Competitive sport and the implications for our understanding of ‘better than’

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1 Abstract

The context of competitive sport is taken as a starting point for the consideration of the transitivity of ‘better than’. The argument made in Nebel (2018) is refuted, with a clarification of the appropriate application of the monotonicity principle. The discussion leads to the introduction of a class of betterness cycle examples, distinct from the well-known spectrum arguments. The transitivity of ‘better than’ is framed as a choice, and an argument made for a preference for a nontransitive notion in competitive sports, and, under certain circumstances, in moral reasoning.
2 Spectrum arguments

This paper concerns the transitivity of ‘better than’.

Transitivity of Better Than

For any value bearers \( A, B, \) and \( C \), if \( A \) is better than \( B \), and \( B \) is better than \( C \), then \( A \) is better than \( C \).

Counterexamples to the transitivity of ‘better than’ are claimed in the form of, so called, spectrum arguments, developed by Rachels (1998) and Temkin (1987, 1996, 2014). Many versions are presented in these works, but following Nebel (2018), we use here an example due to Rachels. Consider

The Bad Spectrum

\[ A: 1 \text{ year of excruciating agony.} \]
\[ B: 2 \text{ years of pain slightly less intense than the pain in } A. \]
\[ C: 4 \text{ years of pain slightly less intense than the pain in } B. \]
\[ \ldots \]
\[ X: 2^{24} \text{ years of pain slightly less intense than the pain in } X. \]
\[ Y: 2^{25} \text{ years of pain slightly less intense than the mild pain in } Y. \]

The pain in \( A \) is often understood to be like that of torture and the pain in \( Z \) to be like that of a hangnail or a mosquito bite. It is argued that stepping through the alphabet, the outcomes seem to get worse at each step, and yet \( Z \) seems better than \( A \). If the relation at each step is as it seems, then ‘better than’ is not transitive.

Nebel (2018) presents an elegant argument against spectrum arguments and therefore in support of the transitivity of ‘better than’. The argument comes in the form of three Combined Spectrum Arguments. Only the first of these, the Bad Spectrum with Ecstasy will be presented here, as they are substantively similar.

It begins by defining an outcome \( A \) that contains a year of excruciating agony and an amount of ecstasy chosen to balance the agony such that \( A \) is neutral - neither good nor bad overall. These amounts may be varied by the reader to meet the condition of neutrality; it is only required that a neutral state is achievable with some amount of ecstasy and some amount of agony. For example suppose that one requires 3 years of ecstasy to balance 1 year of excruciating agony. Now consider
**The Bad Spectrum with Ecstasy**

A: 1 year of excruciating agony and 3 years of ecstasy.

B: 2 years of pain slightly less intense than the pain in A and 3 years of ecstasy.

C: 4 years of pain slightly less intense than the pain in B and 3 years of ecstasy.

... 

X: $2^{24}$ years of pain slightly less intense than the pain in X and 3 years of ecstasy.

Y: $2^{25}$ years of pain slightly less intense than the mild pain in Y and 3 years of ecstasy.

Under the same reasoning as the spectrum argument, as we descend the alphabet the outcomes seem to get worse and yet A seems worse than Z. Now consider

**The Badness Principle**

For any value bearers $x$ and $y$, if $x$ is bad and $y$ is worse than $x$, then $y$ is also bad.

Since A is neutral and the outcomes are consecutively worse, then there is some outcome that is bad, let us say D (this can be adjusted further down the alphabet if needed). But if D is bad, then by the badness principle E is bad, and if E is bad then F is bad, and so on, so that Z is also bad. But A, which is not bad, seems worse than Z. This violates the badness principle.

Therefore, Nebel claims, given the badness principle, the following claims are inconsistent:

1. In the bad spectrum with ecstasy, each outcome is worse than its predecessor.
2. In the bad spectrum with ecstasy, A is worse than Z.
3. Some of the outcomes in the bad spectrum with ecstasy are bad.
4. Not all of the outcomes in the bad spectrum with ecstasy are bad.

He concludes that the combination of (1) and (2) are together less plausible than (3) or (4) and therefore they should be rejected, leading to the rejection of the spectrum arguments.
The paper proceeds in Section 3 with a discussion of ‘better than’ in competitive sport. In Section 4 a working example is introduced and the implications for Nebel’s argument are considered. In Section 5 the relevance to some standard arguments in the literature are discussed. In Section 6, an argument is made for a sense in which one might refute Nebel’s claim that “I cannot see how any agent — let alone a rational one — could desire $x$, prefer $y$ to $x$, and yet not desire $y$.”, and a novel class of betterness cycle examples are introduced. In the final part, Section 7 the transitivity of ‘better than’ is framed as a choice, and an argument is made for a nontransitive notion in competitive sport, and for nontransitive notions, in some circumstances, in wider moral reasoning.

3 ‘Better than’ in competitive sport

The aim of competitive sport is to win. When situations arise that compromise this, for example when a team losing in a preliminary group stage of a tournament is likely to provide them easier progress in the knock-out stages, stakeholders object to the circumstance because it transgresses this principle. Since this is the sole aim of a competitor then an acceptable ‘better than’ relation in a competitive sports context need only be consistent with the principle that winning is better than losing. So for example, I may not claim that Arsenal are a better soccer team than Manchester United because they play more attractively (unless I also claim that the attractiveness of the soccer is the sole determinant of winning). I may claim that Arsenal are (or were) better than Manchester United because they have beaten them, because they have a better record against equivalent opposition, or because they would be expected to beat them, since all of these respect the primacy of winning over losing. As such, multiple definitions of ‘better than’ that may reach different conclusions are permissible. For the purposes of better understanding the nature of some of these, and providing a basis for later widening the scope of the arguments from ‘better than’ to ‘all things considered better than’, it is useful to examine the claims that some of these may have on being an ‘all things considered better than’ notion for competitive sport.

