Warwick Statistics

Risk and Predictability — Where Might Modern Mathematics Take Me?

Offer-holder Visit Day, March 2015

(Prof Mark Steel, Dr Vicky Henderson, Dr Julia Brettschneider)

Welcome to...

Offer-holders for these 3 degree courses:

- Data Science
- Mathematics and Statistics
- MORSE

...and parents or other accompanying persons!

The purpose of today

A varied programme of events, which we hope will:

- inform you
- inspire you
- help you to make the decision that is right for **you**, about which university offer to accept

11:15–12:00	Talk "Risk and Predictability — Where Might Modern Mathematics Take Me?"
12:00–13:00	Lunch Undergraduate Research Project Poster Exhibition Information about Careers, Accommodations, Funding, Admissions and Student-Staff Liaison
13:00–13:45	Talk "How to solve it? Examples from STEP and A-level papers"
14:00–15:00	Campus tour led by current students / Small group meetings with lecturers and professors
from 15:00	Tea, and more information

Where might modern mathematics take me?

Some things to know:

- Mathematics — and especially Statistics — becomes much more interesting at university level.
- The demand for well-rounded maths graduates remains absolutely buoyant, everywhere in the world.
- Demand for our kind of maths, especially so!

Our kind of maths?
Probability, statistics, operational research, mathematical finance, machine learning, ...
These are the most sought after areas of mathematics in the world at large.

In this talk we mention just a few of the exciting application areas for modern mathematics.

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Destinations of our recent graduates

A wide range of

- management consultancy
- investment banking
- medical research
- market research
- ‘big data’ in commerce, science, government, ...
- insurance and actuarial work
- software engineering
- social or economic research
- engineering consultancy
- sport, entertainment

More details on employment statistics and careers in the flyer in your pack

Some recent student projects

A few illustrative examples of what will be presented at lunch today:

- *Behavioural bias in financial decision making*
- *Statistical inference of stochastic differential equations*
- *Does having the right name bring more success?*
- *Comparison of population based Monte Carlo methods*
- *The transition density function of a genetic diffusion process*
- *Modelling of driver performance data*
- *Erdös-Kac theory and Mod-Poisson convergence*
- *Exponential random graphs modelling*
- *On the complexity and behaviour of crypto currencies compared to other markets*

Explaining the growth of countries

- Statistics: dealing with uncertainty.
- Setting: few countries with reliable growth data (n, usually less than 100) and many possible determinants of growth (p, often more than 30).
  Q: any thoughts on what could contribute to growth?
- Hard statistical problem: choose model among many.
  Q: how many different models if p=41 and models are characterized by inclusion or exclusion of each covariate?
  A: $J = 2^{41} = 2.2 \times 10^{12}$.
- In the face of model uncertainty, a formal Bayesian approach is to treat the model index as a random variable (unknown)

  Models $M_j, j = 1, \ldots, J$ in model space $\mathcal{M}$

  Prior $P(M_j)$ on $\mathcal{M}$ and data lead to posterior $P(M_j \mid y)$ where y represents data

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Bayesian Model Averaging (BMA)

Or do you really **have** to choose? Can use BMA:

Inference on quantity of interest, Δ, through mixing

$$P_{\Delta \mid y} = \sum_{j=1}^{J} P_{\Delta \mid y, M_j} P(M_j \mid y)$$

Probabilistic treatment of model uncertainty (like parameter uncertainty)

Use Bayes rule for inference given each model and inference on model space.

Typically J is huge: simulation over $\mathcal{M}$ using Markov chain Monte Carlo, which only tends to visit the most interesting models.

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Some results

We used a sample of average growth data for $n = 72$ countries and $p = 41$ possible covariates. We average over 150,000 models and the best model only has a probability of 1.24% assigned to it.

Important regressors:

- GDP level in 1960 (neg. effect, so convergence)
- Equipment investment (pos. effect)
- Life expectancy

surprising important ones:

- Fraction Confucian (Chinese indicator)

some surprising absences of strong effects:

- Primary school enrollment
- Higher educ. enrollment
- Revolutions and coups

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How you *really* make financial decisions

Psychologists have uncovered a wealth of **behavioural biases** in the way we make decisions under uncertainty.
We are **not** rational !

