An Intransitive ‘Better Than’

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1 Abstract

Many hold that the relation ‘better than’ is transitive. In this paper I argue for an intransitive notion of ‘better than’ in competitive sport. This example is used to explain how we might understand arguments for transitivity based on monotonicity and semantics. I go on to suggest how these considerations may be expanded into the moral realm through the development of a novel class of betterness cycles, so-called unambiguous arguments, distinct from the well-known spectrum arguments.
2 Introduction

Many hold that the relation ‘better than’ is transitive, such that for any value bearers $x$, $y$, and $z$, if $x$ is better than $y$, and $y$ is better than $z$, then $x$ is better than $z$. The spectrum arguments developed by Rachels (1998) and Temkin (1987, 1996, 2014) have provoked an active discussion on the transitivity of ‘better than’. Many responses to the spectrum arguments have considered aspects of incommensurability, incomparability, or vagueness. Others have focused on the particular structure of the spectrum arguments, comparing them to Zeno’s or sorites paradoxes, or questioning the intuitions of the pairwise judgements. In this paper I aim to avoid these considerations by using examples where both the attributes of alternatives and the comparisons of those alternatives are clearly defined, quantitative and independent of any similarity-based reasoning.

I use these examples to address two particular arguments in support of the transitivity of better than. First an argument based on a monotonicity principle, roughly if $x$ is $Q$ and, $y$ is $Q_r$ than $x$, then $y$ is $Q$. I do not seek to refute monotonicity principles, but to clarify their appropriate application and hence to demonstrate why we might not conclude that they must lead to the rejection of intransitivity in the relation ‘better than’. Second an argument based on semantics. As Broome (2004) notes “Some authors write as though the transitivity of betterness is an issue in ethics. It is not; it is an issue in semantics.” I do not here argue whether this claim is correct, but instead argue that even if we allow this, then there is reason to hold that ‘better than’ may be intransitive. In doing this I argue for ‘all things considered better than’ as the dyadic predicate for ‘good’ rather than the transitive-implying understanding of ‘goodness’ as the monadic predicate for ‘all things considered better than’.

Competitive sport provides a useful context for my main working example, as one may appeal to a strong understanding of the context and its features, and those may reasonably be represented and compared quantitatively. While others have addressed the domain of competitive sport (Broome, 2004; Sugden, 1985), my aim in this paper is to demonstrate that a more careful consideration of the case can be illuminating for our understanding of ‘better than’. Additionally I make a suggestion as to how the arguments made in the context of competitive sport might have relevance in the moral realm by suggesting a family of betterness cycles, so called unambiguous arguments, which take as their starting point incontrovertibly intransitive relations.
The paper proceeds in Section 3 with a discussion of what we might understand by ‘better than’ and more particularly ‘all things considered better than’ in the context of competitive sports. In Section 4 the derived intransitive notion is used to discuss the apparent paradox created by holding simultaneously a general monotonicity principle and an intransitive notion of ‘better than’. In Section 5 I discuss the semantic justification for the transitivity of ‘better than’, using ‘later than’ and my notion of ‘better than’ in competitive sport as counterexamples. In Section 6 I consider how the arguments considered in Sections 4 and 5 may have relevance to the moral realm. In the final section I provide some short concluding remarks.

3 ‘Better than’ in competitive sport

The aim of competitive sport is to win. When situations arise that compromise this, for example when a team losing in a preliminary group stage of a tournament is likely to provide them easier progress in the knock-out stages, stakeholders object to the circumstance because it transgresses this principle. Since this is the sole aim of a competitor then an acceptable ‘better than’ relation in a competitive sports context need only be consistent with the principle that winning is better than losing. So for example, I may not claim that Arsenal are a better soccer team than Manchester United because they play more attractively (unless I also claim that the attractiveness of the soccer is the sole determinant of winning). I may claim that Arsenal are (or were) better than Manchester United because they have beaten them, because they have a better record against equivalent opposition, or because they would be expected to beat them, since all of these respect the primacy of winning over losing. As such, multiple definitions of ‘better than’ that may reach different conclusions are permissible. For the purposes of better understanding the nature of some of these, and providing a basis for later widening the scope of the arguments from ‘better than’ to ‘all things considered better than’, it is useful to examine the claims that some of these may have on being an ‘all things considered better than’ notion for competitive sport.

