Blue puzzle: Black and white circle

Question: A circle of radius 1/2 is painted black and white, with the total length of the black region being 1. Show that there must be three white points that form the vertices of an equilateral triangle.

Hints:

- Rephrase the task: You want to show that there is an equilateral triangle inscribed in the circle such that all its vertices coincide with white points (on the circle).
- Rather than actually constructing such a triangle, show that the probability that there is such a triangle is larger than 0. (This way you actually show that there are infinitely many triangles of the kind you are after. In particular that implies there is one of them.)
- Start by choosing at random an equilateral triangle inscribed in the circle.
- Consider that all such triangles are equally likely to be chosen.
- Don't forget the obvious, the opposite of a white point is a black point (in the context of this puzzle).
- What is the probability that one of the triangle's vertices is black? What about all three?

Solution: Suppose we choose an inscribed equilateral triangle uniformly at random. Each vertex has probability $1/\pi$ of being black. Thus, the probability that any one of the vertices is black is no more than $3/\pi$, which is smaller than 1. Thus, there must be some ways of placing the triangle such that all the vertices are white.

Note: The same reasoning can be done without using probabilities. However, this is an example where the use of probabilities makes the solution concise and elegant.