To begin, let us take the ‘better than’ relation ‘has beaten’. It is noted that it is clearly not necessarily transitive. It is common in sports for team A to have beaten team B, team B to have beaten team C, and team C to have beaten team A. One may make at least four objections to using this as a notion of ‘all things considered better than’. First this outcome may be due
to some arbitrariness in the particular instantiation. Upsets are a common feature in sport, and even where one had two teams of the same ability one would expect a result, with many sports not allowing for tied outcomes. This is so much a part of sport that the refrain “may the best team win” is a commonplace statement on the eve of matches. Second, even if one takes a more deterministic approach and denies the possibility of such arbitrariness, one may still point out that this is an outcome under a particular set of external variables. For example weather conditions and officiating decisions may be considered to have an impact on a match outcome and are beyond the attributes of the teams. Third it is possible that team A has beaten team B, and team B has beaten team A. There is something especially uncomfortable in the extreme intransitivity of stating that simultaneously A is better than B and B is better than A. Fourth this is an instantiation from a particular point in the past. To the degree that it may be considered an ‘all things considered better than’ relation, we would need to determine to what time period it applies. It seems reasonable to claim that it can only pertain to the point in the past that the match occurred, and as such cannot be used to determine the present state for example.

An alternative relation would be the predominant ‘better than’ relation used in league sports, namely ‘has aggregated more wins against all other competitors’. The ubiquity and uniformity with which this notion is used might suggest that it forms a commonly accepted ‘all things considered better than’ notion. But there is no reason to believe that because this is the predominant methodology used in the production of league tables that it is expected to correspond to an ‘all things considered better than’ notion. There are several goals of a league tournament, including the creation of a ranking for competitors, making that ranking methodology transparent to stakeholders, and identifying the best team. There is no a priori reason to believe all (or any) of these are possible simultaneously. Definitionally to produce a ranking one requires a transitive notion of ‘better than’. So even if there were an agreed upon notion of ‘all things considered better than’, if it were not transitive then it could not be used for this purpose. As such the requirement for the ‘better than’ notion to be transitive would dominate the requirement to use an ‘all things considered better than’ notion, and so it would be mistaken to conclude that usage necessarily indicates an ‘all things considered’ notion. So we must instead look at the attributes of such a relation. A number of objections may be raised to using this as an ‘all things considered better than’ notion.¹ Let us consider the first two objections raised previously, those of random instantiations and external conditions. While the impacts of these are in a sense averaged over the outcome of all
matches, these are reduced but not eliminated. Statistics tells us that as
the number of matches increases, the biasing effects of chance outcomes or
circumstances on any team are reduced but they do not disappear. Third,
the consideration of time becomes yet more unclear with an aggregation
of outcomes from different time points, so that we cannot even relate the
‘better than’ relation to a particular point in the past.

This may then encourage us to consider the notion ‘would be expected
to aggregate more wins against all other competitors’. In referencing ex-
pected rather than actual instantiations, we are considering this claim over
all possible conditions and randomness in line with the probability that they
occur, and additionally it may be referenced to any particular point in the
past, present or future by using variants ‘was/is/would be expected to ag-
gregate more wins against all other competitors’ and so these objections are
avoided. Under this proposal we might think of this use of expectation as
what accounts for ‘all things considered’. However this has the uncomfort-
able feature that in a multi-team tournament the conclusion of whether A
is better than B depends on the other teams C, D etc. so that whether A is
considered better than B may change without A and B themselves changing.
It is also, as Smead (2019) points out, a non-unique potential aggregation,
with different aggregation methods giving different ‘better than’ relations.

So let us consider the relation ‘expected to beat’. None of the objections
mentioned so far pertains to this — it is not subject to arbitrary outcomes or
circumstances due to its recourse to expectation, the relevant time may be
expressed precisely, it cannot display extreme intransitivity, it is dependent
solely on the qualities of the two teams being compared, and it has a unique
determination. This then, in the context of sports, seems like it may be a
plausible definition of ‘all things considered better than’. As we go on to
show though, this definition turns out to not be necessarily transitive. The
question as to whether this in itself should disqualify it as a ‘better than’
notion is one to which we will return in Section 7.

4 Neutrality and monotonicity

For now let us consider a particular example. Take three american football
teams A, B and C. Let us suppose that their important qualities may be
summarised by their offensive and defensive ability in their running and
passing game, and that points are expected to be scored in a monotonically
increasing way with the difference in the strength of a team’s chosen offence
and the opposition’s corresponding defence. We also assume that they know
their opposition’s qualities, presumably having reviewed their past matches, so they will choose a running or passing offense dependent on where they have greatest advantage. Let us suppose that their qualities are as in Table 1. We can see that in a match between teams A and B, both teams are expected to perform best with a run offense, but team A has an advantage of 2 (= 0 − (−2)) compared to Team B’s advantage of 1 (= 1 − 0), and so team A would be expected to win. Following the same reasoning we would expect team B to beat team C, and team C to beat team A. The relation is therefore nontransitive.

Suppose then we wish to define a neutral team by setting their passing qualities having been given their running qualities. Let such a team be team A with qualities $r_o$, $r_d$, $p_o$, $p_d$ for run offence and defence, and pass offence and defence respectively. Then we may determine a team B with respective qualities $r_o + 1$, $r_d − 2$, $p_o$, $p_d + 1$, and a team C with qualities $r_o − 1$, $r_d − 1$, $p_o + 2$, $p_d$. We will then have the cyclic triad as before, but, following Nebel’s reasoning, B is a ‘bad’ team since it is worse than a neutral team, and C is a ‘good’ team since it is better than a neutral team. But the bad team is better than the good team, since team B would be expected to beat team C.

