# Triangle mosaic

**Question:**

Select a starting point $P_0$ and draw a 1cm long line $c_1$ ending in $P_1$.

From $P_1$, make a 1cm line perpendicular to $\overline{P_0P_1}$. Connect its other end point $P_2$ with $P_0$ to obtain a triangle and call the hypotenuse $c_2$.

From $P_2$, make a 1cm line perpendicular to $\overline{P_0P_2}$ (away from the triangle). Connect its other end point $P_3$ with $P_0$ to obtain a triangle and call the hypotenuse $c_3$.

Keep going. Step $k$ looks like this:

From $P_{k-1}$, make a 1cm line perpendicular to $\overline{P_0P_{k-1}}$ (away from the previous triangle). Connect its other end point $P_k$ with $P_0$ to obtain a triangle and call the hypotenuse $c_k$.

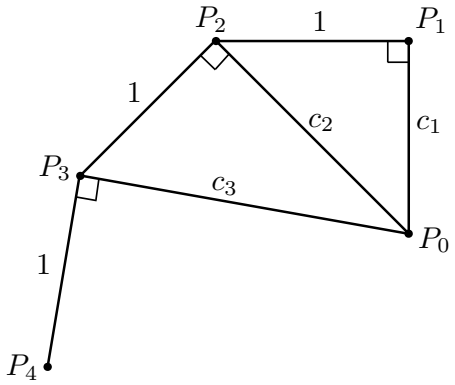
Denote the angles between $c_k$ and $c_{k+1}$ with $\alpha_k$ for $k = 1, 2, 3, \ldots$

Let $c_n$ be the first line to be more than one complete turn away from the starting line $c_1$.

**What is $n$?** Derive formulas for $c_k$ and $\alpha_k$ for $k = 1, 2, 3, \ldots$

**Hints:**

Construct the first few triangles on pencil and paper. Here is a start:




To find a formula for $c_k$ you could derive this for $k = 1, 2$, and 3, guess it for a general $k$, and then prove your conjecture using the technique of *induction*. If you have not learned this or you can not remember it, here are some resources:

- Section 2 in a proof technique handout from Dartmouth: `https://math.dartmouth.edu/~m22x17/misc/LaLonde2012_proof_techniques.pdf`
- Video tutorial by Kimberly Brehm: `https://www.youtube.com/watch?v=TqpNDiqsz7k`
- Guidance for Year 11 and Year 12 (Australian) teachers with many examples including *Tower of Hanoi* and a two-colour problem: `https://www.amsi.org.au/teacher_modules/pdfs/Maths_delivers/Induction5.pdf`

To find a formula for $\alpha_k$ use trigonometry and the formula for $c_k$.