Subthreshold Voltage Gated Channels: Questions

• Q1. Emergence of oscillations

On the P, Q phase diagram it was seen that a system with complex eigenvalues and decaying voltage oscillations, can destabilise to give a spontaneously oscillating neuron if $P < -1$. However, it was noted that for the case of a neuron with only one kind of voltage-gated channel $P = \tau_n/\tau_v$, so that $P > 0$. This showed that a neuron with a single voltage-gated channel cannot go into oscillation. Nevertheless, oscillating voltages are seen in neurons *in vivo*. How can this be explained? Consider a neuron with two voltage-gated channels. The linearised equations are

$$\tau_v \frac{dv}{dt} = \kappa_1 y_1 + \kappa_2 y_2 - v, \quad \tau_{n1} \frac{dy_1}{dt} = v - y_1, \quad \tau_{n2} \frac{dy_2}{dt} = v - y_2 \tag{1}$$

[Q] Assume that the second variable equilibrates very quickly, i.e. set $\tau_{n2} = 0$. Reduce the three dynamic equations to two.
[Q] Rescale time $s = t/\tau_{n1}$. Show that $P = (1 - \kappa_2)\tau_{n1}/\tau_v$.
[Q] What conditions on κ_1 and κ_2 will destabilise the neuron and cause spontaneous oscillations? What kind of currents could this correspond to? What is the physical interpretation of the actions of the currents?

• Q2. H-current effects in the response to square-current pulses

The h-current, or I_h, is a common feature of neurons with large dendritic structure. It is a hyperpolarisation-activated, depolarising current, and as such can protect neurons from too-strong a hyperpolarisation. Its signature is seen in the response to square-current pulses as a non-monotonic overshoot or sag, depending on the sign of the current. The activation time constant of I_h is much longer than the voltage time constant. So $\tau_n/\tau_v \gg 1$ and $\kappa < 0$.
[Q] Using the linear forms of the voltage equation,

$$\tau_v \frac{dv}{dt} = -v + \kappa y + R I_{app} \tag{2}$$

$$\tau_n \frac{dy}{dt} = v - y, \tag{3}$$

show that, in the large τ_n/τ_v, if I_{app} is zero before $t = 0$ and a constant I_0 afterwards then the early-time voltage change is

$$v \simeq R I_0 \left(1 - e^{-t/\tau_v}\right). \tag{4}$$

[Q] For the late-time dynamics the voltage can be assumed to be at its equilibrium, $\tau_v dv/dt = 0$. Use this result to express y as a function of v and solve the equation for y and show that it follows

$$y \simeq \frac{R I_0}{1 - \kappa} \left(1 - e^{-\frac{t}{\tau_n}(1-\kappa)}\right) \tag{5}$$

[Q] Using the formula relating y and v from the $\tau_v dv/dt = 0$ approximation, show that the late-time voltage follows

$$v \simeq \frac{R I_0}{1 - \kappa} \left(1 - \kappa e^{-\frac{t}{\tau_n}(1-\kappa)}\right). \tag{6}$$

[Q] Sketch the form of the voltage response.

• Q3. EIGENVALUES FOR A NEURON WITH THE H-CURRENT

We now derive the eigenvalues in the same limit just considered with $\tau_n/\tau_v \gg 1$.
[Q] In the previous question, what were the two decay time constants identified in this approximation?
[Q] Using the forms for P and Q given in the lecture notes, show that in the large τ_n/τ_v limit both $P \gg 1$ and $Q \gg 1$.
[Q] Expand the eigenvalue equation to show that the two roots take the form

$$\lambda_+ = -\frac{1}{\tau_n}\left(1 - \frac{Q}{P}\right) \quad \text{and} \quad \lambda_= - \frac{1}{\tau_n}\left(P + \frac{Q}{P}\right). \tag{7}$$

[Q] Show that these two eigenvalues are consistent to leading order with the time constants derived above.