To begin, let us take the ‘better than’ relation ‘has beaten’. It is noted that it is clearly not necessarily transitive. It is common in sports for team A to have beaten team B, team B to have beaten team C, and team C to have beaten team A. One may make at least four objections to using this
as a notion of ‘all things considered better than’. First this outcome may be due to some arbitrariness in the particular instantiation. Upsets are a common feature in sport, and even where one had two teams of the same ability one would expect a result, with many sports not allowing for tied outcomes. This is so much a part of sport that the refrain “may the best team win” is commonplace on the eve of matches. Second, even if one takes a more deterministic approach and denies the possibility of such arbitrariness, one may still point out that this is an outcome under a particular set of external variables. For example weather conditions and officiating decisions may be considered to have an impact on a match outcome and are beyond the attributes of the teams. Third it is possible that team A has beaten team B, and team B has beaten team A. There is something especially uncomfortable in the extreme intransitivity of stating that simultaneously A is better than B and B is better than A. Fourth this is an instantiation from a particular point in the past. To the degree that it may be considered an ‘all things considered better than’ relation, we would need to determine to what time period it applies. It seems reasonable to claim that it can only pertain to the point in the past that the match occurred, and as such cannot be used to determine the present state for example.

An alternative relation would be the predominant ‘better than’ relation used in league sports, namely ‘has aggregated more wins against all other competitors’. The ubiquity and uniformity with which this notion is used might suggest that it forms a commonly accepted ‘all things considered better than’ notion. But there is no reason to believe that because this is the predominant methodology used in the production of league tables that it is expected to correspond to an ‘all things considered better than’ notion. There are several goals of a league tournament, including the creation of a ranking for competitors, making that ranking methodology transparent to stakeholders, and identifying the best team. There is no a priori reason to believe all (or any) of these are possible simultaneously. Definitionally to produce a ranking one requires a transitive notion of ‘better than’. So even if there were an agreed upon notion of ‘all things considered better than’, if it were not transitive then it could not be used for this purpose. As such the requirement for the ‘better than’ notion to be transitive would dominate the requirement to use an ‘all things considered better than’ notion, and so it would be mistaken to conclude that usage necessarily indicates an ‘all things considered’ notion. So we must instead look at the attributes of such a relation. A number of objections may be raised to using this as an ‘all things
considered better than’ notion.\textsuperscript{2} Let us consider the first two objections raised previously, those of random instantiations and external conditions. While the impacts of these are in a sense averaged over the outcome of all matches, these are reduced but not eliminated. Statistics tells us that as the number of matches increases, the biasing effects of chance outcomes or circumstances on any team are reduced but they do not disappear. Third, the consideration of time becomes yet more unclear with an aggregation of outcomes from different time points, so that we cannot even relate the ‘better than’ relation to a particular point in the past. Fourth it may not be that we have a complete set of matches on which to compare teams, for example at the mid-point of a season, or in the case of school or college sports where fixtures are often bilaterally arranged.

This may then encourage us to consider the notion ‘would be expected to aggregate more wins against all other competitors’. In referencing expected rather than actual instantiations, we are considering this claim over all possible conditions and randomness in line with the probability that they occur, and additionally it may be referenced to any particular point in the past, present or future by using variants ‘was/is/would be expected to aggregate more wins against all other competitors’ and so these objections are avoided. Under this proposal we might think of this use of expectation as what accounts for ‘all things considered’. However this has the uncomfortable feature that in a multi-team tournament the conclusion of whether A is better than B depends on the other teams C, D etc. so that whether A is considered better than B may change without A and B themselves changing. It is also, as Smead (2019) points out, a non-unique potential aggregation of results, with different aggregation methods giving different ‘better than’ relations.

So let us consider the relation ‘expected to beat’. None of the objections mentioned so far pertains to this — it is not subject to arbitrary outcomes or circumstances due to its recourse to expectation, the relevant time may be expressed precisely, it cannot display extreme intransitivity, it is dependent solely on the qualities of the two teams being compared, and it has a unique determination. It is also, importantly for some, likely to be what one thinks when one is told that “A is better than B”. This then, in the context of competitive sport, seems to be a plausible definition of ‘all things considered better than’. As we go on to show though, this definition turns out to not be necessarily transitive. This is illustrated with the following example.

