A Brief History of Risk

Banks and other financial institutions have to look after (and ideally profit from) money in risky environments. They somehow have to manage the risk: something which has never been more apparent to experts or to the general public; it is certain that specialists in the mathematics of risk will be highly in demand for years to come. Financial mathematics provides the tools required to manage risk effectively.

From Farming to Futures

Risk has been present since the dawn of human civilisation. In the first days of agriculture it became apparent that in some years crops fail whilst in others there may be a glut. Farmers found that both extremes worked against them: in times of famine they had nothing to sell and in times of excess the value of their crops fell to almost nothing. Both merchants and farmers found it to be to their advantage to draw up contracts which provided both sides with a measure of security by fixing a price for grain prior to its harvest. With the passage of time, this idea of trading in "futures" has widened: from grain to metals, coffee, sugar and even concentrated orange juice. Indeed, much more abstract "financial instruments" have been derived from the same basic ideas.

Nobel Prizes

Major theoretical advances have transformed this type of trading.

Physics:
In 1905, Einstein's miraculous year, he published five revolutionary papers. One of these developed the mathematical model of Brownian motion; a model for the motion of individual molecules. In 1921 he received the Nobel prize for his contributions to physics. Many famous mathematicians have worked on this model developing a rich and powerful theory of random motion which can be applied to the price of financial instruments as easily as to the positions of molecules.

Economics:
Theoretical economists in the 50's and 60's studied the contracts that were being used to hedge risk. In 1973, Black and Scholes published a powerful formula for pricing these contracts using Einstein's Brownian Motion to model the underlying random phenomena. Scholes received the Nobel prize in 1997.

Trading in Risk

It's now commonplace for trading to encompass complex derivative contracts based on share prices, currencies, interest rates or debt. Recently credit derivatives (especially "insurance" against defaults) have been of interest to the general public and much discussed by the mainstream media. The trade in such contracts has reached trillions of dollars worldwide and financial institutions must, as we have all seen, manage their exposure to all types of risk (and often the best way to eliminate risk is to make it worthwhile for another party to acquire it): they need highly skilled individuals who know how to use the theory in the right way.

Risk and Reward

With uncertainty comes both risk and the potential for profit.

Consider the following game:

a coin is tossed and you gain one pound if heads is showing and nothing otherwise. What would it be fair to pay to enter this game? If heads and tails are equally likely then you might expect to make about fifty pence each time you play (averaged over a large number of games). This suggests that paying more than fifty pence per game would be extravagant whilst paying less would seem to be a good deal. A price of fifty pence, the so-called *expected payoff*, seems about right.

Generalising this idea to real world situations can be a first step towards dealing with uncertainty, but it can be far from easy! Consider a stock instead of a coin. The value of the stock S_T at a future time, T, is unknown : it's a random quantity. In a "European call option" you pay an amount V_0 to have the *option* to buy one unit of the stock at time T for an amount K. If it turns out that $S_T>K$ then you will be able to gain a profit of S_T-K; otherwise you won't *exercise the option*. The overall gain at time T will be the positive part of S_T-K.

What is the fair price V_0 for this option? The answer is not as obvious as you might think having considered the coin problem. In the coin problem the profit was immediate; in the stock market problem it is not and we have to consider the profit we could make by investing K elsewhere. Black and Scholes produced a marvellous answer to this problem in 1973.

Under suitable assumptions, they showed that the fair price in this setting is $S_0\Phi(d)-Ke^{rT}\Phi(d-\sigma\sqrt{T})$ where σ measures the rate at which the stock price varies randomly, Φ is the cumulative distribution function for the standard normal random variable, r is the compound interest rate and $d\sigma\sqrt{T}=ln(S_0/K)+(r+\sigma^2/2)T$

The "suitable assumptions" do not often apply in reality. However, variations of this famous formula are still used today in many financial situations.

What's it about?

Sophisticated tools are needed to capture the influence and interaction of time and uncertainty. One key area is probability theory. Topics such as martingales (things which on average neither increased nor decrease), Brownian motion (a process which characterises pure diffusion) and stochastic calculus (the mathematical framework used to produce the Black-Scholes formula) were once studied exclusively by postgraduates but have now found their way into the undergraduate syllabus of many universities worldwide – including Warwick.

There has been an explosion of demand in the Finance Industry for suitably-trained graduates ranging from those with a specialist Master's degree in Financial Mathematics to those with a PhD in the mathematical sciences. This makes for a dynamic, challenging and exciting environment in which to work. Finance has also developed rapidly as an academic discipline and there are consequently many opportunities for mathematicians seeking a career in academia.