The apparent paradox relies on three assumptions:

1. we may have a nontransitive ‘better than’ relation
2. we may define a neutral alternative
3. the strong monotonicity principle applies to the relation

where the strong monotonicity principle may be defined as:

**The Strong Monotonicity Principle**

For any property $P$, with the opposite property $Q$, if $x$ is not $P$ (i.e. it is $Q$ or neutral) and $y$ is $Q$er than $x$, then $y$ is $Q$. 

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<table>
<thead>
<tr>
<th>Team</th>
<th>Run Offense</th>
<th>Run Defense</th>
<th>Pass Offense</th>
<th>Pass Defense</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>+1</td>
<td>−2</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>C</td>
<td>−1</td>
<td>−1</td>
<td>+2</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Qualities of teams A, B and C
The paradox exists whether ‘expected to beat’ is an ‘all things considered’ notion or not. One must only concede that it is a notion of ‘better than’. As was discussed in the first part it seems entirely reasonable in the context of competitive sports for ‘expected to beat’ to be a ‘better than’ notion, perhaps even an ‘all things considered better than’ notion. As has been demonstrated, the relation may be nontransitive so, if the paradox is to be resolved, then one of the other two conditions must be refuted.

In considering defining the neutral alternative, there is nothing particularly special about neutrality on the good-bad spectrum. Neutrality is used here, as it is in Nebel’s *Bad Spectrum with Ecstasy* and *Good Spectrum with Agony* arguments, as there exists readily available language to describe the situation: names for the point on the spectrum (‘neutral’) and for the two sides it separates (‘good’ and ‘bad’). But if one could define a point that lies at the transition from ‘bad’ to ‘very bad’ then one could make the same argument and conclude that ‘very bad’ is better than ‘bad’. We might even note that *Combined Spectrum Arguments* are not really necessary. A spectrum argument of the canonical type with one of the outcomes identified as a particular point on the good-bad spectrum would lead to the same conclusion given the same reasoning process. So the point to be refuted is that one may identify any particular point on the good-bad spectrum.

There does seem to be some general meaning to what a ‘good’ team or a ‘bad’ team would be with regard to its qualities. A ‘good’ team would be one with high scores across its qualities, and a ‘bad’ team one with low scores across its qualities. This does not imply an ability to summarise the quality of a team in a single value however, and the qualities have value only in so far as they increase the expectation of a team winning, in line with the aim of sport. It is for example possible for a team D to be worse based on all standard statistics summarising their qualities e.g. mean, median, mode, maximum, minimum, but still be expected to beat a team E. For example as summarised in Table 2

<table>
<thead>
<tr>
<th>Team</th>
<th>Run Offense</th>
<th>Run Defense</th>
<th>Pass Offense</th>
<th>Pass Defense</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>+2</td>
<td>−1</td>
<td>−9</td>
<td>−1</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+10</td>
</tr>
</tbody>
</table>

Table 2: Qualities of teams D and E

Suppose additionally that the mean value for each quality across the league was 0, then we might consider that team E is ‘good’ based on an
evaluation of its qualities given that it is at or above average levels in every quality and far above in one, whereas Team D might be considered ‘bad’ given that they are below average in three of the four qualities, far below in one of those, and only slightly above in the one where they are above.

Alternatively it may be appealing to consider that a team with mean values for each of the individual qualities would be a team of neutral quality. But consider the example presented in Table 3. Here the four teams in the tournament each have the same mean of 0 across their qualities, and each of the qualities has the same mean of 0. However team F would be expected to lose all their matches. It therefore seems unsatisfactory to consider them a team of neutral quality.

<table>
<thead>
<tr>
<th>Team</th>
<th>Run</th>
<th>Pass</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Offense</td>
<td>Defense</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>−1</td>
<td>−1</td>
</tr>
<tr>
<td>H</td>
<td>−1</td>
<td>−1</td>
</tr>
<tr>
<td>I</td>
<td>+2</td>
<td>+2</td>
</tr>
</tbody>
</table>

Table 3: Qualities of teams F-I

These examples suggest the difficulties in defining a neutral team, or even a good or bad team, based directly on their qualities, even having considered them relative to the summary statistics of the appropriate comparison class. They suggest that instead a neutral team ought to be defined with respect to an aggregation of their pairwise comparisons to the other members of the appropriate comparison class. In this setting one plausible choice for the qualities would be one that meant the team would be expected to win as many matches as they lose.² This is consistent with our observation that the determination of ‘good’ or ‘bad’ must be made with respect solely to the outcome of matches. It makes ‘good’ and ‘bad’ explicitly dependent on the alternatives, which is not unintuitive. Norwich may be a ‘bad’ team in the context of the English Premier League, but are ‘good’ if we are taking all club teams, in any league, as our comparison class. In our example in Table 1, team A is a neutral team by this definition (as are teams B and C). Given a reference set of other alternatives then it will be possible to define a neutral team, one that would be expected to win as many matches as it would lose. It is then true that a team with high quality scores is likely to be considered ‘good’ but only in so far as these elevated qualities result in a higher proportion of wins.
The discussion though gives a lead as to why we might be prepared to relinquish the strong monotonicity principle, or at least to believe that it may be misapplied. Our definition of neutral had recourse to an aggregation of pairwise comparisons, our definition of ‘better than’ only to a single pairwise comparison. The essentially pairwise nature of the ‘expected to beat’ relation and the aggregated nature of ‘good’ or ‘bad’ come into conflict with the notion of monotonicity. That is they are not necessarily compatibly applied within the strong monotonicity principle. We may consider notions of ‘good’ and ‘bad’ with a nontransitive ‘better than’ within the strong monotonicity principle only with respect to the two competitors being compared, and may not assume that those labels of ‘good’ or ‘bad’ will persist when the team is considered within a wider comparison class.\textsuperscript{3} One might consider this a rejection of the strong monotonicity principle. More precisely I would view the strong monotonicity principle as correct but only insofar as the concepts of ‘\textit{Q}er than’, ‘\textit{P}’ and ‘\textit{Q}’ (specifically ‘better than’, ‘good’ and ‘bad’) that it has recourse to are consistent, and that in the context of nontransitive ‘better than’ notions it will not necessarily be consistent in its application to any more than the two-item set in the pairwise comparison.\textsuperscript{4}

