

STEP and 'How to solve it'.

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21st/22nd March 2016

Purpose of this talk

- Give you a feel for a lecture at university.
- Look at solving some problems based on material from A-level maths – hopefully useful since you have exams coming up!

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Comments:

- No need to take notes – material will be on the web later. What I write on the blackboard is not a complete answer.
- Don't worry if you get lost.

A question for you:

Is mathematics a creative subject?

'How to solve it'

- 'How to solve it' is a short book describing strategies for solving maths problems.
- It was written by famous Hungarian mathematician George Pólya in 1945.
- It has sold more than 1 million copies.



*George Pólya*¹

¹ image from <http://cmc-math.org>

'How to solve it'

Pólya give 4 principles for problem solving:

- ① Understand the problem.
- ② Devise a plan.
- ③ Carry out the plan.
- ④ Review / extend.

1. Understand the problem

- This step is very important, and often neglected.
- What information have you got?
- What do you need to find / show?
- Draw a picture or diagram.

2. Make a plan

- This is the part requiring most **creativity**.
- Choosing a plan gets easier with experience.
- How have you solved other similar problems?
- Pólya discusses several potential strategies.

2. Make a plan

Pólya's strategies:

- Guess and check
- Make an orderly list
- Eliminate possibilities
- Use symmetry
- Consider special cases
- Use direct reasoning
- Solve an equation
- Look for a pattern
- Draw a picture
- Solve a simpler problem
- Use a model
- Work backwards
- Use a formula
- Be ingenious

3. Carry out the plan

This is the deductive part, and it's usually easier than making the plan.

- Use care and patience.
- Persist with the plan until you are sure that it won't work.
- If it doesn't work, choose another plan.

This is how degree-level mathematics is done – several failures followed by one success.

4. Review / extend

- Does your answer make sense?
- Can you check your answer?
- Look back at which plans worked and which didn't.
- Did you need all of the assumptions that you made?
- Can you use this problem to solve more complex problems?

1998 STEP III, q12

The villages A , B , C and D lie at the vertices of a tetrahedron, with each pair joined by a road. After a snowfall the probability any road is blocked is p and is independent of the conditions of any other road.

Show that the probability that it is possible, after a snowfall, to travel from any village to any other village by some route is

$$P = 1 - p^3(6p^3 - 12p^2 + 3p + 4).$$

1999 STEP II, q14

You play the following game. You throw a six-sided fair dice repeatedly. You may choose to stop after any throw, except you must stop if you throw a 1. Your score is the number obtained on the last throw. Determine the strategy you should adopt in order to maximise your expected score.



Both these questions are “toy examples” of really interesting and important applications of modern mathematics.

- 1 Connectedness of a random network.
 - Routing information via the internet
 - Gene expression and cell signalling in cancer
- 2 Optimal stopping strategies playing risky games.
 - Modern finance – should I buy / sell these shares now or wait until later?

My personal tips for STEP

- Practice trying to solve these kinds of problems.
 - Past STEP papers are available on-line!
- Read through the formula book with a teacher.
- Many STEP questions involve:
 - Binomial Theorem
 - Differentiation
 - Integration