Cable Theory: Questions

- Q1. Input resistance at a node

An experimentalist manages to patch an electrode directly on the point on the dendritic tree where three long dendrites meet, each with distinct properties. At that point she injects a constant current I_{app} into the cell. The electrotonic length constants in each dendrite are $\lambda_1, \lambda_2, \lambda_3$ and the resistances for an electrotonic length's worth of dendrite are $R_{\lambda 1}, R_{\lambda 2}, R_{\lambda 3}$. Use a coordinate system on each dendrite where the point where the dendrites meet is at $x = 0$ and where x increases away from the node.
[Q] What is the general solution to the steady-state cable equation in the dendrites?
[Q] Noting that the voltage cannot jump at any point, write down an equation relating the v_ks at the injection site in terms of the injection-site voltage v_0
[Q] Now consider the conservation of current. Let $I_k(0)$ be the current flowing down dendrite k at the point of the node $x = 0$. Write an equation relating these currents.
[Q] Use this result to solve the voltage distribution in each dendrite.
[Q] Show that the input resistance is consistent with the three dendrites acting as resistors in parallel.

- Q2. Voltage-clamp and synaptic current

A common method for measuring synaptic current is to use a technique called *voltage clamp*. In this method the voltage at the point of injection is fixed at some value v_0 by injecting an appropriate current I_{app}. If the voltage is to be constant it means that I_{app} must balance any internal currents at that point and hence I_{app} is equal in magnitude to the internal currents flowing into the injection point. This provides a method for measuring the internal currents in a neuron.

Consider a dendrite of length L closed at each end; $x = 0$ is the point of current injection where the voltage is fixed, and at $x = L$ there is a synapse. The synapse has a reversal potential E_s and an absolute conductance G_s. By application of a drug to the bathing solution containing the neuron, the synapse is kept permanently open; it is a steady-state situation. The voltage at $x = 0$ is clamped to the resting voltage E_L and we measure all voltages from this value $v = V - E_L$.
[Q] Write down the general steady-state solution to the cable equation. How many free constants are there?
[Q] Apply the boundary condition at $x = 0$ and write the solution in terms of a hyperbolic trigonometric function.
[Q] What is the voltage at L in terms of the unknown constant?
[Q] What is the v-dependent form of the incoming synaptic current into the dendrite at L?
[Q] Match this to the current in the dendrite at L and fix the last free constant.
[Q] Calculate the magnitude of the current that is being injected at $x = 0$ to keep the voltage there at $v = 0$.
[Q] Under what circumstances does the applied current give an accurate measure of the the current flowing into the synapse?

- Q3. Dynamics of Charge Spread on a Long Dendrite

If a charge Q is injected at position $x = 0$ at a time $t = 0$ on an infinitely long dendrite, the voltage distribution is of the form

$$V = E_L + \frac{Ae^{-t/\tau_L}}{\sqrt{4\pi t/\tau_L}} \exp\left(-\frac{x^2}{4\lambda^2 t/\tau_L}\right) \qquad (1)$$

[Q] Show that this form satisfies the cable equation for $t > 0$.
We now want to fix the prefactor A (what units does this have?). Charge is related to voltage by $Q = CV$.
[Q] Consider the voltage distribution close to the point of injection just after $t = 0$ and use it to fix the prefactor A in terms of Q, R_λ and τ_L. You will need to know how to calculate the integral of a Gaussian to solve this.