A matter of life and death: mortality estimation and prediction

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Why mortality matters

Mortality estimates forecasts are vitally important

- Tax and Expenditure
- Healthcare
- Pensions and Insurance

- **Goal**: Forecast future mortality with a realistic quantification of uncertainty.

The life table

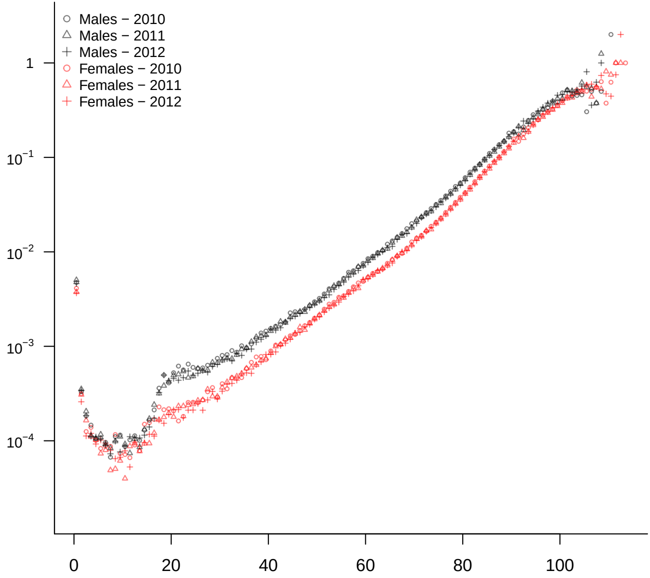
A static summary of the distribution of age at death for a population:

English Life Tables No 17

Period expectation of life
Based on data for England and Wales for the years 2010-2012

Age				Males				
x	m_x	q_x	l_x	d_x	L_x	T_x	μ_x	e_x
0	0.004757	0.004746	100000	475	99576.3	7896837		78.97
1	0.000306	0.000306	99525	30	99510.2	7797072	0.000369	78.34
2	0.000207	0.000207	99495	21	99484.6	7697562	0.000246	77.37
3	0.000147	0.000147	99474	14	99467.0	7598078	0.000172	76.38
4	0.000115	0.000115	99460	12	99453.9	7498612	0.000128	75.39
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
109	0.676172	0.491440	8	4	5.5	11	0.661588	1.43
110	0.701065	0.503943	4	2	2.8	5	0.685990	1.39
111	0.725677	0.516003	2	1	1.4	3	0.709972	1.34
112	0.750015	0.528125	1	1	0.7	1	0.733841	1.30

Crude mortality rates 2010-2012



A basic smoothing model

We estimate the *Central Mortality Rate*

$$m_x \equiv \frac{\text{Expected number of deaths aged x in 1 year}}{\text{Population aged x at risk}}$$

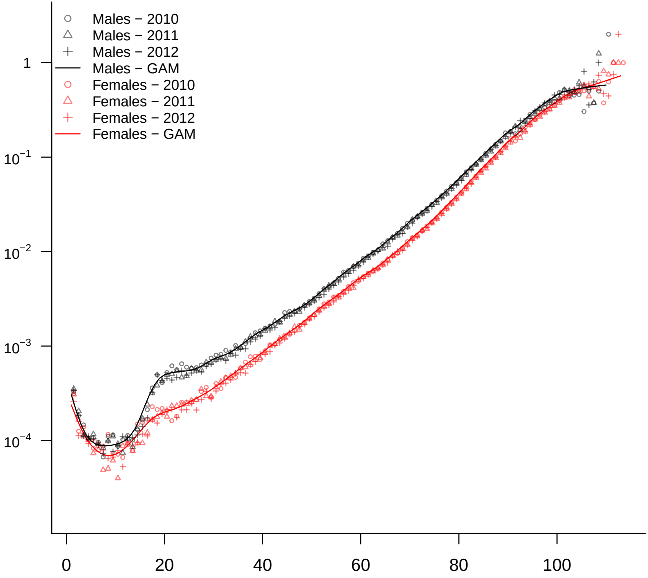
In a generalised additive (smooth) statistical model, we estimate the m_x by

$$\hat{m}_x = \exp s(x)$$

where s is a function which 'trades off' smoothness as a function of age with how closely the $\hat{m}_x$ are to the corresponding observed rates

$$\frac{\text{Observed number of deaths aged x in 1 year}}{\text{Population aged x at risk}}$$

Smooth mortality rates 2010-2012



Models for older ages and extrapolation

To obtain a more robust fit at older ages, and to extrapolate the mortality function m_x beyond the range of the observed data, one might use the log-linear Gompertz model