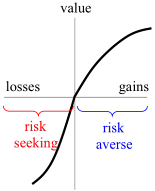
- BBC2 *Horizon* programme "How you *really* make decisions"
- *Thinking Fast and Slow*, D Kahneman (Nobel Prize, 2002)
- Government has a *Behavioural Insights Team* to provide policy recommendations

How do Mathematics, Statistics and Probability contribute?

- Identify potential biases — Analyze data & design statistical tests
- Develop stochastic models to capture human behaviour under biases: to explain and predict how we might behave — in particular, in a dynamic setting

Experimental and Empirical Evidence suggests....

Tend to prefer a certain £500 to a 50% chance of £1000 **risk averse over gains**
But prefer a 50% chance of losing £1000 to a certain loss of £500 **risk seeking over losses**



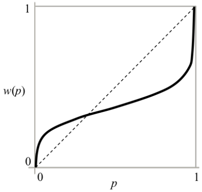
Averse to gambles such as (£110, 50%; $-$£100, 50%) **loss averse**
Use **reference points**, mental accounts, framing
Delay realization of losses (relative to gains) - *disposition effect*

Why do people buy lottery tickets and insurance?

Tend to prefer a
$\frac{1}{1000}$ chance of £5000 to a certain £5

But prefer a certain loss of
£5 to a $\frac{1}{1000}$ chance of losing £5000

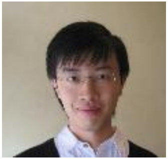
We tend
to **over-weight** small probabilities

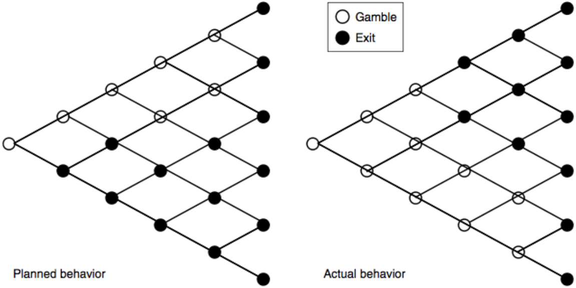


The National Lottery
Life changing®

How can students get involved? PhD level research

PhD student **Alex Tse**
is incorporating probability weighting
into stochastic trading models.
Time-inconsistent behaviour emerges.
Casino gambling.





What Research can I do as an Undergraduate?

Fourth year MMORSE student **Nikesh Lad** is analyzing individual investor behaviour with a very large dataset of trades - 158,000 accounts over a five year period.

Third year student **Rosie Ferguson** will be doing an 8 week **URSS** project with me this summer.

Behavioural Biases in Financial Decision Making

Warwick Statistics

NIKESH LAD | DEPARTMENT OF STATISTICS | SUPERVISORS: DR J. BRETTSCHNEIDER, DR V. HENDERSON

Introduction

Traditional economic theory postulates that investors are "wealth maximisers". However, emotion and psychological factors influence our decisions. Behavioural finance attempts to fill the void of phenomena in stock markets that cannot be described plausibly in models based on rationality.

Project aim:

- Investigate individual investor behaviour using real trading data.
- Explore whether the propensity to sell a stock is positively related to whether the stock has attained its historical high price.

Literature

Descriptive theory

- Heuristics: a mechanism or strategy which people use (often unconsciously) to reduce the complexity of tasks.
  - ∗Often leads to biases, e.g. framing and availability.
- Loss aversion: refers to the asymmetric motives people have to strongly prefer avoiding losses to acquiring gains.
- Disposition effect: a paradox where investors tend to "sell winners too early and rise losers too long."

Theoretical models

- Prospect theory: value function on the domain of gains and losses.
  - ∗Replaces expected utility with probability weighting function.
  - ∗Reflects the human tendency to overweight small probabilities and underweight high probabilities.

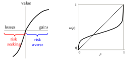


Figure 1: (a) value function (b) probability weighting function

Literature describes two broad categories of investor behaviour.

1. Time-consistent models

Threshold models: optimal strategy is to sell the stock the first time it reaches a threshold level; property known as time-consistency.

∗Example – realization utility.

2. Non time-consistent models

Regret models: investors observe the maximum price of a stock and 'gamble for resurrection.' Wait until the stock price reaches this historical high price again before selling – will not sell below this price.

Data

Use trading data from a US discount brokerage firm (Odean, 1998).

- January 1991 to December 1996.
- 78,000 unique households collectively with 158,034 accounts.
- Filter data for trades common stocks; leaves 10,373 stocks.
- A random sample of 10,000 households is taken for analysis.

Data has three main demographic categories: active trader, affluent households and general households.