5 Bald men and sorites

The argument presented here against accepting the strong monotonicity principle as it is applied in Nebel (2018) is of the same nature but may be of a different type to the arguments presented in Temkin (2014, p.284-294) in response to the bald man example of Wasserman (2005). In this example, Wasserman considers a man with a small bald spot at the centre of his head and sufficient hair that one would not consider him bald. He then compares the first man to a second man with a very slightly larger bald spot but sufficiently more hair such that one would consider the first man balder than the second man. By continuing in this way he eventually arrives at a man with an incredibly dense concentration of hair in a single spot at the back of his head, such that his head is almost all bare and one would consider him balder than the first man.

The arguments are the same in that in both cases the ‘\textit{Q}er than’ relation described (‘better than’, ‘balder than’) is incompatible with the considerations that are included to conclude that something is ‘\textit{Q}’ (‘good’, ‘bald’) and thus the strong monotonicity principle is misapplied. However in the case of ‘balder than’ this is because the distribution of hairs seems to be ignored in the pairwise comparison — because it is not significantly different — so that

\textsuperscript{10}
the number of hairs becomes the sole determinant whereas in the sports case
the determinant qualities never change. It is tempting to see the bald man
example as being due to the vagueness of the quality ‘distribution’. Aside
from the statistician’s qualm that there are senses in which ‘distribution’ as
it is discussed here may be measured, we can readily provide an example
to show that this vagueness is not really the cause. Suppose I wish to buy
a car. My primary determinant for preferring one car over another is fuel
efficiency but if two cars have a similar fuel efficiency then I will make a
determination based on top speed. In this case both the fuel efficiency and
top speed are single determinable values. I may even express the preference
relationship in a formulaic manner. Representing this mathematically, let
our teams $X$ and $Y$ be represented by vectors of their qualities $x = (x_1, x_2)$
and $y = (y_1, y_2)$, $x, y \in \mathbb{R}^2$ then we may consider a comparison of the two
to be represented by a function $f : \mathbb{R}^2 \times \mathbb{R}^2 \mapsto \mathbb{R}$, where $X$ is ‘better than’
$Y$ if and only if
$$f(x, y) > 0,$$
and define for example
$$f(x, y) = (x_1 - y_1)^3 + (x_2 - y_2).$$
Now consider cars A, B and C with respective qualities $a = (1, 1)$, $b =$(1.5, 0.75), $c = (2, 0.5)$ then it is clear that we have a cyclical triad, and also
that we could add any number of extra options so that this may be cyclic
of order $n$. As with the bald man example if the difference in the first
parameter (fuel efficiency, hair distribution) is small then it is the difference
in the second parameter (top speed, number of hairs) that dominates in the
pairwise comparison, but given a set of items that span a wide set of the
parameters it is the first parameter that will correlate more highly to being
‘$Q$er than’ (‘more desirable than’, ‘bald than’) a greater number of other
alternatives (cars, pates), and thus is more likely to take primacy in defining
what is considered $Q$ (‘desirable’, ‘bald’). We should also note that as well
as not being vague these items are clearly not incommensurable, and their
value is quantitative.

It is tempting to consider that the spectrum arguments may be a kind
of sorites paradox due to their similar structure. Both begin with a plausi-
ble premise, before applying a series of seemingly plausible reasoning steps,
where the small difference in some property is crucial, to establish a conclu-
sion that goes against intuition. One may object however, as Temkin (2014,
p.278) does, that the steps are very different in nature. In the spectrum
arguments, an immediately succeeding item is determined to be ‘$Q$er than’
the last due to an increase(decrease) in the second property despite a slight
decrease(increase) in the first property. In the sorites paradox an immediately
succeeding item is determined to be ‘Q’ due to the preceeding item
being ‘Q’ and the change in the underlying property being insufficient to
have caused a change to ‘not Q’. The two examples should not therefore be
equated.

But there is a sense in which a spectrum argument may lead to a sorites
paradox. Take the Bad Spectrum. We are dealing with two dimensions,
intensity and duration of pain. The step between any consecutive outcomes
is made due to the plausible intuition that a small pain intensity difference is
offsettable by some sufficiently large duration difference. On the other hand
the conclusion that Z is better than A is due to the plausible intuition that a
sufficiently large pain intensity difference, for example between torture and a
hangnail, is not offsettable by any duration difference. And so by adding in
an appropriate soritical premise on the offsettability of any particular pain
intensity difference, we have the sorites paradox,

Initial premise: A small pain intensity difference is offsettable.
Soritical premise: If a pain intensity difference \(d\) is offsettable, then a
pain intensity difference slightly larger than \(d\) is off-
settable.
Conclusion: Any pain intensity difference is offsettable.

