

## 2. EXPANDING BASE II

What is

$$c_n = \sum_{k=n+1}^{\infty} \frac{k-1}{k!}?$$

### Extensions

- (1) Show that every rational  $x$  in  $(0,1)$  can be written as

$$\sum_{k=2}^{\infty} \frac{y_k}{k!},$$

with  $y_k \in \{0, 1, \dots, k-1\}$  for each  $k$ , in exactly *two* ways: one in which all but finitely many of the  $y_k$ 's are 0 and the other in which all but finitely many of the  $y_k$ 's take the value  $k-1$ .

- (2) Show that  $e - 2$  is not a rational number (and hence  $e$  is not rational).