Ranking Methodologies
(and a few extra bits)

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This is where I started...

Coventry
Partly Cloudy - 8°C
11:59 AM

Quick facts

Coventry is a city in central England. It's known for the
...then I lived here...
...and here...
Singapore, an island city-state off southern Malaysia, is a global financial center with a tropical climate and multicultural
...and the one place on earth I swore I’d never end up was here. (fail)
Bradley Terry

In the context of tournaments, the probability that team $i$ beats team $j$ is given by

$$P(i \succ j) = \frac{\pi_i}{\pi_i + \pi_j}$$

where $\pi_i$ is positive-valued, and can be thought of as a parameter reflecting the strength of team $i$.

Zermelo (1929), Bradley & Terry (1952)
Extension to include ties

\[
P(i \succ j) = \frac{\pi_i}{\pi_i + \pi_j + \nu \sqrt{\pi_i \pi_j}}
\]

\[
P(i \approx j) = \frac{\nu \sqrt{\pi_i \pi_j}}{\pi_i + \pi_j + \nu \sqrt{\pi_i \pi_j}}
\]

Davidson (1970)
Extension to account for home advantage (order effects)

\[
P(i \succ j) = \frac{\pi_i}{\pi_i + \gamma \pi_j + \nu \sqrt{\pi_i \pi_j}}
\]

\[
P(i \prec j) = \frac{\gamma \pi_j}{\pi_i + \gamma \pi_j + \nu \sqrt{\pi_i \pi_j}}
\]

\[
P(i \approx j) = \frac{\nu \sqrt{\pi_i \pi_j}}{\pi_i + \gamma \pi_j + \nu \sqrt{\pi_i \pi_j}}
\]

Davidson & Beaver (1977)
Applying to 3 for a win, 1 for a draw

\[ P(i \succ j) = \frac{\pi_i}{\pi_i + \pi_j + \nu(\pi_i \pi_j)^{1/3}} \]

\[ P(i \approx j) = \frac{\nu(\pi_i \pi_j)^{1/3}}{\pi_i + \pi_j + \nu(\pi_i \pi_j)^{1/3}} \]

See: alt-3.uk

Firth (2017)
Q: Wouldn’t it be nice if there was a sport with which I was familiar, where the points system was just a bit more complicated?
Extensions

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A: Rugby union!
Q: Wouldn’t it be nice if there was a sport with which I was familiar, where the points system was just a bit more complicated, where there was a system of matches that do not make up a full round robin?
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Schools rugby!
Q: Wouldn’t it be nice if there was a sport with which I was familiar, where the points system was just a bit more complicated, where there was a system of matches that do not make up a full round robin, and there was an actual tournament based on the results of these matches?
Extensions

Q: Wouldn’t it be nice if there was a sport with which I was familiar, where the points system was just a bit more complicated, where there was a system of matches that do not make up a full round robin, and there was an actual tournament based on the results of these matches?

A: Daily Mail Trophy!
Extensions

Q: Wouldn’t it be nice (for me, at least) if there was a sport with which I was familiar, where the points system was just a bit more complicated, where there was a system of matches that do not make up a full round robin, and there was an actual tournament based on the results of these matches, and the methodology they currently use could do with some serious improvement?
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A: Full house!
Rugby union scoring rule

- 4 points for a win
- 2 points for a draw
- 0 points for a loss
- 1 bonus point for losing by less than seven points
- 1 bonus point for scoring four or more tries
RASR (pronounced ‘razor’) - Ranking Algorithm for Schools Rugby

Part one: result outcome

\[ P(\text{team } i \text{ beats team } j \text{ by wide margin}) \propto \tau^4 \pi_i^4 \]
\[ P(\text{team } i \text{ beats team } j \text{ by narrow margin}) \propto \kappa \tau^3 \pi_i^4 \pi_j \]
\[ P(\text{team } i \text{ draws with team } j) \propto \nu \pi_i^2 \pi_j^2 \]
\[ P(\text{team } j \text{ beats team } i \text{ by narrow margin}) \propto \frac{\kappa \pi_i \pi_j^4}{\tau^3} \]
\[ P(\text{team } j \text{ beats team } i \text{ by wide margin}) \propto \frac{\pi_j^4}{\tau^4} \]
Part two: try bonus outcome

\[ P(\text{team } i \text{ and team } j \text{ both gain try bonus point}) \propto \theta \pi_i \pi_j \]
\[ P(\text{only team } i \text{ gains try bonus point}) \propto \tau \pi_i \]
\[ P(\text{only team } j \text{ gains try bonus point}) \propto \frac{\pi_j}{\tau} \]
\[ P(\text{neither team gains try bonus point}) \propto \phi \]
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Part three: Add a prior
Example Methodological offshoot

A robust mean

$$\mu = \frac{m}{\sum_{i} \frac{1}{1 + \pi_i}} - 1$$

idea due to David Firth
Areas of potential further study 1

Ranking theory

- Influential edges within B-T ranking
- Investigation of prior
  - IQR over time
  - Simulated same ability teams ranking
  - Empirical similar ability teams ranking
- Home advantage – distance vs fixed
- Extending violations beyond pairwise to sub-tournaments of size $>2$
- PageRank $\approx$ RASR (Build on David Selby work, if he doesn’t want to!)
- B-T explained in terms of moving down parameters of family of loss functions
- Is there an iterative form of LPPM that gives B-T?
- Philosophy of retrodictive vs predictive
Areas of potential further study 2

Ranking applied

- European rugby (website)
- County championship cricket
- Influential liquidity providers in market system
- Time series of citations ranking (the rise of the economists)
- Schools ranking by pupil movements
- University Teaching ranking by pairwise survey comparison
- Using pairwise ranking to improve secular ranking e.g. Thomson Reuters Women
- Soft power index search in one country give website in another
- Twitter influence through Bradley-Terry