One objection to this expression of the Bad Spectrum as a sorites paradox
would be that the Bad Spectrum does not make the claim that a sufficiently
large pain intensity difference is not offsettable by any duration difference.
It makes the more specific claim that the very large intensity difference of
A and Z, that between torture and a hangnail, is not offsettable by the
duration difference of A and Z, that between 1 year and \(2^{25}\) years. This
is a reasonable objection if the relation was of the nature defined for car
preference. There, in the comparisons of A and B, and of A and C, the
difference in fuel efficiency is small enough such that it is offsettable by a
difference in top speed, but in comparing A and C the difference in fuel
efficiency was not offset by the difference in top speed. However it was not
that this difference in fuel efficiency was not offsettable by any difference
in top speed. Indeed for any fuel efficiency difference there would be a
(perhaps realistically unachievable) difference in top speed that could offset
it. This perhaps emphasises how considering the two dimensions as being
ones of degree and kind is unhelpful to understanding ‘better than’. They
might better be seen as two dimensions on which a comparison will depend.
With that understood, it would seem to be arbitrarily restrictive that the differences in any dimension will be aggregated in a purely linear way, so that the difference in the first dimension will just be added to the difference in the second dimension. It seems more likely that the dimensional differences will impact the comparative relation in ways that may vary by dimension.

However others might claim that in the \textit{Bad Spectrum} the reason that we might consider that the difference between 1 year and 2^{25} years is not sufficient to offset the pain intensity difference between torture and a hangnail, is because no duration difference is capable of offsetting the pain intensity difference between torture and a hangnail. If this view is accepted then we do have a sorites paradox. This need not disqualify the arguments however. There is a highly plausible resolution to the paradox, that also serves as an indication of how we might more properly reconcile notions of ‘good’ and ‘better than’. Let us take the rich version of the paradox as our working example as Nebel (2018) does. One resolution to the paradox offers that there is no single point on the wealth scale beyond which someone may be considered rich, but that instead it is more properly thought of as a probability distribution of richness, so that if someone is one dollar wealthier than the last person then we may only say that the probability that they are rich is higher.\footnote{As we proceed along this scale the probability that someone may not be considered rich becomes vanishingly small. While as a matter of efficient rhetoric, and possibly even heuristic reasoning, we often prefer to consider that an item is either ‘Q’ or ‘not Q’, it seems more likely to be the case that many concepts are distributional rather than binary. The recent successes in creating artificial intelligences that use Bayesian techniques that are inherently distributional might be thought of as providing evidence for this. Under this view the notion of an item, in this context, being ‘Q’ may be thought of as some acceptably small probability that an item is ‘not Q’ based on an updating of this probability through the individual pairwise comparisons. These pairwise comparisons should then be thought of as constituting individual pieces of evidence that may or may not be consistent with the overall judgement. So, to use a sports analogy in a different context, the misapplication of the strong monotonicity principle is like claiming that some poor player is better than some good player, because in a single match they performed better.\footnote{10}}
6 Unambiguous arguments

Returning to our sports argument, it is tempting to try to make a link to a moral argument directly. Suppose there is an altruistic but compulsive gambler, who cannot resist gambling but will donate all his winnings to good causes. The bookmakers have mistakenly assessed all teams as being equal and so he gets the same odds for a bet on any match (perhaps they have used a mistaken ‘better than’ notion that relies on the average of the qualities of the teams!). Then his choice of which team to back should be defined by the ‘is expected to beat’ relation, and using our example from Table 1, preferring team A in a comparison with team B, team B in a comparison with team C, and team C in a comparison with team A will achieve the most good in moral terms. In this sense, A is better than B, B is better than C, and C is better than A.

It is rare for our moral choices to be so fundamentally pairwise, but these situations may exist, as in the case of the altruistic gambler. Some may object to the use of gambling in an example demonstrating moral betterness, but that is not an essential feature. It may instead be seen as an example from a class of betterness cycle examples that take as their starting point the three part intuition that:

1. ‘better than’ is a complex relation,
2. complex relations are multi-dimensional,
3. multi-dimensional relations may be (and often are) intransitive.

This suggests that one may consider examples based on unambiguously intransitive relations in some non-moral space and relate them to a moral outcome. Such examples also have as an essential feature that there is some sense in which the fundamental unit of comparison is of a smaller dimension, typically pairwise, than that of the entire choice set. So for example, suppose there are only three types of vehicle. Their performance is known but unreliable. The time in hours that they will take to travel 100 miles is with equal probability of a third: 3, 5 or 7 hours for vehicle A; 1, 6, or 8 hours for vehicle B; and 2, 4 or 9 hours for vehicle C. Now suppose there is an evil-doer who has identified a target 100 miles away and will take a vehicle to get there, and a good actor who wishes to thwart the evil-doer by taking another vehicle in order to get there first. The probability that a vehicle of type A beats one of type B is 5/9, that a type B beats a type C is 5/9, and that a type C beats a type A is 5/9. These probabilities being a
direct result of the probabilistic speed of each. Now suppose that there are only two vehicles available, but they are of as yet unknown type. The good actor is hurrying to the site of the two vehicles, and is expected to arrive momentarily before the evil-doer. We have to advise her of the betterness relation of her choice set, so that when she arrives and is able to identify them, she is able to make a correct selection. I claim that, with respect to the choice facing her, one may properly say that A is better than B, B is better than C, and C is better than A, and that this betterness is of a moral type.