\textbf{The Intransitive Football Teams}
Take three American football teams A, B and C. Let us suppose that their important qualities may be summarised by their offensive and defensive ability in their running and passing game, and that points are expected to be scored in a monotonically increasing way with the difference in the strength of a team’s chosen offence and the opposition’s corresponding defence. We also assume that they know their opposition’s qualities, presumably having reviewed their past matches, so they will choose a running or passing offense dependent on where they have greatest advantage. Let us suppose that their qualities are as in Table 1.

<table>
<thead>
<tr>
<th>Team</th>
<th>Run Offense</th>
<th>Run Defense</th>
<th>Pass Offense</th>
<th>Pass Defense</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>+1</td>
<td>-2</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>C</td>
<td>-1</td>
<td>-1</td>
<td>+2</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Qualities of teams A, B and C

As the coach of team A, you wish to select your most effective offense. You note that team B’s run defense is rated at −2 and so with your run offense, rated at 0, you have a net advantage of 2 (= 0 − (−2)) if playing a run offense. Your pass offense is rated at 0 and your opposition’s pass defense is rated at +1 and so you have a net advantage of −1 (= 0 − (−1)) if playing a pass offense. You therefore choose a run offense as it brings you greater net advantage. Team B’s coach makes a similar calculation and finds that a run offense gives a net advantage of +1 (= +1 − 0) and a pass offense a net advantage of 0 (= 0 − 0). They therefore also choose a run offense. However Team A’s net advantage from their selected offense of +2 is greater than team B’s net advantage from their selected offense of +1. Team A is therefore expected to beat team B. Following the same reasoning we would expect team B to beat team C, and team C to beat team A. The relation is therefore intransitive.
4 Monotonicity

There are forceful arguments for the transitivity of ‘better than’ based on monotonicity principles. The particular example presented here is derived from the elegant account of Nebel (2018). Consider a monotonicity principle defined as follows.

**The strong monotonicity principle**

For any property $P$, with the opposite property $Q$, if $x$ is not $P$ (i.e. it is $Q$ or neutral) and $y$ is $Q$er than $x$, then $y$ is $Q$.

Suppose we define a neutral team by setting their qualities. Let such a team be team A with qualities $r_o$, $r_d$, $p_o$, $p_d$ for run offence and defence, and pass offence and defence respectively. Then we may determine a team B with respective qualities $r_o + 1$, $r_d - 2$, $p_o$, $p_d + 1$, and a team C with qualities $r_o - 1$, $r_d - 1$, $p_o + 2$, $p_d$. We will then have the cyclic triad as before, but by the strong monotonicity principle, B is a bad team since it is worse than a neutral team, and C is a good team since it is better than a neutral team. But the bad team is better than the good team, since team B would be expected to beat team C, which violates the strong monotonicity principle, and seems counter to general reason.

The apparent paradox relies on three assumptions:

1. we may have an intransitive ‘better than’ relation
2. we may define a neutral alternative
3. the strong monotonicity principle holds

The paradox exists whether ‘expected to beat’ is an ‘all things considered’ notion or not. One must only concede that it is a notion of ‘better than’. As was discussed in Section 3 it seems highly plausible in the context of competitive sports for ‘expected to beat’ to be a ‘better than’ notion, perhaps even an ‘all things considered better than’ notion. As has been demonstrated, the relation is intransitive. If the paradox is to be resolved, then we must consider if another of the assumptions may be refuted, or if indeed the rejection of the intransitivity of ‘better than’ is the most plausible resolution.

In considering defining the neutral alternative, there is nothing particularly special about neutrality on the good-bad spectrum. Neutrality is
used here as there exists readily available language to describe the situation: names for the point on the spectrum (‘neutral’) and for the two sides it separates (‘good’ and ‘bad’). But if one could define a point that lies at the transition from ‘bad’ to ‘very bad’ then one could make the same argument and conclude that ‘very bad’ is better than ‘bad’. We might even note that the Combined Spectrum Arguments that Nebel (2018) introduces are not really necessary. A spectrum argument of the canonical type with one of the outcomes identified as a particular point on the good-bad spectrum would lead to the same conclusion given the same reasoning process. So the point to be refuted is that one may identify any particular point on the good-bad spectrum.