Analysis

Holding times

Investigate three different holding times to develop a picture of investor behaviour.

Buy-to-sell – how long does an investor hold a stock for?

- Gamma curve fits the features of distribution well, verified by goodness-of-fit tests. Represents waiting time until the rth event.
  - ∗Event: the investor faces a *sell* versus *hold* decision.
- Interpret the shape parameter as characterising the investors level of patience which determines their waiting time.
- Would expect the shape parameter for active traders to be less than for affluent or general households.

Holding time	Median (days)	Mean (days)	Shape
Buy-to-sell	169	342	0.768
Active trader	163	312	0.707
General household	218	356	0.819
Affluent household	298	427	1.027
Maximum-to-sell	49	167	0.428
Buy-to-maximum	63	175	0.486

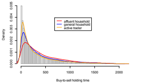


Figure 2: histogram of buy-to-sell holding time with demographic densities

Maximum-to-sell – does the observance of a maximum price increase propensity to sell?

- 42.1% of stock trades have maximum-to-sell holding time of less than 28 days.

Consider holding time *relative* to the buy-to-sell holding time.

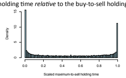


Figure 3: histogram of the distribution of scaled maximum-to-sell holding time

Produces interesting result, after noticing maximum price investors:

- Found to be selling stocks very promptly
- Found to be waiting a long time to sell; here maximum price happens very shortly after stock purchase.

Buy-to-maximum – how long does the investor wait to observe a historical high price?

- The longer the investor waits to realise a maximum price, the higher the median return, see Figure 6.

Return analysis

Analyse the returns of a stock trade, defined as

$$return = \frac{sell\ price - buy\ price}{buy\ price}.$$

- Large proportion of investors making *small* gains or losses, with 30.2% of trades with returns between −0.1% and 0.1%.
- Distribution of returns is leptokurtic with a large positive skew, distribution is not Normally distributed.

Demographic	Active	General	Affluent
Median return (%)	0.035	0.056	0.049

- A scaled-t distribution is found to provide an adequate fit.

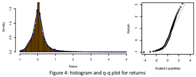


Figure 4: histogram and q-q plot for returns

Difference between maximum and sell price

Define the maximum price as the highest price that occurs since the stock was purchased and until the stock was sold (note that the maximum price can occur at the sell time itself).

- Investors typically observed to sell at a price just below the maximum price of the stock trajectory, since the stock was bought.
- 61.1% of stock trades sold within a price range of \$0 to \$5 below the maximum price.
- 12.6% of stock trades sold at the maximum price itself.

Relationships between returns and holding times

- Positive returns are best realised when the investor observes the maximum price and reacts promptly.

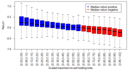


Figure 5: boxplots of returns versus scaled maximum-to-sell holding time

- Median return is negative if the investor waits roughly less than 20 days to observe the maximum price.

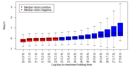


Figure 6: boxplots of returns versus buy-to-maximum holding time

Conclusions

A representation of a typical investor.

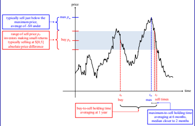


Figure 7: illustrative stock price trajectory

Propensity to sell seems to be higher if the investor observed a historical high price of the stock price trajectory and is dependent on a number of factors:

- Whether the stock is making a positive or negative return.
  - ∗Selling occurs at a prompter rate for positive returns.
- If the maximum price occurs at a time which is not close to when the stock was purchased (Figure 5).
- The longer the investor waits to realise a maximum price, the higher the median return – greater chance of experiencing maxima of greater magnitudes (Figure 6).
- The type of investor.
  - ∗On average, active traders have shorter buy-to-sell holding times and yield lower returns.
  - ∗Consistent with idea that active investment strategies can underperform passive strategies.

Is this behaviour time consistent?

- Not in the classical sense – large proportion of investors are selling stocks just below the maximum price and *not* the first time the price reaches some pre-determined level.