A transitivist might argue that in both examples, the altruistic gambler and the racing evil-doer, these are actually three different ‘better than’ relations, pertaining to the three choice sets \{A,B\}, \{B,C\}, and \{C,A\} respectively. But the relation being applied is entirely consistent in each case, those relating to the expected moral outcome from the expected match outcome and from the increased probability of arriving first respectively. This consistency is indicated indeed by being able to use merely the phrase ‘better than’ without clarification. The argument is even stronger when considering ‘all things considered better than’ since it seems reasonable that one of the things that should be relevant in an ‘all things considered’ judgement is the dimension of the most appropriate unit of comparison, in this case pairwise, from the structure of the scenario. This is an argument for giving primacy to the dimension of the unit of comparison over the dimension of the choice set in our determination of what the ‘better than’ relation ought to mean.

I have called arguments of this class *Unambiguous Arguments* because, in contrast to spectrum arguments, the transitivity of the relation at the centre of the example is unambiguous. It is clearly not dealing with incommeasurable, incomparable or vague alternatives, it is not due to a failure of intuition, and there is no ambiguity from the competing claims of degree and kind. What some may argue is that moral betterness has a status that invalidates the direct conversion from other unambiguously intransitive relations into moral ‘better than’ relations.

Indeed a claim that one can make this conversion may have a number of arresting conclusions. Reverting back to the altruistic gambler example, if the league were additionally packed with a dozen other teams all with the same abilities as team B, then it would be true to say that, for the altruistic gambler, team A was a good team to choose, and team C was a bad team to choose, but in a single pairwise comparison he should prefer to choose team C rather than team A. This would seem to be a counterexample to Nebel’s claim that “I cannot see how any agent — let alone a rational one — could desire \(x\), prefer \(y\) to \(x\), and yet not desire \(y\).” It may be argued
that it is a false move to claim this, because the desirability of A and B (x and y in Nebel’s quote), and the preference for one over the other are being used incompatibly in the examples of A, B and C. But this is precisely the point. If we require that these are compatible when the desirability is determined in relation to a comparison class beyond just A and B, then we are requiring that the ‘better than’ notion is transitive. So our choice is whether we allow nontransitive ‘better than’ notions at the cost of no longer being able to apply monotonicity principles to reach conclusions on the wider set of choices. Or we insist that ‘better than’ notions are transitive at the cost of refusing to allow that some of the notions of ‘better than’, of the type used in this paper (‘expected to beat’, ‘more desirable’ (cars), Unambiguous Arguments), are really ‘better than’ notions, and at the cost of making the assessment of whether A is better than B dependent on the other alternatives C, D, E etc..

7 ‘Better than’ preferences

In framing the insistence on the transitivity of ‘better than’ as a choice, we are now in the uncomfortably iterative position of trying to determine when one ‘better than’ notion is better than another ‘better than’ notion. In competitive sports we are able to ground this preference in two features of the situation. First in contrast to other areas where defining absolute and relative value — ‘good’, ‘bad’, or ‘better than’ — may be less clear, in sports we have a fundamental principle on which we may base any ‘better than’ notion, that winning is better than losing. With this in mind, as I argued at the start, any ‘better than’ notion consistent with this principle seems plausible. Second the definitional unit of comparison in sports competition is essentially pairwise (at least in the sports scenarios we are considering here). Matches involve just two teams. We do not have matches of three, or four, or five teams. So giving primacy to the pairwise comparison in our understanding of ‘better than’ over the necessarily aggregationist concept of ‘good’ or ‘bad’ seems inkeeping with this fundamental unit. It is this second insight that also suggests that the ‘expected to beat’ notion of ‘better than’ may be reasonably thought to be an ‘all things considered better than’ notion in the context of sports where matches necessarily consist of just two competitors. To the degree that this is considered true, the arguments made here for nontransitivity then pertain to ‘all things considered better than’ relations as well as ‘better than’ relations.

Let us now consider the question of what might make one ‘better than’
relation better than another ‘better than’ relation in questions of morality. In considering the preference for transitive or non-transitive interpretations of ‘better than’ in the moral realm, it is difficult to see how one would go about comparing the ‘goodness’ of ‘better than’ relations. Perhaps one way of doing so would be to use the relation that embodies the most truth. In that the nontransitive relation may tell us about the outcomes of all pairwise comparisons it is more informative and may be argued to represent a greater amount of truth. More clearly, it seems at least plausible to suggest that one ‘better than’ relation may be preferred to another ‘better than’ relation if the choices it leads to are expected to be better.

Taking the racing evil-doer example, if we insist that ‘better than’ is a transitive relation then we may only say that the alternatives are equally good since amongst the choice set \{A, B, C\} they have equal probability of being first to reach the target against an unknown alternative. However if we allow the ‘better than’ relation to be transitive then we can advise the good actor that A is better than B, B is better than C, and C is better than A. She is therefore expected to make a choice leading to a better outcome when the nontransitive ‘better than’ relation is employed.

A much wider claim for using a nontransitive notion that gives primacy to the pairwise comparisons might make the argument that the semantic structure of ‘Qer than’ is essentially pairwise, and so an interpretation of ‘Qer than’ that obtains for all pairwise comparisons should be preferred in favour of one that is transitive across the choice set. “A is Qer than B” is a phrase that permits only the comparison of two items. One might seek to add others by, for example, saying “\{A,B\} is Qer than \{C,D\}” or “A is Qer than B and C” but these are still essentially pairwise. In “\{A,B\} is Qer than \{C,D\}” the items are now sets rather than individuals but there are still only two of them, and “A is Qer than B and C” is the conjunction of the two pairwise comparisons “A is Qer than B” and “A is Qer than C”. Under such a view, the essentially pairwise nature of the comparison that I argued was a result of the match scenario or the existence of just two available vehicles, is actually present in all relations of the form “Qer than” simply due to the semantic structure of the term.