There does seem to be some general meaning to what a ‘good’ team or a ‘bad’ team would be with regard to its qualities. A ‘good’ team would be one with high scores across its qualities, and a ‘bad’ team one with low scores across its qualities. This does not imply an ability to summarise the quality of a team in a single value however, and the qualities have value only in so far as they increase the expectation of a team winning, in line with the aim of sport. It is for example possible for a team D to be worse based on all standard statistics summarising their qualities e.g. mean, median, mode, maximum, minimum, but still be expected to beat a team E. For example as summarised in Table 2

<table>
<thead>
<tr>
<th>Team</th>
<th>Run</th>
<th>Pass</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Offense</td>
<td>Defense</td>
</tr>
<tr>
<td>D</td>
<td>+2</td>
<td>−1</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Qualities of teams D and E

Alternatively it may be appealing to consider that a team with mean values for each of the individual qualities would be a team of neutral quality. But consider the example presented in Table 3. Here the four teams in the tournament each have the same mean of 0 across their qualities, and each of the qualities has the same mean of 0 across the four team league. However team F would be expected to lose all their matches. It therefore seems unsatisfactory to consider them a team of neutral quality.

These examples suggest the difficulties in defining a neutral team, or even a good or bad team, based directly on their qualities, even having considered
them relative to the appropriate comparison class. They suggest that instead a neutral team ought to be defined with respect to an aggregation of their pairwise comparisons to the other members of the appropriate comparison class. In this setting one plausible choice for the qualities of a neutral team would be one that meant the team would be expected to win as many matches as they lose. This is consistent with our observation that the determination of ‘good’ or ‘bad’ must be made with respect solely to the outcome of matches. It makes ‘good’ and ‘bad’ explicitly dependent on the alternatives, which is not unintuitive. Norwich may be a ‘bad’ team in the context of the English Premier League, but are ‘good’ if we are taking all club teams, in any league, as our comparison class. In our example in Table 1, team A is a neutral team by this definition (as are teams B and C). Given a reference set of other alternatives then it may be possible to define a neutral team, one that would be expected to win as many matches as it would lose. Thus it seems there are reasonable grounds for not rejecting the assumption of being able to define a ‘neutral’ team.

The discussion though gives a lead as to why we might be prepared to relinquish the strong monotonicity principle, or at least to believe that it may be misapplied. Our definition of neutral had recourse to an aggregation of pairwise comparisons, our definition of ‘better than’ only to a single pairwise comparison. The essentially pairwise nature of the ‘expected to beat’ relation and the aggregated nature of ‘good’ or ‘bad’ come into conflict with the notion of monotonicity. That is they are not necessarily compatibly applied within the strong monotonicity principle. We may consider notions of ‘good’ and ‘bad’ with an intransitive ‘better than’ within the strong monotonicity principle only with respect to the two competitors being compared, and may not assume that those labels of ‘good’ or ‘bad’ will persist when the team is considered within a wider comparison class. One might consider this a rejection of the strong monotonicity principle. More precisely I would view

<table>
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<tr>
<th>Team</th>
<th>Run</th>
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<th>Defense</th>
<th>Pass</th>
<th>Offense</th>
<th>Defense</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>−1</td>
<td>−1</td>
<td>0</td>
<td>+2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>H</td>
<td>−1</td>
<td>−1</td>
<td>0</td>
<td>+2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>I</td>
<td>+2</td>
<td>+2</td>
<td>−4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3: Qualities of teams F-I
the strong monotonicity principle as correct but only insofar as ‘Qer than’, ‘P’ and ‘Q’ (specifically ‘better than’, ‘good’ and ‘bad’), that it has recourse to, are consistent, and that in the context of intransitive ‘better than’ notions it will not necessarily be consistent in its application to any more than the two-item set in the pairwise comparison.

