2. Expanding Base II

What is

$$c_n = \sum_{k=n+1}^{\infty} \frac{k-1}{k!}?$$

Hint What is $c_1 = \sum_{k=2}^{\infty} \frac{k-1}{k!}$?

Extensions

(1) Show that every rational x in (0,1) can be written as

$$\sum_{k=2}^{\infty} \frac{y_k}{k!},$$

with $y_k \in \{0, 1, \ldots, k-1\}$ for each k, in exactly *two* ways: one in which all but finitely many of the y_k's are 0 and the other in which all but finitely many of the y_k's take the value $k-1$.

Hint Prove this by contradiction: so suppose that $x = \frac{m}{n!} = \sum_{k=2}^{\infty} \frac{y_k}{k!}$, with infinitely many of the y_k's not being zero and infinitely many not being $k-1$ and deduce that x cannot be of the form $\frac{m}{n!}$

(2) Show that $e-2$ is not a rational number (and hence e is not rational).

Hint What is the power series for e^x?