I am not persuaded of this view. I see it in the same vein of semantic dogmatism as the arguments in favour of transitivity that assert, as Broome et al. (2004, p. 61) puts it, “[t]he basis of the argument is simply that ‘better than’ is the comparative of the monadic predicate ‘good’.” If ‘better than’ is truly to matter in our consideration of choice then it must relate to the decisions that we take, and so to base the understanding on the semantics, rather than vice versa, seems backward. In the racing evil-doer
example above, the ‘goodness’ approach had a minimal amount to offer in interpreting which ‘better than’ notion to use, and led us astray in advising the good actor. In the same way, if we envisage a similar example but this time with a larger class of thousands of vehicles that follow a roughly, though not entirely, transitive betterness relation then it may be morally better, in a consequentialist sense at least, to employ a simpler transitive ‘better than’ notion than one that permits all the pairwise ‘better than’ relations, as the time to communicate the full relations would be so greatly larger that the chances of a good decision would be reduced and so the probability of thwarting the evil-doer decreased. As such, the transitive notion of ‘better than’ would then be better than the nontransitive notion of ‘better than’.

More generally, suppose that we have choices A, B, and C, where a cyclic preference relationship exists for the paired comparisons — A is preferred to B in a consideration of A and B, B is preferred to C in a consideration of B and C, C is preferred to A in a consideration of C and A. Suppose we make our choice of the appropriate ‘better than’ relation to use and write down our summary of the items’ relative merits. In the first choice we just note that they are equal, in the second choice we explain the cycle. Now suppose someone comes along who has not independently observed the comparisons or the comparators, and only has this information to go on. They are asked to choose one, but told that alternative C is no longer available. Then the second summary is clearly more useful for their practical reasoning. This suggests, for cases where there are a finite, possibly small, number of discrete alternatives then informationally richer nontransitive ‘better than’ notions may be preferable. For larger sets of alternatives or continuous alternatives, especially of high dimension, it may make sense instead to insist on transitivity for the purposes of practical reasoning, since any single pairwise understanding of ‘better than’ becomes a tiny, perhaps infinitesimal, amount of information in understanding the overall choice set, and in foregoing monotonicity principles we have lost the powerful ability to extrapolate relative relations to a much wider set, something that may in itself have moral value. Beyond the identification of the nature of the choice facing us in understanding ‘better than’, the sports context has allowed an explication for why we might challenge the interpretation of a monotonicity principle as it is used in Nebel (2018), and this challenge would seem to have relevance to wider considerations. More tentatively, even if the Unambiguous Arguments are found not to be persuasive, perhaps the plausibility of a nontransitive ‘better than’ notion in the sports context allows for a greater appreciation of how nontransitive ‘better than’ relations might work in a moral context. In this way perhaps being able to establish less disputably the existence
of nontransitive ‘better than’ relations in one environment forces us to not reject out of hand nontransitive ‘better than’ relations in a moral context.

Notes

1. Temkin (2014, p.468-471) discusses what he calls the ‘sports analogy’. Here he considers ‘wins most games against all other teams during the season’ as an ‘all things considered better than’ notion as a means of preserving the Axiom of Transitivity while maintaining the Essentially Comparative View in the context of wider practical reasoning. He rejects it on four grounds. I believe it mostly has no bearing on the arguments discussed in this paper, but while I agree with his conclusion in rejecting it, I find his arguments mostly unpersuasive. First he states: “to determine the best candidate among \( n \) applicants, each applicant must be compared to every other. As noted in section 13.1, this would require \( \frac{n}{2} \times (n - 1) \) separate comparisons.” This is simply statistically incorrect. As he himself seems to later acknowledge there are methods of ordering the comparisons such that significantly fewer pairwise comparisons are likely to be required. More substantially perhaps, even if it were correct, it would not invalidate the approach for all matters of practical reasoning, for example where a relatively small number of discrete alternatives were being compared. Second he rejects it on the grounds of not readily being able to identify the appropriate comparison class. Again it seems likely that there are numerous scenarios where that is unlikely to be an impediment. Third he rejects it based on wins not accounting for all aspects that should be considered, but numerous sports exist where points are awarded for outcomes other than just wins, for example rugby union, ice hockey, and county championship cricket. If there is an aspect that stakeholders prize enough for it to be considered within their determination of ‘better than’ then they may change the points system, so that instead of considering ‘has aggregated more wins against all other competitors’ they may consider ‘has aggregated more points against all other competitors’. Fourth he notes that the qualities that one might wish to use for a three-way comparison may be different to those one might wish to use for a two-way comparison. I find this more persuasive, but it seems to be an argument against using the sports analogy, or indeed any method, as a uniform notion applicable to all scenarios. There could however be scenarios where the appropriate factors to consider in a multi-item comparison are the same as those in a two-way comparison, and for those scenarios this argument would not allow us to reject a sports analogy approach.

2. Smead (2019) argues that social choice theory provides us with other possible methods of aggregating the pairwise comparisons. Which of these is used is unimportant to the argument in this paper however. We only require that a reversion to an aggregationist approach of some type is required in order to form concepts of ‘good’ and ‘bad’.

3. It is tempting to see this simply as the transitivity of aggregation compared to the potentially nontransitive pairwise comparison unit. But this would be a mistake since aggregations, even those seemingly respecting the primacy of winning over losing, need not be transitive. The so-called ‘minimum violations’ method where teams are ranked to minimise the number of occurrences of a lower ranked team having beaten a higher ranked team is an example of such a nontransitive aggregationist approach that respects the principle of winning being better than losing.