5 Semantics

Some hold that the transitivity of ‘better than’ is a logical truth. They understand ‘better than’ to be the comparative of the monadic predicate ‘goodness’, with this being a semantic fact. Under this stipulation ‘better than’ is indeed transitive. One may argue for this stipulation on the basis that cyclical relations do not have the structure ‘Qer than’ or ‘more Q than’, and for relations that do have this structure there is always an associated value of ‘Qness’ that is being compared. So for example if considering people sitting in a circle then ‘to the left of’ is a cyclical relation. But there is not an associated meaning to ‘person A is lefter than person B’, nor that ‘person A is left’.

Broome (2004) considers a number of possible exceptions to this, including ‘later than’. One may say ‘1 a.m. is later than midnight’ and ‘2 a.m. is later than 1 a.m.’ and so forth until we have that ‘midnight is later than 11 p.m.’ completing the cycle. Here ‘later than’ does not seem to be related to ‘lateness’. Perhaps we might understand 11 p.m. to be late and 6 a.m. to be early. But then we can start at a time that is late, continue via times that are later than the previous one, and end at a time that is early. Broome (2004) seeks to resolve this by distinguishing between the ‘historical lateness’ that is being used in ‘later than’ and the ‘contextual lateness’, referring to its position in the day, that is being used in ‘late’/‘early’. This though still leaves the ‘historical lateness’ comparative as cyclical, which he deals with by asserting, based on a monotonicity principle, that one of ‘1 a.m. is later than midnight’, ‘2 a.m. is later than 1 a.m.’ ... ‘midnight is later than 11 p.m.’ must indeed be false. There is therefore some hard cut-off point (though it may be vague), let us call it x, before which is ‘late’ and after which is ‘early’, and it would be incorrect to claim ‘ten minutes after x is later than ten minutes before x’.

An alternative interpretation of ‘historical’ and ‘contextual’ lateness is offered by the discussion of neutrality in the previous section. There I ar-
gued that ‘neutral’, and by extension any point on the good-bad spectrum, should be seen as an aggregate of pairwise comparisons, such that it is the comparative that is the predicate. Such an understanding provides a neat linkage of our intuitions for ‘historical’ and ‘contextual’ lateness. Whereas insisting on the monadic predicate ‘late’ resulted in the unintuitive claim of our misuse of ‘later than’, taking the dyadic predicate ‘later than’ explains why ‘early’ and ‘late’ are located where they are in ‘contextual’ time, and also why they are vague. Due to the nature of human sleep cycles most of us have more experience of 6 a.m. being ‘earlier than’ than ‘later than’ in relevant pairwise comparisons with other times of the day, and of 11 p.m. being ‘later than’ rather than ‘earlier than’ in relevant pairwise comparisons. An aggregation of these comparisons leads to our identification of these times as ‘early’ or ‘late’ respectively. But there is no clear deterministic function for how these experiences should be aggregated, we have varying experiences over time, relatively little experience for sleep hours, and experiences vary across individuals, and so the resulting understanding is vague.

Taking the comparative as the predicate also readily explains why an alternative, whose intrinsic qualities do not change, may be viewed properly as ‘good’ in some choice sets, and ‘bad’ in others. In a tournament with ten other teams all with the qualities of team B, then team A would be ‘good’, since they would be expected to win all their pairwise comparisons, but in a tournament with ten other teams all with the qualities of team C, then team A would be ‘bad’, since they would be expected to lose all their pairwise comparisons.

Suppose we accept this account and determine that a pairwise ‘better than’ is the dyadic predicate for ‘good’. How then should we understand the comparative that acts between two items based on the relative degree of their derived ‘goodness’? It seems highly reasonable, at least as a matter of semantics, that this should take the label ‘better than’. And so we seem to now have two notions of ‘better than’ that might reasonably apply, but lead to different conclusions. For the sake of clarity let us refer, consistently with the ‘later than’ / ‘late’ example, to ‘contextual better than’ as that which relates to the relative degree of ‘goodness’, where that ‘goodness’ has been determined based on pairwise ‘better than’ comparisons. As I noted in the ‘later than’ / ‘late’ example, this goodness is very possibly vague even where the underlying ‘better than’ notion is not. But to the extent that these aggregations are deterministic then it could be the case that the ‘contextual better than’ is transitive. This is notable for being the case without rely-
ing on what Temkin (2014) calls an Internal Aspect View, roughly that the ‘goodness’ of an alternative depends solely on the degree to which it entails different attributes and the degree to which each of those attributes matter. Some may even accept my account that this ‘contextual better than’ is derived, through an identification of ‘goodness’, from pairwise comparisons, but might insist that these should properly be seen as something else, preference relations perhaps, with the ‘contextual better than’ representing the ‘all things considered better than’ notion due to it taking account of the entire choice set. In countering this, within the context of a semantic argument, I would make three arguments, one general and two domain specific.