$$\log m_x = \beta_0 + \beta_1 x, \qquad x \geq x_0$$

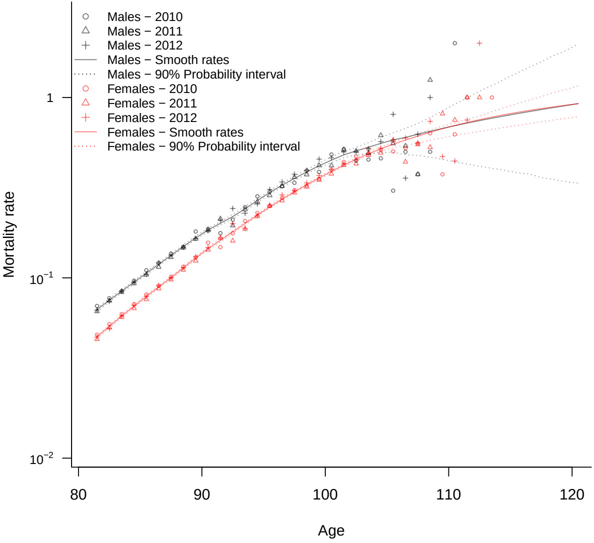
where x_0 is a suitable threshold

or

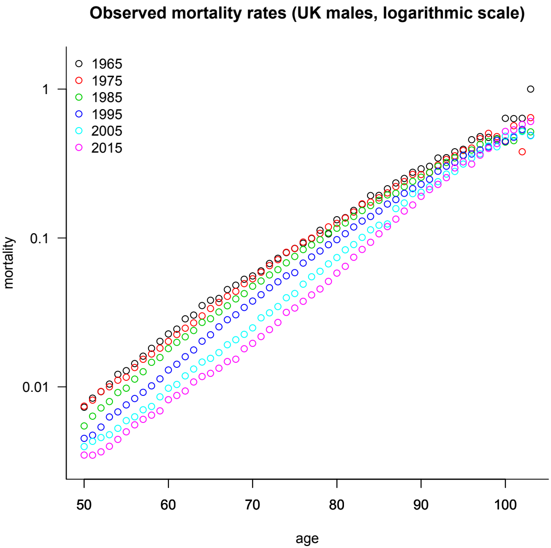
$$m_x = \frac{\beta_2 \exp(\beta_0 + \beta_1 x)}{1 + \exp(\beta_0 + \beta_1 x)}, \qquad x \geq x_0$$

where mortality rates flatten off, converging to the limit β_2 as $x \to \infty$.

ELT17 modelling at high ages



Mortality improvement



Dynamic models and projection

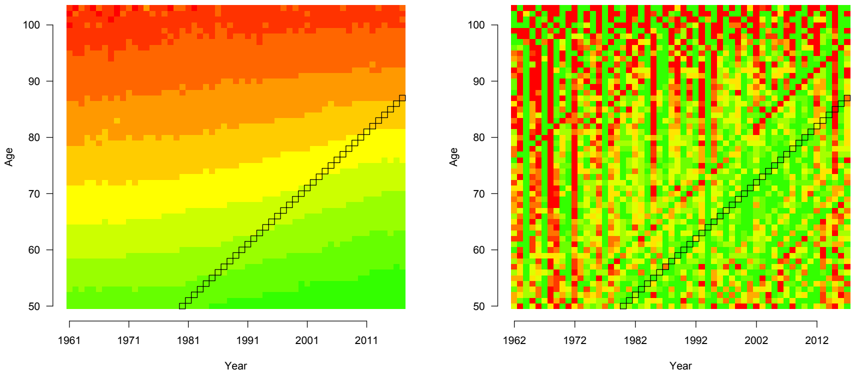
Now mortality varies not just by age, but over time as well.

We denote by m_{xt} the central mortality rate aged x, in year t, in population of interest, for $t = 1, \ldots, T =$present.

Statistical mortality models provide a framework for projecting m_{xt} etc for $t = T + 1, T + 2, \ldots$

Hence providing us with the information we need for planning.

The three time dimensions



A cohort is a subpopulation sharing a common birth-year. (1930 birth cohort identified above)

The model

Age-period-cohort (APC) GAM for mortality improvements

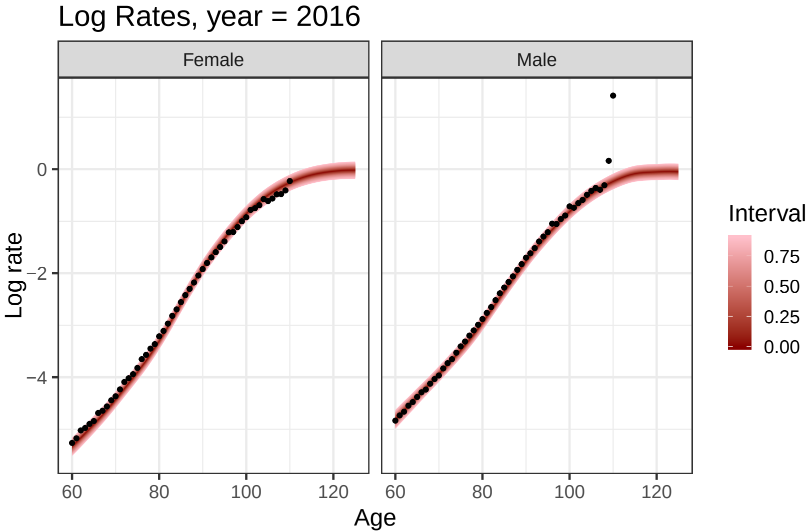
$$\log \frac{m_{xt}}{m_{x\,t-1}} = s_\alpha(x) + \kappa_t + s_\gamma(t-x)$$

For the highest ages $x > x_0$, use the structured model

$$m_{xt} = \frac{\beta \exp\left(\mu_0 + \mu_1 x + (\alpha_0 + \alpha_1 x)t\right)}{1 + \exp\left(\mu_0 + \mu_1 x + (\alpha_0 + \alpha_1 x)t\right)} \exp\left(\kappa_t + s_\gamma(t-x)\right)$$

Forecast (with uncertainty)

Fit on data up to 2006, 10 year projection.



Forecast life expectancy (with uncertainty)

Fit on data up to 2006.