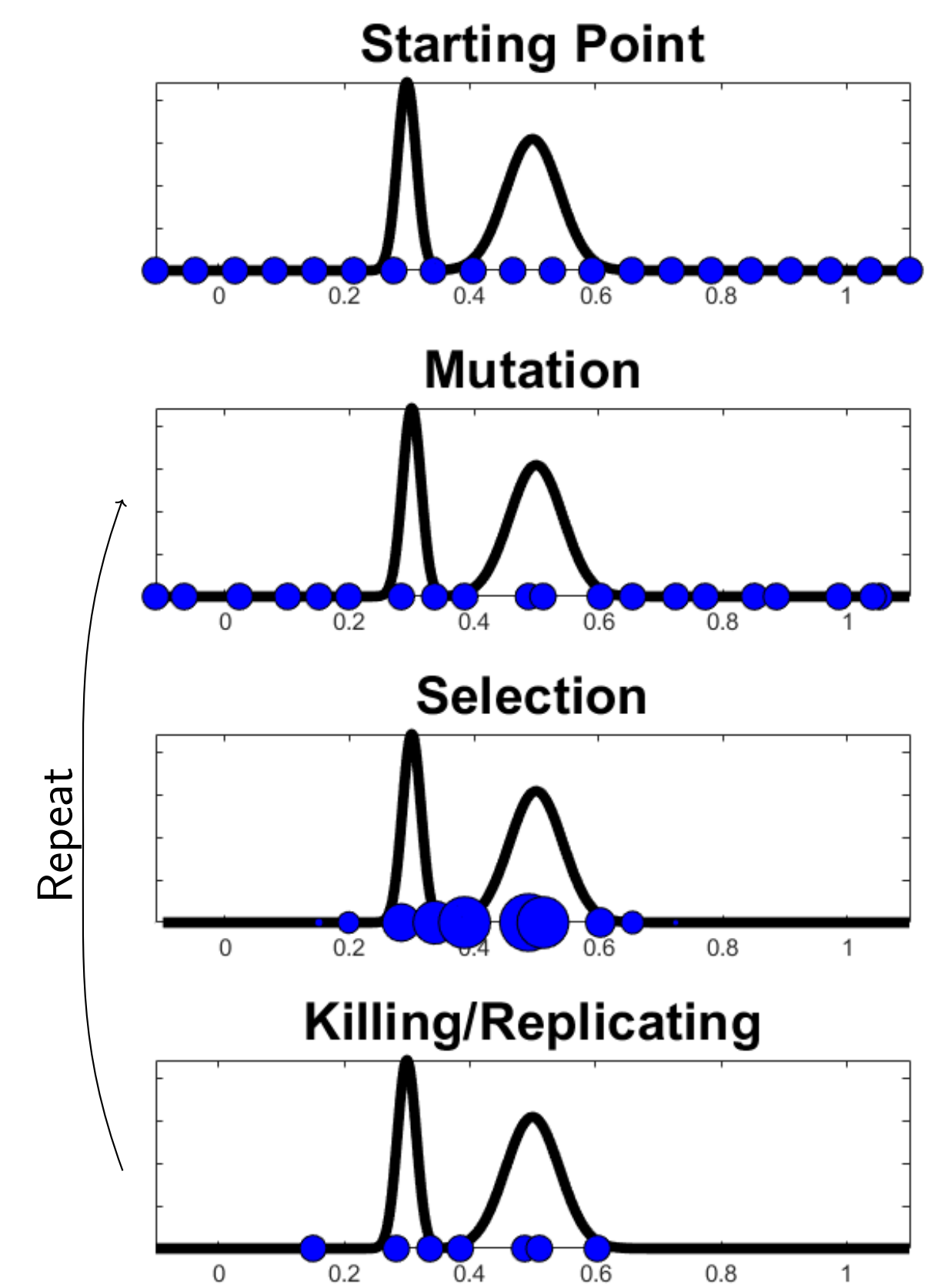► a new population is originated, fitter mutations replicate, while the other individuals die out.