First that the semantic structure of ‘Qer than’ is essentially pairwise, and thus this pairwise nature should be given primacy in determining the nature of ‘better than’. “A is Qer than B” is a phrase that permits only the comparison of two items and gives no indication of the total choice set other than the two alternatives themselves. If only these two items are to be considered, that one is ‘good’ and the other ‘bad’ in the context of the total choice set is not a relevant point of information for the determination of ‘better than’. One might seek to challenge this by expanding it from being purely pairwise by, for example, saying “\{A,B\} is Qer than \{C,D\}” or “A is Qer than B and C” but these are still essentially pairwise. In “\{A,B\} is Qer than \{C,D\}” the items are now sets rather than individuals but there are still only two of them, and “A is Qer than B and C” is the conjunction of the two pairwise comparisons “A is Qer than B” and “A is Qer than C”.

Second that if one accepts that a ‘contextual better than’ is based on an aggregation of ‘pairwise comparisons’ then the pairwise comparisons are foundational. In cases such as competitive sport, as Broome (2004) notes “goodness of a ... team is a complex matter involving the ability to do well against a variety of opponents.” And so the ‘contextual better than’ is dependent on these foundational comparisons. Crudely, whether a team is ranked higher in a league is a function of the multiple pairwise comparisons not vice versa. There would seem to be some sense in which a fundamental comparison of relative merit has greater claim on a title of ‘all things considered’ than a comparison derived from it.

Third that in cases such as the instransitive football teams the unit of comparison is specifically pairwise. Matches are played between two teams, not three, or four, or five. This argument would seem to be even stronger for ‘all things considered better than’, as the natural unit of comparison would seem to be a relevant factor that should be accounted for within the notion
of ‘all things considered’.

In many circumstances it may be that there are other subsets of the entire choice set, to which comparatives are applied, that constitute the foundational comparisons for the determination of the ‘contextual better than’. So for example ‘6 a.m. is the earliest time I would be willing to get up’ may be a foundational comparison when determining times that are ‘early’ or ‘late’. Here the choice set is not explicitly defined but is likely to be greater than a two item set but smaller than the entire clock. Even in these cases the first argument holds, and pairwise comparisons may be a sufficiently significant part of these foundational comparisons for the domain-specific arguments also to have force.

Perhaps some would accept these arguments in the limited case where the fundamental unit of comparison is pairwise. If such circumstances were very rare, as I believe them to be, this may also be a contributory factor to why some people’s intuition in favour of a transitive ‘better than’ is so strong.

6 What of morality?

It is tempting to try to make a link to a moral argument directly. Consider the following example.

The altruistic gambler

Suppose there is an altruistic but compulsive gambler, who cannot resist gambling but will donate all his winnings to good causes. He is betting on matches between the teams from Table 1. The bookmakers have mistakenly assessed all teams as being equal and so he gets the same odds for a bet on any match (perhaps they have used a mistaken ‘better than’ notion that relies on the average of the qualities of the teams!).

Then his choice of which team to back should be defined by the ‘is expected to beat’ relation, preferring team A in a comparison with team B, team B in a comparison with team C, and team C in a comparison with team A will achieve the most good in moral terms. In this sense, as it pertains to his betting decisions, A is better than B, B is better than C, and C is better than A, and since his winning is linked to a moral good, this ‘better than’ notion lies in the moral realm.
Some may object to the use of gambling in an example demonstrating moral betterness. Perhaps more significantly others may object that it is not precisely identifiable what ‘better than’ actually means here. One might suggest something like ‘more likely to be a winning bet than’, but then it is not readily apparent that this is related to the pairwise comparison rather than in a consideration of all alternatives and a randomly chosen match. In this sense my use does not accord in this context with the natural language meaning of ‘better than’, even if I might claim that it does accord with an underlying notion of ‘all things considered better than’.

