Same birthdays

Question:

(1) There are 27 students in your Maths class. What is the probability that at least one of your classmates has the same birthday as you?
(2) Suppose there are n students in a class. What is the probability that at least two students in the class have the same birthday?

For simplicity assume there are no leap years: each year has 365 days.

Hints:

- Sometimes it is easier to calculate the probability of the opposite of an event. What is the opposite of "at least one"? What is the opposite of "at least two are the same"?
- Can you make use of the fact that the birthdays of different students are independent? If you do not remember what independence of events implies for probabilities, here is a tutorial: `https://www.mathplanet.com/education/pre-algebra/probability-and-statistic/probability-of-events`

Solution:

Part (1):
Instead of considering the probability of

$$A = \text{“at least one classmate has the same birthday as you”}$$

we consider the opposite event

$$A^c = \text{“none of the other classmates has the same birthday as you”}.$$

Birthdays are equally distributed over the year. (In reality this is nearly correct, though slight variation depending on season an events have been observed, but we ignore those for simplicity.) Hence the answer to the question is the same for whatever your date your birthday is. For each of your classmates, the probability to not have the same birthday as you is 364/365. Birthdays occur independently of each other, hence the probability factorises which yields

$$P(A^c) = \left(\frac{364}{365}\right)^{26}$$

and therefore

$$P(A) = 1 - \left(\frac{364}{365}\right)^{26} = 1 - 0.931154 = 0.068846.$$

So, the probability that at least one of your classmates has the same birthday as you is about 6.9%.

Part (2):
We use similar arguments and a similar strategy as in the previous part. Instead of considering the probability of

$$B_n = \text{“at least two of } n \text{ classmates have the same birthday”}$$

we consider the opposite event

$$C_n = \text{“all } n \text{ classmates have different birthdays”}.$$

Order all the students in the class. The answer to the question does not depend on what order you assigned nor what the birthday of the first student is. We also use the simplified assumptions that all birthdays are independent of each other.

The probability that the second students birthday is different from the first one's is

$$P(C_2) = 1 - \frac{1}{365}.$$

For the third classmate to not have the same birthday as the first two, they need to avoid both of these days. So, the probability for the first three classmates to all have different birthdays from each other is

$$P(C_3) = \left(1 - \frac{1}{365}\right) \cdot \left(1 - \frac{2}{365}\right).$$

By the same token, the probability for the first forth classmates to all have different birthdays from each other is

$$P(C_4) = \left(1 - \frac{1}{365}\right) \cdot \left(1 - \frac{2}{365}\right) \cdot \left(1 - \frac{3}{365}\right)$$

If you keep going, the probability for n classmates to not have the same birthday as any of the other ones is

$$P(C_n) = \left(1 - \frac{1}{365}\right) \cdot \left(1 - \frac{2}{365}\right) \cdot \dots \cdot \left(1 - \frac{n-1}{365}\right).$$

Notes:

- *Have you tried to calculate the probabilities of A or B_n directly? This is more cumbersome that the strategy we used.*
- *Calculate the $P(B_n)$ for some examples. You may want to find the smallest n for which the probability of B_n is bigger than the probability of its opposite. Guess first, and the compute it with the formula found in Part (2). How good was your guess?*