Fig. 4: Schematic representation of SMC

Sequential Monte Carlo for PET

An SMC implementation of (2) using the data in Figure 3 as input a gives a sequence of increasingly-good approximations of the reference image in Figure 2:
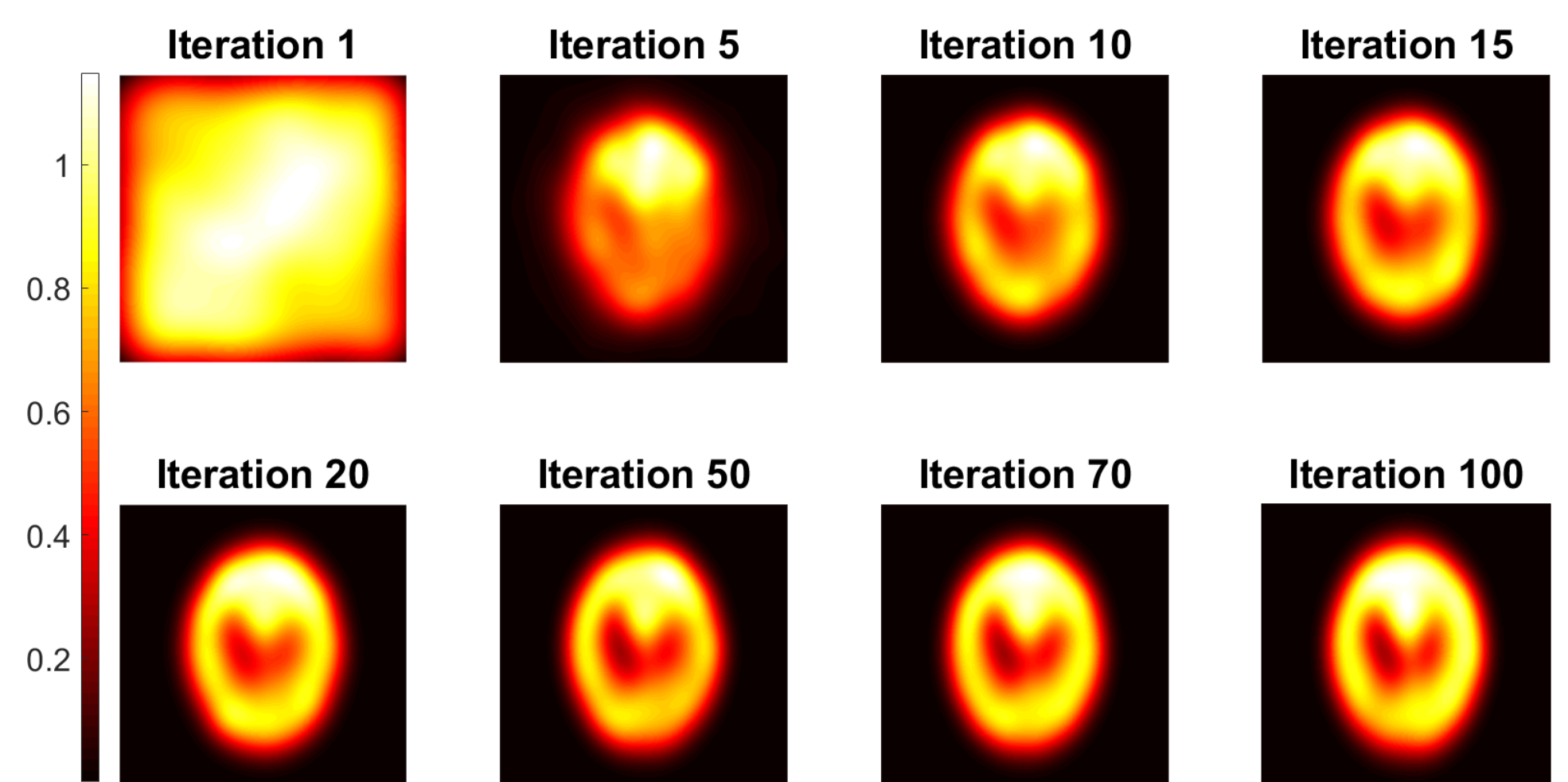

Fig. 5: 100 steps of the reconstruction process through SMC.

References

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