But perhaps we can instead usefully see it as a particular example from a class of betterness cycle examples that take as their starting point the three part intuition that:

1. ‘better than’ is a complex relation,
2. complex relations are multi-dimensional,
3. multi-dimensional relations may be (and often are) intransitive.

This suggests that one may consider examples based on unambiguously intransitive relations in some non-moral space and relate them to a moral outcome. And as I noted in Section 5, these might have further force if there is an essentially pairwise comparison on which they are based. Consider the following example.

**The racing evil-doer**

Suppose there are only three types of vehicle. Their performance is known but unreliable. The time in hours that they will take to travel 100 miles is with equal probability of a third: 3, 5 or 7 hours for vehicle A; 1, 6, or 8 hours for vehicle B; and 2, 4 or 9 hours for vehicle C. Now suppose there is an evil-doer who has identified a target 100 miles away and will take a vehicle to get there, and a good actor who wishes to thwart the evil-doer by taking another vehicle in order to get there first.

Now suppose that there are only two vehicles available, but they are of as yet unknown type. The good actor is hurrying to the site of the two vehicles, and is expected to arrive momentarily before the evil-doer. We have to advise her of the betterness relation of her choice set, so that when she arrives and is able to identify them, she is able to make a correct selection.
The probability that a vehicle of type A beats one of type B is $5/9$, since with an equal $1/9$ probability we have the time pairs $(1 < 2), (1 < 4), (1 < 9), (6 > 2), (6 > 4), (6 < 9), (8 > 2), (8 > 4), (8 < 9)$. Likewise the probability that a type B beats a type C is $5/9$, and that a type C beats a type A is $5/9$. These probabilities being a direct result of the probabilistic speed of each.\(^5\) I claim that, with respect to the choice facing her, one may properly say that A is better than B, B is better than C, and C is better than A, and that this betterness is of a moral type.

A transitivist might argue that these are actually three different ‘better than’ relations, pertaining to the three choice sets \{A,B\}, \{B,C\}, and \{C,A\} respectively. But the relation being applied is entirely consistent in each case, that relating to the expected moral outcome from the relative probability of arriving first. This consistency is indicated indeed by being able to use merely the phrase ‘better than’ without clarification. The argument is even stronger when considering ‘all things considered better than’ since the inherent unit of comparison is pairwise.\(^6\)

I have called arguments of this class unambiguous arguments because, in contrast to spectrum arguments, the transitivity of the relation at the centre of the example is unambiguous. It is clearly not dealing with incommensurable, incomparable or vague alternatives, the intransitivity is not due to a failure of intuition, there is no ambiguity from the competing claims of degree and kind, and they are not open to challenge as Zeno’s or sorites paradoxes. It is perhaps also informative in doubting our transitive intuitions in the moral realm that despite being a mathematical fact the transitivity of the ‘expected to arrive before’ relation may be counter-intuitive for many.

7 Concluding Remarks

I have argued that ‘better than’, perhaps even ‘all things considered better than’ in the context of competitive sport is intransitive. The sports context facilitated an explication for why we might challenge prominent arguments for the transitivity of ‘better than’ as an analytic or logical truth. This challenge would seem to have relevance to wider considerations, and in particular led to the proposal of a family of betterness cycle examples, the so-called unambiguous arguments. Even if the unambiguous arguments are found not to be persuasive, it is to be hoped that the plausibility of an intransitive ‘better than’ notion in the sports context allows for a greater appreciation of how
intransitive ‘better than’ relations might be understood in a moral context.

Notes

1. Broome (2004, p.52) dismisses the sports example as a potential counterexample to the transitivity of ‘better than’ as follows: “The very fact that the relation ‘can regularly beat’ may be intransitive amongst football teams should make you realize it is not equivalent to ‘is better than’.” Thus he recognises the potential intransitivity of the sports scenario but assumes that this in itself must disqualify it from any claim to being a ‘better than’ relation. This seems a somewhat tautological claim — an intransitive counterexample to the transitivity of ‘better than’ cannot be an example of a ‘better than’ relation because it is intransitive.

