Integrate-and-Fire Models: Questions

• Q1. Quadratic Integrate-and-Fire (QIF) Model

The quadratic integrate-and-fire model obeys

$$\frac{dV}{dt} = q(V - V_0)^2 + \mu \tag{1}$$

which is identical to the canonical IF model, except that here the reset $V_{re} < V_0$ and threshold $V_{th} > V_0$ are finite. The model spontaneously oscillates when $\mu > 0$.
[Q] Show that the period T is equal to

$$T = \frac{1}{\sqrt{q\mu}} \left(\arctan\left((V_{th} - V_0)\sqrt{\frac{q}{\mu}} \right) - \arctan\left((V_{re} - V_0)\sqrt{\frac{q}{\mu}} \right) \right). \tag{2}$$

[A] Divide equation (1) by its RHS and integrate between V_{re} and V_{th} which are the voltages at the beginning and end of a period. It helps to rescale time and voltage.
[Q] Show that just above the critical point, when μ is small and positive, the firing rate of the canonical Type I model is recovered which is $r = \sqrt{q\mu}/\pi$.
[A] The argument of the first arctan approaches $+\infty$ and the second $-\infty$ so that the sum of the terms in the parenthesis becomes π - this yields the required firing rate.

• Q2. Behaviour of the Exponential IF Model near the Critical Point

The exponential integrate-and-fire model obeys the equation

$$\frac{dV}{dt} = \frac{F(V)}{\tau_L} = \frac{E_L - V + \Delta_T e^{(V - V_T)/\Delta_T} + I/g_L}{\tau_L}. \tag{3}$$

With a threshold at $V_{th} = \infty$ and $V_{re} < V_T$ finite.
[Q] Show that the minimum of F(V) is at a voltage V_T.
[A] Straightforward differentiation.
[Q] Expand $F(V)$ around V_T to second order in voltage to show that near V_T

$$\frac{dV}{dt} \simeq \frac{1}{2\Delta_T \tau_L}(V - V_T)^2 + \frac{E_L + \Delta_T - V_T + I/g_L}{\tau_L} \tag{4}$$

[Q] Identify the critical current I^*.
[A] $I^* = g_L(V_T - \Delta_T - E_L)$.
[Q] By comparison with the form for the QIF, show that for an applied current just above the critical value the firing rate of the EIF model is

$$r = \frac{1}{\pi \tau_L} \sqrt{\frac{I - I^*}{2\Delta_T g_L}}. \tag{5}$$

[A] This follows directly from $q = 1/2\Delta_T \tau_L$ and $\mu = (I - I^*)/g_L \tau_L$.