Blue puzzle: Black and white circle

Question: A circle of radius $1/2$ is painted black and white, with the total length of the black region being 1. Show that there must be three white points that form the vertices of an equilateral triangle.

Solution: Suppose we choose an inscribed equilateral triangle uniformly at random. Each vertex has probability $1/\pi$ of being black. Thus, the probability that any one of the vertices is black is no more than $3/\pi$, which is smaller than 1. Thus, there must be some ways of placing the triangle such that all the vertices are white.

Note: The same reasoning can be done without using probabilities. However, this is an example where the use of probabilities makes the solution concise and elegant.

Extensions:

(1) Is it necessarily possible to find four white points that form a square?
(2) Is it necessarily possible to find four white points that form a rectangle?

Solutions to extensions:

(1) No. For example, if we paint a whole quadrant of the circle black then there is no white square. The quadrant has length $\pi/4 < 1$.
(2) Yes. We can choose a random rectangle ABCD by choosing two independent uniform random diameters AC and BD. By the same reasoning as above, each diameter has probability at least $1 - 2/\pi$ of being white, so the probability of all four vertices being white is at least $(1 - 2/\pi)^2 > 0$.