2. Temkin (2014, p.468-471) discusses what he calls the ‘sports analogy’. Here he considers ‘wins most games against all other teams during the season’ as an ‘all things considered better than’ notion as a means of preserving the Axiom of Transitivity while maintaining the Essentially Comparative View in the context of wider practical reasoning. He rejects it on four grounds. I believe it mostly has no bearing on the arguments discussed in this paper, but while I agree with his conclusion in rejecting it, I find his arguments mostly unpersuasive. First he states: “to determine the best candidate among $n$ applicants, each applicant must be compared to every other. As noted in section 13.1, this would require $(n/2) \times (n - 1)$ separate comparisons.” This is simply statistically incorrect. As he himself seems to later acknowledge there are methods of ordering the comparisons such that significantly fewer pairwise comparisons are likely to be required. More substantially perhaps, even if it were correct, it would not invalidate the approach for all matters of practical reasoning, for example where a relatively small number of discrete alternatives were being compared. Second he rejects it on the grounds of not readily being able to identify the appropriate comparison class. Again it seems likely that there are numerous scenarios where that is unlikely to be an impediment. Third he rejects it based on wins not accounting for all aspects that should be considered, but numerous sports exist where points are awarded for outcomes other than just wins, for example rugby union, ice hockey, and county championship cricket. If there is an aspect that stakeholders prize enough for it to be considered within their determination of ‘better than’ then they may change the points system, so that instead of considering ‘has aggregated more wins against all other competitors’ they may consider ‘has aggregated more points against all other competitors’. There may also be scenarios where ‘wins’ do account for all relevant aspects. Indeed it might be argued that their relevance to ‘wins’ is what makes them worthy of consideration. Fourth he notes that the qualities that one might wish to use for a two-way comparison may be different to those one might wish to use for a three-way, four-way etc. comparison. I find this more persuasive, but it seems to be an argument against using the sports analogy, or indeed any method, as a uniform notion applicable to all scenarios. There could however be scenarios where the appropriate factors to consider in a multi-item comparison are the same as those in a two-way comparison, and for those scenarios this argument would not allow us to reject a sports analogy approach.
3. Smead (2019) argues that social choice theory provides us with other possible methods of aggregating the pairwise comparisons. Which of these is used is unimportant to the argument in this paper however. We only require that a reversion to an aggregationist approach of some type is required in order to form concepts of ‘good’ and ‘bad’ across a choice set of greater than two items.

4. It is tempting to see this simply as the transitivity of aggregation compared to the potentially intransitive pairwise comparison unit. But this would be a mistake since aggregations, even those seemingly respecting the primacy of winning over losing, need not be transitive. The so-called ‘minimum violations’ method where teams are ranked to minimise the number of occurrences of a lower ranked team having beaten a higher ranked team is an example of such an intransitive aggregationist approach that respects the principle of winning being better than losing.

5. Readers may recognise this example as relating to intransitive dice examples such as Efron’s dice.

6. I have mostly considered two potential notions of ‘better than’ here – the ‘pairwise better than’ that is intransitive, and the ‘contextual better than’ which is transitive. One way we might want to think about the question as to which is an ‘all things considered better than’ notion is in the iterative form of whether ‘pairwise better than’ is better than ‘contextual better than’. It seems at least plausible to suggest that one ‘better than’ notion will apply rather than another ‘better than’ notion if the choices it leads to are expected to be better. Taking the racing evil-doer example, if we take ‘better than’ to be a transitive relation then we may only advise that the alternatives are equally good since amongst the choice set \{A, B, C\} they have equal probability of being first to reach the target against an unknown alternative. However if the ‘better than’ relation can be intransitive then we can advise the good actor that A is better than B, B is better than C, and C is better than A. She is therefore expected to make a choice leading to a better outcome when the intransitive ‘better than’ relation is employed. I would consider however that this is not an argument for the nature of the relation, but rather for what we ought to hold as our concept of ‘all things considered better than’ if we accept ‘all things considered better than’ to be a guide to our actions.

References