Bibliography

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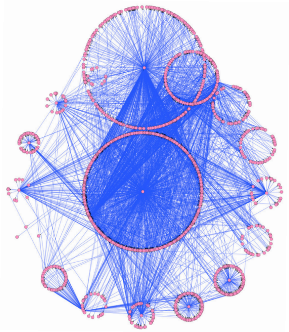
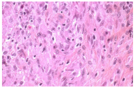
Big data in genomics and medicine

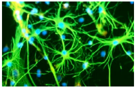
- Novel high-throughput molecular measurement technologies
- Genome-wide perspective
- Hope: New avenues for scientific research
- Medical applications in complex genetic diseases: etiology, prognosis, treatment
- **Challenges for mathematical sciences:**
  - Extract *information* from data
  - Ensure reproducible results
  - Model biological processes



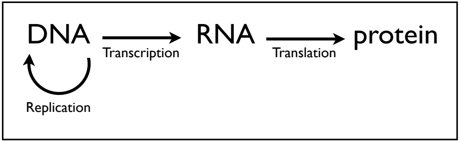
Example: Microarray Gene Expression Data

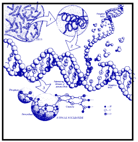
- DNA is the *blueprint* of the organism
- Your liver and your brain?



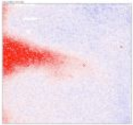


- Gene expression:

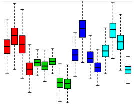




- Microarray: Expression of tens of thousands of genes simultaneously

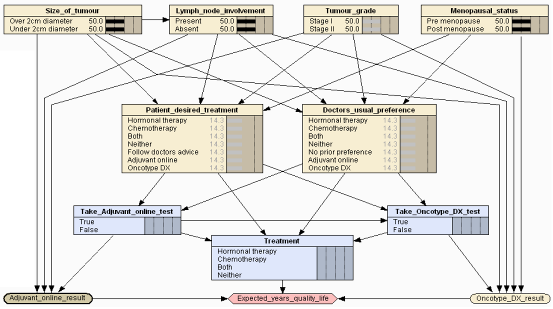


- Mathematical and statistical challenges:
  - High-dimensional noisy data
  - Models (e.g. preprocessing, networks)
  - Methodology to scaled up (e.g. multiple testing)



Example: Decision making in Cancer Recurrence Prevention

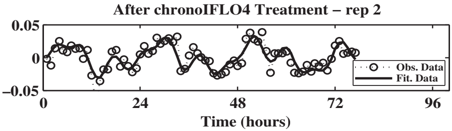
- Adjuvant treatment?
- Recurrence risk based on gene expression panel
- Complex decision under uncertainty
- Bayesian networks



Example: Chronotherapy

- Maximising treatment efficacy while minimising side effects
- Medication aligned with circadian clock (time series analysis)

$$\hat{x}_t = \sum_{j=1}^{\hat{N}} \hat{a}_j^{\hat{N}} \sin\left(2\pi t/\hat{p}_j\right) + \hat{b}_j^{\hat{N}} \cos\left(2\pi t/\hat{p}_j\right)$$



Mathematics as language of sciences and social sciences

"The instrument that mediates
between theory and practice,
between thought and observation,
is mathematics;
it builds the connecting bridge and
makes it stronger and stronger."

David Hilbert
Mathematician
1862-1943
23 Problems at
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- Seen today: economics, finance, genomics/medicine.
- More: sociology, psychology, demography, actuarial sciences, epidemiology, physics, chemistry, geology, geography, agriculture, engineering, communication, traffic, music, sports, astronomy, business analytics and more
- Brochure in your pack: *Warwick Statistics Research Spotlights*

Studying at Warwick Statistics

- Medium size department, still growing
- Design of interdisciplinary degrees, teaching committee, student staff liaison committee (SSLC), personal tutor system
- Senior scholarship, Prizes (4 graduation, 4 UG, STEP)
- Learning happens in lectures, exercise classes, tutorials, labs, projects, library study, problem solving, student teams
- Diverse student body



What else happen's in a day?

- 270+ student societies such as Argentine Tango, Science Fiction, Debating, Hindu, Music ensembles... - or set up your own!
- 73 sports clubs, 100+ competitive teams, world class facilities
- Art Centre (2 theatres, cinema, concert hall, art gallery)
- **Employability skills:** communication, problem solving, planning & organisation, time management, team work
- Also: Enjoy performances, parties & relax
- Sample schedule (UG websites)

Questions?

What next?

now Lunch: Undergraduate Research Project Poster Exhibition, Careers, Funding, Accommodation, Admissions, Student-Staff Liaison (Daniel Wison-Nunn & Pieris Christofi)

1pm Talk by Dr Jon Warren: "How to solve it!" (for students only) (Alternative event for accompanying persons: campus tour)

2pm Campus tour and small group meetings (for students only)

3pm Tea