4. This argument is similar to the one that Broome et al. (2004, p.52-53) proposes in dis-
cussing lateness, where he draws a distinction between the ‘historical time’ that is the basis for the comparative and the ‘contextual time’ that is the basis for the absolute, and so concludes that the monotonicity principle is improperly applied to infer intransitivity in the understanding of ‘later than’.

5. Following the same notation, a mathematical expression of Temkin’s Internal Aspects View would then be that there exists some function $g : \mathbb{X} \rightarrow \mathbb{R}$ such that

$$f(x, y) = g(x) - g(y)$$

We may also use this notation to highlight that an Internal Aspects View is not necessarily for transitivity to hold. For example, suppose that the qualities of $X$ and $Y$ may be summarised by a single real-valued quantity. Then the function

$$f(x, y) = (x - y)^3 = x^3 - x^2 y + xy^2 - y^3$$

defines a transitive relationship that is not consistent with the Internal Aspects View.

6. Our discussions in this paper are all with respect to multi-dimensional examples, but this notation is also useful for clarifying an element of the discussion on transitivity in a uni-dimensional frame. Suppose we define $X$ and $Y$ as having single qualities of value $x$ and $y$ respectively, with $x, y \in \mathbb{R}$ and define $f$ by

$$f(x, y) = 4(x - y) - (x - y)^2$$

and consider teams $A$, $B$ and $C$ with qualities summarisable by the values $6$, $3$, and $0$ respectively. Then $A$ is better than $B$ since $f(6, 3) = 3$ and $B$ is better than $C$ since $f(3, 0) = 3$ but $A$ is not better than $C$ since $f(6, 0) = -12$.

One could readily put a real world moral interpretation to this. Suppose that we believe that equality is the crucial factor for defining the quality of a society, so that a society is better if equality is higher. In considering the ‘better than’ relation for the policies that pertain in a society we consider not just the current generation but future generations also. A change to policies that increase equality therefore may generally be expected to produce a society that is better than before the change. However suppose that if these policies are too aggressive in increasing equality in the present generation then the political backlash to the pace of this change brings reduced equality in future generations. Then this future effect, dependent on the absolute size of the change, may outweigh the present effect.

Initially one might object that this is simply a reference frame effect. A is not better than $C$ since $f(6, 0) = -12$, but $C$ is not better than $A$ since $f(0, 6) = -60$. We have a base case in each comparison and that is not the same in all the comparisons so the relation is different. Hence it is not reasonable to compare these conclusions. One might in turn challenge that by claiming that an existing situation is a common situation in practical reasoning, and that it is reasonable for that existing situation to have an impact on the evaluation of ‘better than’, and especially ‘all things considered better than’ where it may be regarded as one of the things to be considered. However the sports analogy perhaps once again helps us to see this more clearly. In that context we would think of it as a home team advantage. So that ‘expected to beat in a match played at A’s home’ and ‘expected to beat in a match played at B’s home’ are simply different notions of ‘better than’, and neither may reasonably be considered ‘all things considered better than’.

7. In fact one may define a uni-dimensional quantitative cyclic relationship if the dimension in which the value quantifications are made is itself cyclic. For example the angular coordinate in a polar coordinate system.
8. This makes the argument being made here different to the seminal work of Rachels (1998) where he states in conclusion “The rejection of Transitivity implies that we should not conceive value quantitatively.” While a rejection of a quantitative conception of value may indeed be correct, it seems to be a uni-dimensional concept of value that is being argued against, not a quantitative one.

9. There would seem to be at least four possible ways to motivate a distributional view. First by appealing to group usage, which since people will use the term differently provides a probability distribution. Second that for any individual any point that they identify only becomes a point at the moment of identification. So had we asked at even a momentarily different time or under slightly different circumstances we may have received a different response, and in this sense any individual in fact holds a distributional view that only becomes a single point on inspection. Third that the property is simply distributional independent of usage, due to the complexities of defining it. This is widely accepted for many properties when we are thinking of them in a future state, even measurable quantitative ones. It seems plausible therefore that qualitative properties considered in a quantitative way (for example in arguing for a point notion such as ‘neutral’) may be distributional. Fourth that the true property is something more complex altogether but that the best workable approximation is the distributional one, allowing us to reason, but with an appropriate degree of uncertainty.

10. We may also wonder if ‘neutral’ should more properly be considered as a distribution rather than a point. It is not immediately clear what that would mean for these arguments. One version might hold that, even if we think of pleasure and pain as commensurable, existing on a common unidimensional spectrum, they themselves may be probability distributions, so when one is asked to name the duration of ecstasy to balance a year of excruciating pain, this is impossible since one is highly unlikely to be able to match the probability distribution of ‘neutral’ through a weighted sum of the probability distributions of pleasure and pain. There is no reason to believe that the probability distributions are of a form that would allow this. An alternative would then be to attempt to match a summary statistic of the ‘neutral’ probability distribution — perhaps most obviously the mean or mode — but then we are not recreating a ‘neutral’ outcome but rather an approximation of it that is located at a similar place.

11. Readers may recognise this example as relating to nontransitive dice examples such as Efron’s dice.

12. Broome et al. (2004, p.52) discusses a very similar sports example but offers only this argument ‘The very fact that the relation ‘can regularly beat’ may be intransitive amongst football teams should make you realize it is not equivalent to ‘is better than’.” Thus he recognises the intransitivity of the sports scenario but assumes that this in itself must disqualify it from any claim to being a ‘better than’ relation. This seems a somewhat tautological claim — an intransitive counterexample to the transitivity of ‘better than’ cannot be an example of a ‘better than’ relation because it is intransitive.

References


