Basic Electrophysiology: Questions

• Q1. The Charge on the membrane

The membrane capacitance is $10\text{nF}/\text{mm}^2$. A certain neuron can be approximated as a sphere of radius 20μm and a resting potential of -65mV.
[Q] What is the total capacitance of the cell?
[A] $A = 4\pi r^2 = 5 \times 10^{-9}\text{m}^2 = 5.03 \times 10^{-3}\text{mm}^2$, so that $C = cA = 50.3$pF
[Q] What is the membrane charge when the neuron is at its resting potential?
[A] $Q = CV$ so that $Q = 3.27 \times 10^{-12}C$.
The charge on an electron is $1.602 \times 10^{-19}C$.
[Q] How many ions does the membrane charge correspond to?
[A] 2.04×10^7 ions.
The molarity of internal free charge carriers is given in table 1.
[Q] What is the total number of charge carriers in the cell?
[A] Total ionic Molarity is 50 + 397 + 40mM= 0.487M. Total volume of the cell $4\pi r^3/3 = 3.35 \times 10^{-14}\text{m}^3 = 3.35 \times 10^{-11}$litres. Number of moles of ions in this volume is 1.63×10^{-11}. Number of free charge carriers $\sim 10^{13}$.
[Q] What is the ratio of charge carriers on the membrane to those in the bulk of the cell?
[A] About 10^{-6}.

• Q2. Diffusion in a Channel

An ion of concentration N, with fixed internal N^i and external N^e concentrations is diffusing through a trans-membrane channel. The internal side of the membrane is at $x = 0$ at V^i and the external at $x = a$ at V^e. The protein forming the channel is itself charged, so that the voltage $V(x)$ inside the channel cannot be assumed linear and so the induced drift velocity v is not necessarily constant. The density N obeys the diffusion equation and continuity equations

$$\frac{\partial N}{\partial t} = D\frac{\partial^2 N}{\partial x^2} - \frac{\partial}{\partial x}(vN) \quad \text{and} \quad J = vN - D\frac{\partial N}{\partial x} \tag{1}$$

[Q] Use this equation to derive the general steady-state, zero-flux (when $J = 0$) density of ions in the channel as a function of $V(x)$ using the forms for D and v given in the lecture.
[A] The quantities required are $D = \mu k_B T$, $v = -\mu q z(dV/dx)$ and from setting the current equation to zero

$$\frac{1}{N}\frac{\partial N}{\partial x} = \frac{-z}{V_T}\frac{\partial V}{\partial x} \quad \text{with solution} \quad N(x) = N^i e^{-z(V(x)-V^i)/V_T}. \tag{2}$$

[Q] If the voltage changes linearly along the channel, how does the density change with position down the channel?
[A] Exponentially.
In the derivation of the GHK equation a linear change of the voltage, between the inside and outside, was assumed.
[Q] Is the derivation of the Nernst equation for the equilibrium potentials conditional on this assumption?
[A] No it is not.
[Q] How does a non-linear $V(x)$ affect the derivation of the Goldman current and GHK equation for ions with the same z.
[A] It is not a problem as the denominator is the same.

[Q] What about for mixed ions with $z = \pm 1, \ \pm 2$ etc?
[A] Not possible because the denominators have different functional forms.

• Q3. The Nernst and GHK equations

Table 1 gives the molarities for Sodium, Potassium and Chlorine in neural tissue.
[Q] What are their equilibrium potentials at 37°C?
[A] The Nernst equation is

$$E = \frac{V_T}{z} \log \left(N^e / N^i \right) \tag{3}$$

so that $E_{Na} = 58$mV, $E_K = -80$mV and $E_{Cl} = -70$mV.
[Q] Using table 1, calculate the resting potential of the membrane using the GHK equation derived in the lectures.
[A] The GHK equation is

$$E_{rest} = V_T \log \left(\frac{P_K N_K^e + P_{Na} N_{Na}^e + P_{Cl} N_{Cl}^i}{P_K N_K^i + P_{Na} N_{Na}^i + P_{Cl} N_{Cl}^e} \right) \tag{4}$$

Inserting the results gives $E_{rest} = -70.25$mV.
The linearised form of the Goldman current for an ion s is

$$I_s = g_s(V - E_s) \quad \text{where} \quad g_s = \frac{P_s z_s^2 q}{V_T} N_s^e N_s^i \left(\frac{\log(N_s^e / N_s^i)}{N_s^e - N_s^i} \right). \tag{5}$$

[Q] What are the relative values of g_s for the ionic species? (Drop the q/V_T factor.)
[A] $g_{Na} : g_K : g_{Cl}$ is 2.45 : 62.9 : 227
Now consider the sum of the linear, ohmic approximations which give the leak current

$$g_L(V - E_L) = g_{Na}(V - E_{Na}) + g_K(V - E_K) + g_{Cl}(V - E_{Cl}) \tag{6}$$

[Q] What is the form of E_L as a function of the conductances and equilibrium potentials?
[A] It is

$$E_L = (E_{Na} g_{Na} + E_K g_K + E_{Cl} g_{Cl}) / (g_{Na} + g_K + g_{Cl}) \tag{7}$$

[Q] What does the ohmic assumption predict for the resting potential?
[A] The leak reversal E_L is equal to -71.2

ion	external $[N^e]$ (mM)	internal $[N^i]$ (mM)	relative permeability
Na^+	437	50	0.02
K^+	20	397	1
Cl^-	556	40	2

Table 1: Molarity in mM of ionic species for the squid giant axon. Molarity M is the number of moles (where one mole of a quantity is 6.02×10^{23} units) per litre of fluid. There are one thousand litres in a cubic metre.